Noise Considerations in Low Resistance NIS Tunnel Junctions

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Abstract

We discuss noise sources in normal-insulator-superconductor (NIS) tunnel junctions relevant for their X-ray and particle detection applications. The electric current shot noise and correlated tunneling power shot noise are calculated. The thermodynamic noise due to the electron-phonon coupling is derived for the general case $T_e \neq T_p$.

Introduction

Normal-insulator-superconductor (NIS) tunnel junction devices are presently being developed for X-ray and particle detection applications ([1], [2], [3], [4], [5]). An important consideration is the devices' sensitivity limit as imposed by intrinsic noise sources. We present an analysis of these noise sources under only three approximations: all distribution functions are thermal, backtunneling is negligible, and the substrate and S electrode are well heat-sunk. Comparison of shot noise and sensitivity push one to low R_n, so we take a typical junction to have $R_n = 0.5\Omega$ with N electrode volume $10^4\mu\text{m}^3$, voltage biased with SQUID readout.

NIS Junction Fundamentals

We quickly review the theory of NIS junctions. We define the sign of the bias voltage by $U = U_N - U_S$, where N and S are the two electrodes. Quasielectrons ($E > 0$) and quasiholes ($E < 0$) can tunnel in both directions. The quasiholes are transformed to negative energy for convenience. If we define (for $|E| > \Delta$ only)

$$g_1(E) = \frac{\mathcal{N}(E)}{|e|^2 R_n} f_n(E - eU)\,[1 - f_s(E)]$$

$$g_2(E) = \frac{\mathcal{N}(E)}{|e|^2 R_n} f_s(E)\,[1 - f_n(E - eU)]$$

then tbe number currents are $\Gamma_{ns}(E > 0) = g_1(E)$, $\Gamma_{sn}(E > 0) = g_2(E)$, $\Gamma_{ns}(E < 0) = g_2(E)$, and $\Gamma_{sn}(E < 0) = g_1(E)$. $\frac{1}{|e|^2 R_n} = \frac{2\pi}{\hbar}|M|^2 \mathcal{D}_n(0)\mathcal{D}_s(0)$ contains the tunneling matrix element and density of states factors. $e < 0$ is the charge of the electron. $\mathcal{N}(E) = \frac{|E|}{\sqrt{E^2-\Delta^2}}$ is the BCS modification to the density of states and f_n and f_s are the N and S electron distribution functions (referred to their respective Fermi levels). The electric current is

$$I = -|e| \int_{|E|>\Delta} dE\, sgn(E)\,[\Gamma_{ns}(E) - \Gamma_{sn}(E)]$$

Numerical integration of the above expression is preferable to analytic approximations (those in the literature can be off by 50%). The sensitivity parameter, $S = \frac{dI}{dT}$ at fixed U, can be calculated by finite difference of the numerical integral.

There is a power associated with the tunneling current when the lead to the N electrode is a superconductor with $\Delta \gg k_b T$ because tunneling particles must be replaced by Cooper pairs at the Fermi level from the lead to maintain charge neutrality. Therefore, particles tunneling out of the N electrode remove energy $\varepsilon = sgn(E)(E - eU)$, while those tunneling in bring this energy; hence the power entering the N electrode is

$$P_{tun} = -\int_{|E|>\Delta} dE\, sgn(E)(E - eU)\,[\Gamma_{ns}(E) - \Gamma_{sn}(E)]$$

Again, approximations to the integral are inaccurate.

Because of this power flow, the N electrode and substrate temperatures may differ. For this reason, others are investigating the use of the NIS tunnel junction as a microrefrigerator ([6], [7]). To determine the exact steady state, we should solve coupled Boltzmann equations for the N electrons, the S electrons, and the phonons, fully accounting for energy transfer between the three systems via tunneling hot electrons and athermal phonons. Instead, we make three approximations to render the problem tractable:

- The distribution functions of all three systems are thermal on all timescales (infinitely fast thermalization within each system)
- There is no quasiparticle backtunneling or athermal phonon transmission to the N electrode, which can return tunneling power to the N electrode rather than deposit it in the heat sink.
- The thermal impedance from the S electrons and phonons to the heat sink is negligible, and hence $T_S = T_p = T_{sink}$.

Relaxation of these approximations is discussed by Jochum, *et al.*, [5]. We use these approximations as a "best case" scenario for the noise while maintaining the salient aspects of electrothermal feedback. With this, the N electron temperature is set by equating the electron-phonon power ($P_{ep} = \Sigma V(T_e^5 - T_p^5)$, $\Sigma \sim 1-5$ nW $\text{K}^{-5}\mu\text{m}^{-3}$ typically) to the the tunneling power and solving for T_e numerically. For low resistance junctions, T_e is significantly decreased below the substrate (phonon) temperature. This can yield gains in sensitivity due to reduction in heat capacity of the N electrons (C_e) and increased S.

For our noise analysis, we require the junction current's response to energy deposition in the N electrons. This is found by solving the dynamic energy conservation equation in the standard way; the result is

$$\delta I(\omega) = \frac{S}{G} P_{dep}(\omega) \left[1 + i\omega\tau\right]^{-1}$$

where $P_{dep}(\omega)$ is the Fourier transform of the power deposition ($P_{dep}(\omega) = E$ for a X-ray-like $P_{dep} = E\delta(t)$ deposition), $G_{ep} = -\frac{dP_{ep}}{dT_e}$, $G_{tun} = -\frac{dP_{tun}}{dT_e}$, $G = G_{ep} + G_{tun}$ is the total conductance, and $\tau = \frac{C_e}{G}$ is the decay time. Note that the tunneling cooling path (G_{tun}) reduces τ.

Noise Sources

The intrinsic noise sources are:

- Shot noise due to the tunneling current.
- "Thermodynamic" noise due to shot noise on the tunneling power; this is correlated with the tunneling current shot noise.
- "Thermodynamic" noise due to fluctuations of the electron-phonon power flow.

The tunneling power fluctuation is $\delta P(t) = [-\sum_k \varepsilon_k \, \delta(t - t_k)] - \langle P \rangle$ where ε_k is the energy removed by the kth tunneling particle. Note the sign convention. We apply the response function, add the current shot noise fluctuations, and Fourier transform to find

$$\delta I_{tun}(\omega) = \sum_k \left[q_k - \frac{S}{G}\varepsilon_k \left[1 + i\omega\tau\right]^{-1}\right] e^{-i\omega t_k} \\ -\frac{S}{G} 2\pi \langle P \rangle \, \delta(\omega) \left[1 + i\omega\tau\right]^{-1} - 2\pi \langle I \rangle \, \delta(\omega)$$

The correlation of the power and current fluctuations is incorporated by the matched phase factor. For different tunneling events, the phases are uncorrelated, so cross terms between different events vanish in the averaging done to derive the noise power spectral density (PSD):

$$J_{tun}(\omega \neq 0) = \frac{\Gamma}{1+\omega^2\tau^2} \left[\left(\frac{S}{G}\right)^2 \langle \varepsilon^2 \rangle - 2 \, \frac{S}{G} \langle q\varepsilon \rangle \right] + \Gamma \langle q^2 \rangle$$

where $\Gamma = \Gamma_{ns} + \Gamma_{sn}$ is the total number current. Explicitly, the three terms are

$$\Gamma \langle q^2 \rangle = |e|^2 \int_{|E|>\Delta} dE \left[\Gamma_{ns}(E) + \Gamma_{sn}(E)\right]$$

$$\Gamma \langle \varepsilon^2 \rangle = \int_{|E|>\Delta} dE \, (E - eU)^2 \left[\Gamma_{ns}(E) + \Gamma_{sn}(E)\right]$$

$$\Gamma \langle q\varepsilon \rangle = -|e| \int_{|E|>\Delta} dE \, (E - eU) \left[\Gamma_{ns}(E) + \Gamma_{sn}(E)\right]$$

The noise PSDs of different directions and energies add as they do because they are uncorrelated. In typical conditions, one type of carrier dominates the current; in such cases, $\Gamma \langle q\varepsilon \rangle$ is evidence of anticorrelation between the current shot noise and the current induced by the power shot noise (for $\omega < \tau^{-1}$). This is a form of the electrothermal feedback discussed by Mather [8] and more recently by Irwin [9]. While such effects modify the noise PSD, the overall resolution is unchanged because the noise amplitude decrease coincides with a noise bandwidth increase.

The δ-function phonon emission and absorption that yield the electron-phonon power also yield a power shot noise. From the standard form for this power [10], we find the power noise PSD is

$$J_{ep}^{P}(\omega \neq 0) = \int_0^\infty dE_{\vec{k}} \; f(E_{\vec{k}}) \; \mathcal{D}(E_{\vec{k}}) \int \frac{V d^3 q}{(2\pi)^3} \; \varepsilon_{\vec{q}}^2 \; \frac{2\pi}{\hbar} |M|^2 \\ \times \left\{ \left[1 - f(E_{\vec{k}-\vec{q}})\right] \delta(E_{\vec{k}} - E_{\vec{k}-\vec{q}} - \varepsilon_{\vec{q}}) \left[n(\vec{q}) + 1\right] \right. \\ \left. + \left[1 - f(E_{\vec{k}+\vec{q}})\right] \delta(E_{\vec{k}} - E_{\vec{k}+\vec{q}} + \varepsilon_{\vec{q}}) \left[n(\vec{q})\right] \right\}$$

$\varepsilon_{\vec{q}}$ is squared as usual for a shot noise. The two terms correspond to phonon emission and absorption, added here as uncorrelated noises. Integration of this expression and application of the response function yields

$$J_{ep}(\omega \neq 0) = \frac{(S/G)^2}{1+\omega^2\tau^2} \left[k_b T_e^2 G_e \frac{\zeta(6)}{\zeta(5)} + k_b T_p^2 G_p \; \xi\left(\frac{T_p}{T_e}\right)\right]$$

where $G_e = 5\Sigma V T_e^4$ ($= G_{ep}$ defined above) and $G_p = 5\Sigma V T_p^4$ are the standard conductances and

$$\xi\left(\frac{T_p}{T_e}\right) = \frac{1}{5\Gamma(5)\zeta(5)} \int_0^\infty \frac{y^5 dy}{e^y - 1} \left[e^{y\frac{T_p}{T_e}} + 1\right] \left[e^{y\frac{T_p}{T_e}} - 1\right]^{-1}$$

In general, there is no closed form for ξ; it must be calculated numerically. However, for $T_p = T_e$, the integral simplifies and one finds $\xi = \frac{2\zeta(5)-\zeta(6)}{\zeta(5)}$, yielding the standard equilibrium result $2k_b T^2 G$. For $T_p \gg T_e$, one finds $\xi = \frac{\zeta(6)}{\zeta(5)}$, yielding $k_b T_p^2 G_p \frac{\zeta(6)}{\zeta(5)}$ and thus making the noise symmetric in the two extreme cases.

The contributions J_{tun} and J_{ep} are uncorrelated and thus the total noise power spectral density is their sum. For further information, including plots of the currents and noise PSDs and optimal energy resolution calculations, please go to

http://cfpa.berkeley.edu/golwala

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