Pion decays and $R_{e/\mu}$ ph135c talk

We want to consider decays of π^+ $\left\{ \begin{array}{l} m_{\pi^+} \simeq 140 \text{ MeV} \\ \tau = \dfrac{1}{\Gamma(\pi^+ \to \text{anything})} \simeq 3 \times 10^{-8} s \end{array} \right.$

two main decay channels

$\pi^+ \to e^+ \nu_e$ ($m_e \simeq .511$ MeV)

$\pi^+ \to \mu^+ \nu_\mu$ ($m_\mu \simeq 107$ MeV)

(assume $m_{\nu_e} = m_{\nu_\mu} = 0$)

define $\quad R_{e/\mu} \equiv \dfrac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)}$

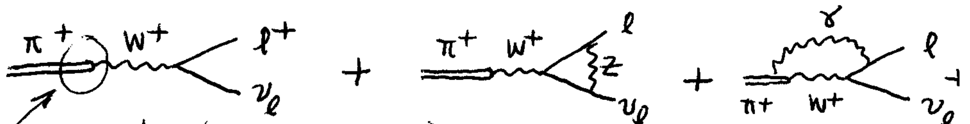
plan:
(i) calculate $R_{e/\mu}$ in SM as accurately as possible
(ii) measure $R_{e/\mu}$ " ______
(iii) look for deviations between $R_{e/\mu}^{SM}$ and $R_{e/\mu}^{expt}$
calculate deviations due to your favorite model of "Beyond-the-SM" new physics (e.g. SUSY)

i. $R_{e/\mu}$ in the SM

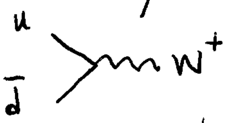
First of all, why consider a ratio $R_{e/\mu}$?

Suppose we just wanted the ~~branching ratio~~ decay rate for $\pi^+ \to \ell^+ \nu_\ell$ $(\ell = e, \mu)$

$\Gamma(\pi^+ \to \ell^+ \nu_\ell) =$



tree-level   radiative corrections



theoretical difficulties from QCD: pion decay const. $F_\pi \simeq 93$ MeV (3% from lattice)

also includes pion-structure ~~dependent~~ effects, mostly independent of ℓ

These QCD effects bring large theoretical uncertainties to the calculation of $\Gamma(\pi^+ \to \ell^+ \nu_\ell)$ since theorist can't accurately calculate QCD effects at low energies

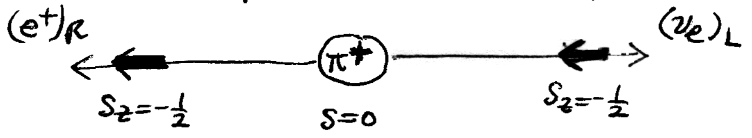
<u>However</u> these effects are mostly independent of ℓ (lepton flavor) and cancel out of the ratio $R_{e/\mu}$ even though $\Gamma(\pi^+ \to e^+ \nu_e)$ and $\Gamma(\pi^+ \to \mu^+ \nu_\mu)$ can't be calculated accurately, $R_{e/\mu}$ can!

$$R_{e/\mu}^{SM} = \frac{m_e^2}{m_\mu^2} \underbrace{\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2}_{\text{phase space}} \left(1 + \begin{matrix}\text{small}\\ \text{radiative}\\ \text{corrections}\end{matrix}\right) = (1.2352 \pm 0.0005) \times 10^{-4}$$

uncertainty comes from lepton-flavor <u>dependent</u> pion structure effects (which are unknown & estimated)

helicity suppression

$W^\pm$ only couple to left-handed particles (& RH antiparticles)



but this violates angular momentum conservation: need $\pi^+ \to (\nu_e)_L \, (e^+)_L$

mass term mixes $(e^+)_L$ and $(e^+)_R$, so decay can proceed for $m_e \neq 0$, but at the cost of picking up a factor of m_e^2

ii. <u>measurement of $R_{e/\mu}$</u>

$R_{e/\mu}$ was measured in early 1990's by two labs:

PSI (Paul Scherrer Institute), Switzerland

TRIUMF (Tri-university meson Facility), British Columbia

places are huge national labs with collaborators from all over the world. measurement of $R_{e/\mu}$ was just one experiment out of many.

measurements had comparable precision and agree within errors

Average: $R_{e/\mu}^{expt} = (1.230 \pm 0.004) \times 10^{-4}$

~~Although~~

Although $R_{e/\mu}^{expt}$ and $R_{e/\mu}^{sm}$ agree, clearly there is much room for better experiments to make a more precise test of SM

~~Future~~

Upcoming experiments at PSI & TRIUMF will remeasure $R_{e/\mu}$ with precision increased by a factor of 10 — comparable to theoretical uncertainty in $R_{e/\mu}^{sm}$

iii. Beyond the SM

New heavy particles may contribute to $R_{e/\mu}$ through virtual effects

(just as π^+ decays through virtual W^+)

even though available energy in decay is $m_\pi \simeq 140$ MeV $\ll M_{New}$

Example: SUSY

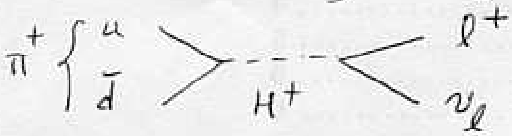
(1) charged Higgs H^+

recall: SM — 1 neutral Higgs $H \rightarrow h^0$

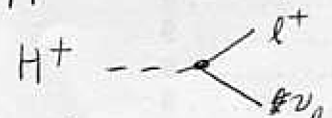
MSSM — 3 neutral Higgs & 1 charged Higgs

$H_u, H_d \longrightarrow h^0, H^0, A^0, H^\pm$

pion can decay through H^+ exchange at tree-level

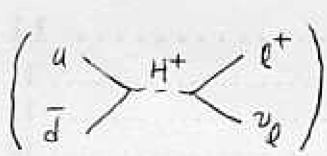


because H^+ is a scalar $(s=0)$, there is no Helicity suppression (good)

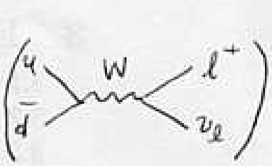
but  interaction has ~~strong~~ strength $y_\ell \sim \frac{m_\ell}{100\,\text{GeV}}$

(bad)

can show:



$$= \frac{m_\pi^2 \tan\beta \, (m_u \cot\beta - m_d \tan\beta)}{(m_u + m_d)\, M_{H^+}^2}$$



~~need large $\tan\beta$~~

note: independent of m_ℓ

$\Rightarrow$ cancels from $R_{e/\mu}$!

in general, $R_{e/\mu}$ can give strong constraints on new scalars due to absence of helicity suppression

e.g. leptoquarks from GUT's

(2) new R-parity violating interactions

recall: R-parity is a postulated symmetry of MSSM

quarks, leptons, gauge & Higgs bosons have $P_R = +$

superpartners have $P_R = -1$

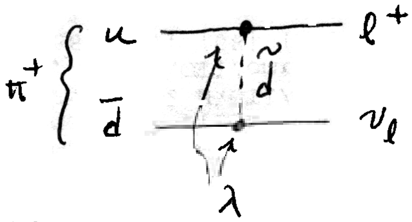
purpose: keeps proton from decaying

e.g. $\tau(p \to e^+ \pi^0) > 10^{33}$ yrs

allows 1 stable superpartner (DM candidate)

it is possible to get rid of R-parity without allowing p-decay (but now no stable DM)

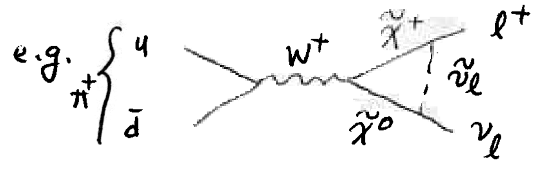
with RPV interaction, π^+ can decay ~~by~~ through intermediate virtual down squark $\tilde{d}$



measurement of $R_{e/\mu}$ provides limits on magnitude of RPV interaction strength $|\lambda|^2$.

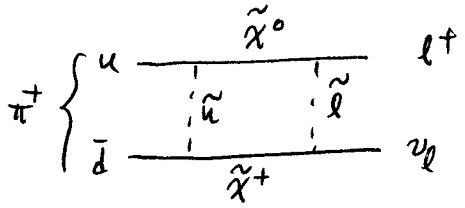
n/12 0$ as $\OML/cmm/m/it/12 ^^M \OMS/cmsy/m/n/12 ; 1$\OT1/cmr/m/n/12 , and $[]
\OML/cmm/m/it/12 ,[]\OT1/cmr/m/n/12 (\OML/cmm/m/it/12 ^^M\OT1/cmr/m/n/12) \OM
S/cmsy/m/n/12 ;
[103] [104] [105] [106] [107] [108] [109] [110] [111] [112] [113] [114]
Overfull \hbox (6.8363pt too wide) detected at line 5798

(3) 1-loop contributions to $R_{e/\mu}$ in MSSM (w/ R-parity)



$\tilde{\chi}^+$ chargino
$\tilde{\chi}^0$ neutralino
$\tilde{\nu}_\ell$ sneutrino

vertex correction (penguin)



box graphs

expectations:

1. all MSSM contributions vanish if Wino is very heavy
   $\Rightarrow$ want at least one chargino & neutralino ~~must be~~ light
2. MSSM contribution to $\pi^+ \to e^+ \nu_e$ same as to $\pi^+ \to \mu^+ \nu_\mu$
   if $m_{\tilde{e}} = m_{\tilde{\mu}}$ i.e. MSSM corrections cancel from $R_{e/\mu}$

   $\Rightarrow$ want $\dfrac{m_{\tilde{e}}}{m_{\tilde{\mu}}} \ll 1$ or $\gg 1$

LaTeX Warning: Label `]' multiply defined.

Leptonic Pion Decay

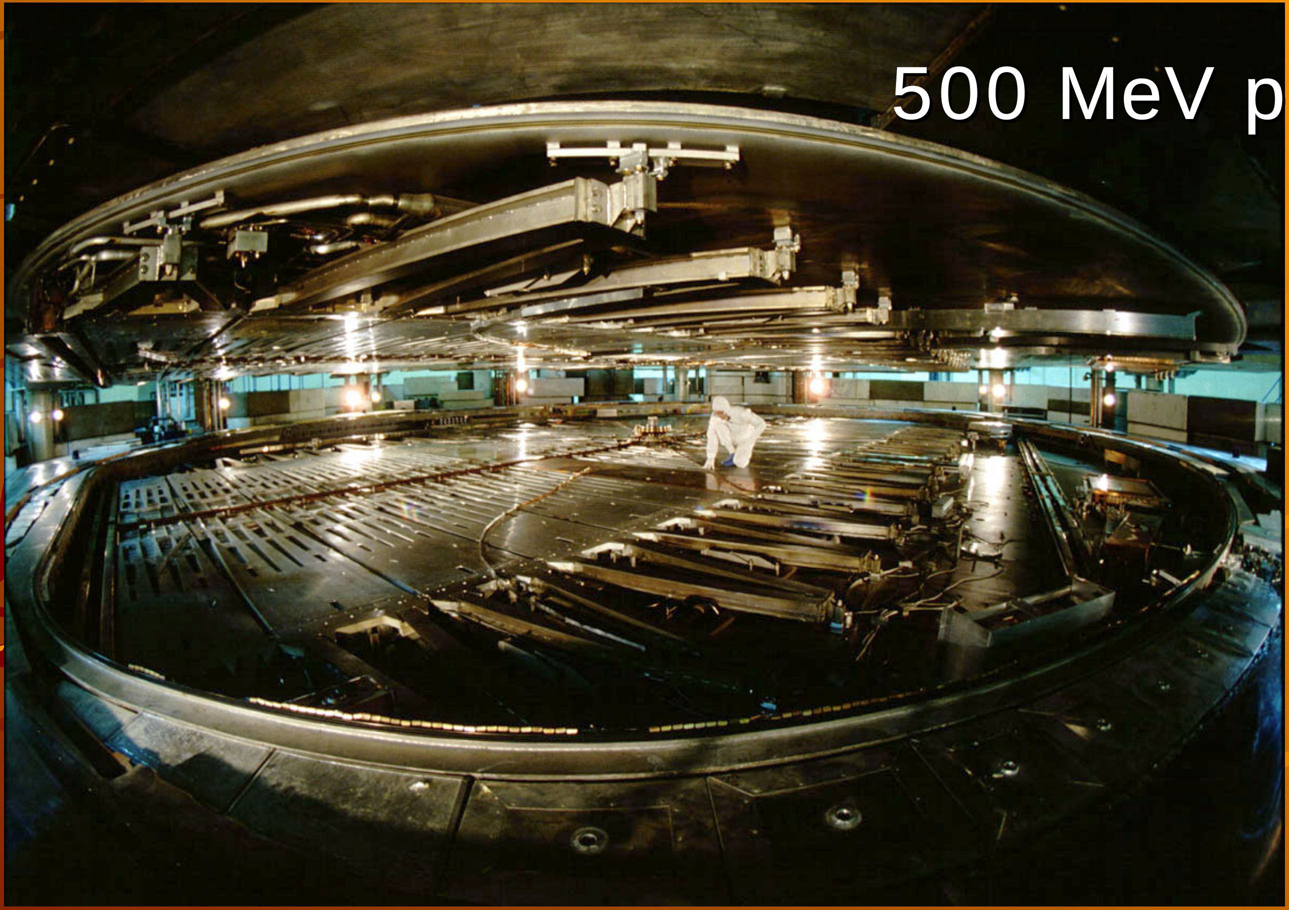
Experimental Aspects

TRIUMF

Pion Decay Experiments

Cyclotron

The TRIUMF Cyclotron



500 MeV protons

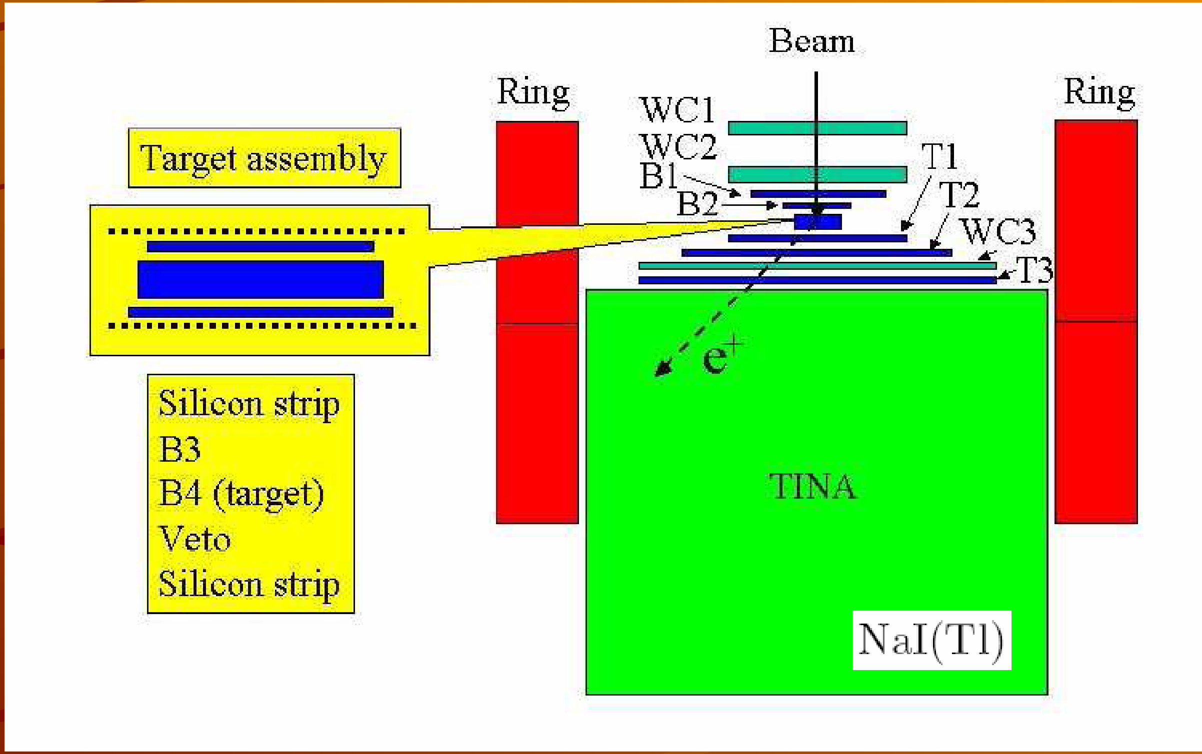
PIENU Experiment

- Early 90s:

  measured $R^{exp}_{e/\mu} = 1.2265 \pm 0.0034(stat) \pm 0.0044(syst) \times 10^{-4}$

- Current Experiment: Similar to original experiment but
  - More statistics
  - Better MC to understand energy dependence of multiple e^- scattering
  - Narrower π momentum distribution
  - Better dE/dx measurements in target

Current Experiment



π beam:

$\sim 5 - 10 \times 10^4$ pions/s.

75 MeV/c π^+ beam

Counting $\pi \rightarrow e\nu$ vs $\pi \rightarrow \mu\nu \rightarrow e\nu\nu$

- All π^+'s decay to final state e^+
- Only the π^+ and e^+ is detected (not the μ^+)
  - So how do we distinguish between the two decay channels?

- Solutions:
  - Energy Spectrum of e^+

$$\pi^+ \rightarrow e^+\nu \ (T_{e^+} = 69.3 \ \mathrm{MeV}) \qquad \mu^+ \rightarrow e^+\nu\bar{\nu} \ (T_{e^+} = 0 - 52.3 \ \mathrm{MeV})$$

  - Time of e^+ shower in detector

μ lifetime is long: ~2 μs

Counting $\pi \rightarrow e\nu$ vs $\pi \rightarrow \mu\nu \rightarrow e\nu\nu$

Plots from past TRIUMF experiment

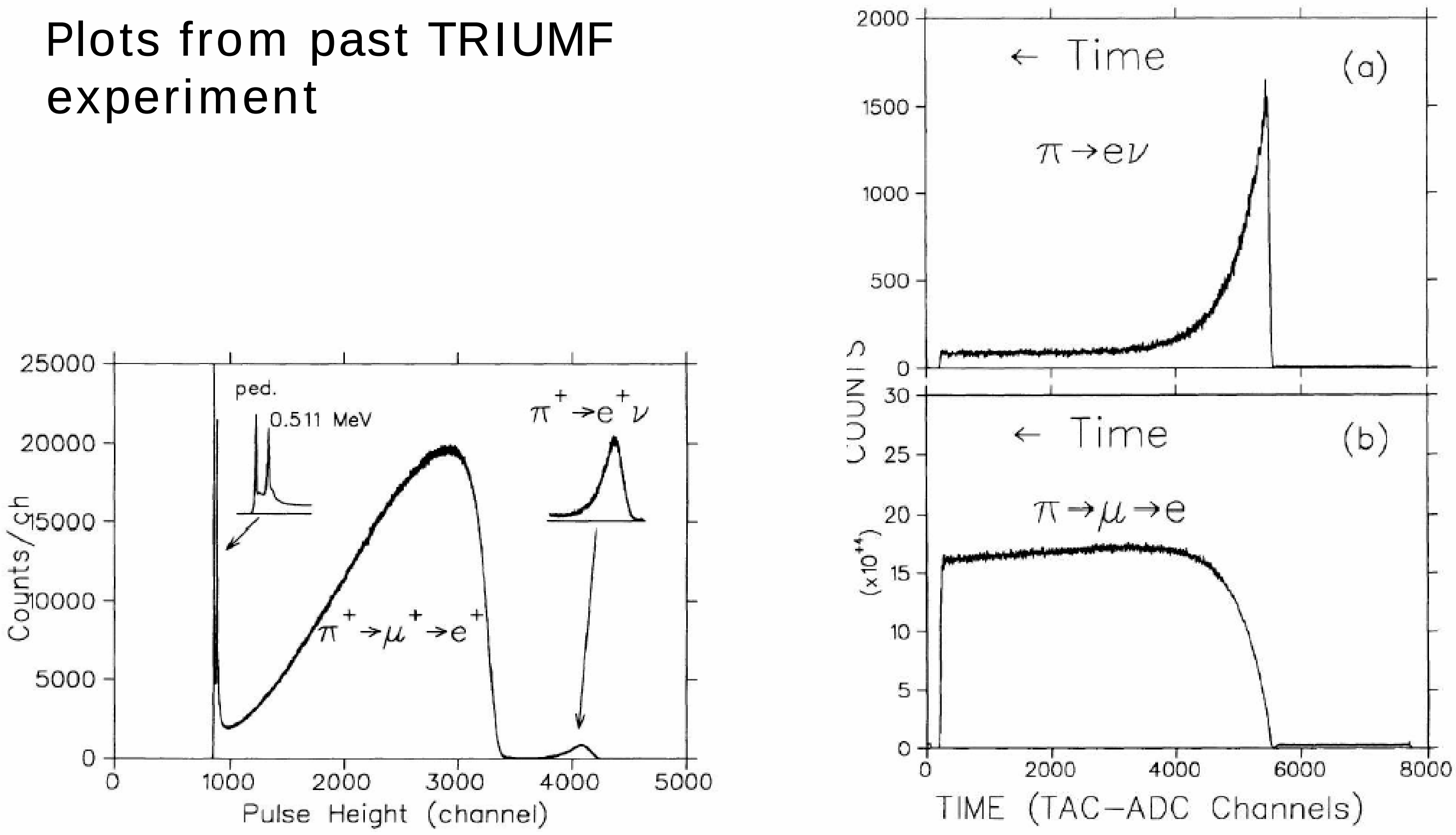
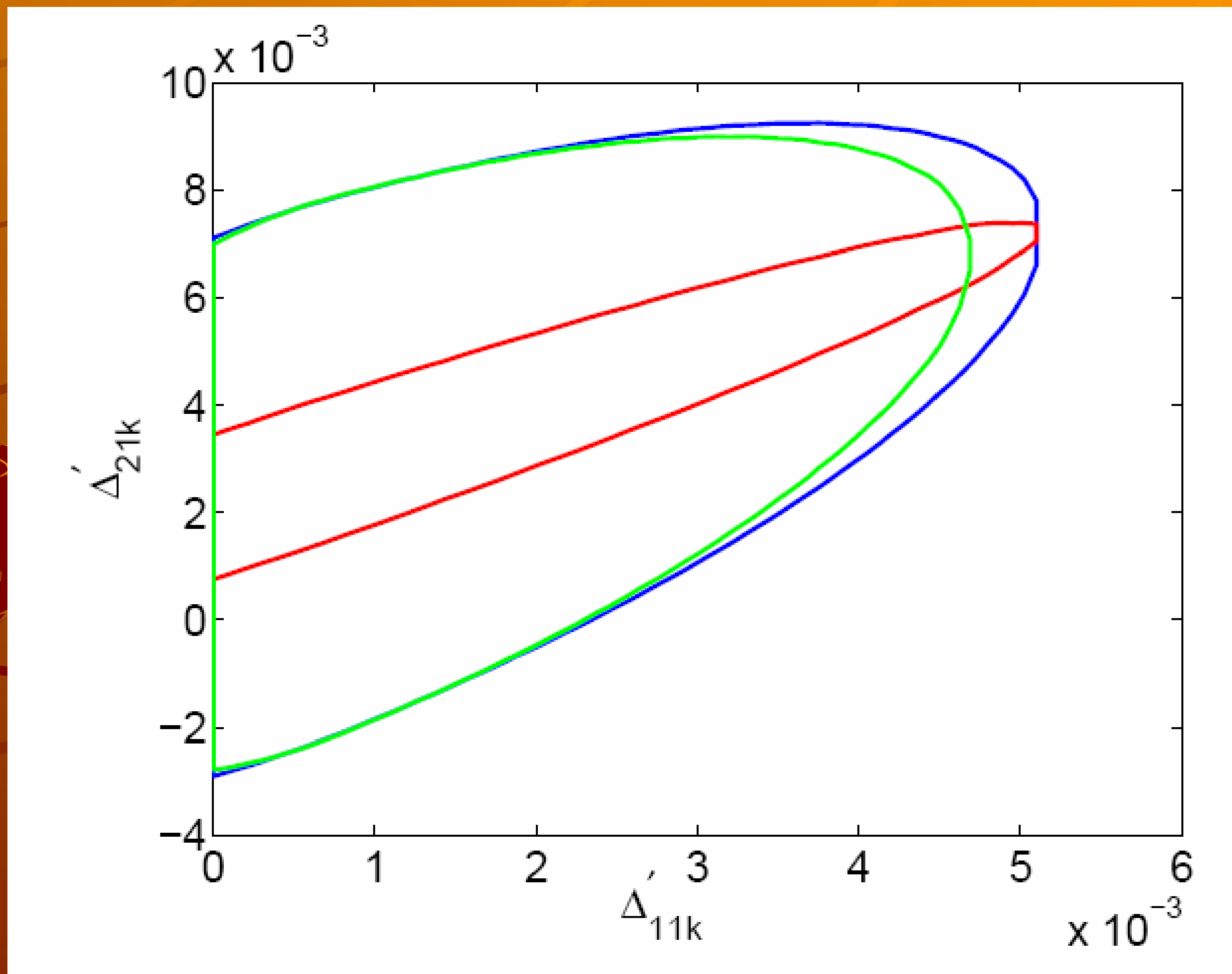


Fig. 2 Left: Energy spectrum of e^+'s in the early time window. Right: Time spectra for events in the (a) $\pi^+ \rightarrow e^+\nu$ and (b) $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ energy regions.

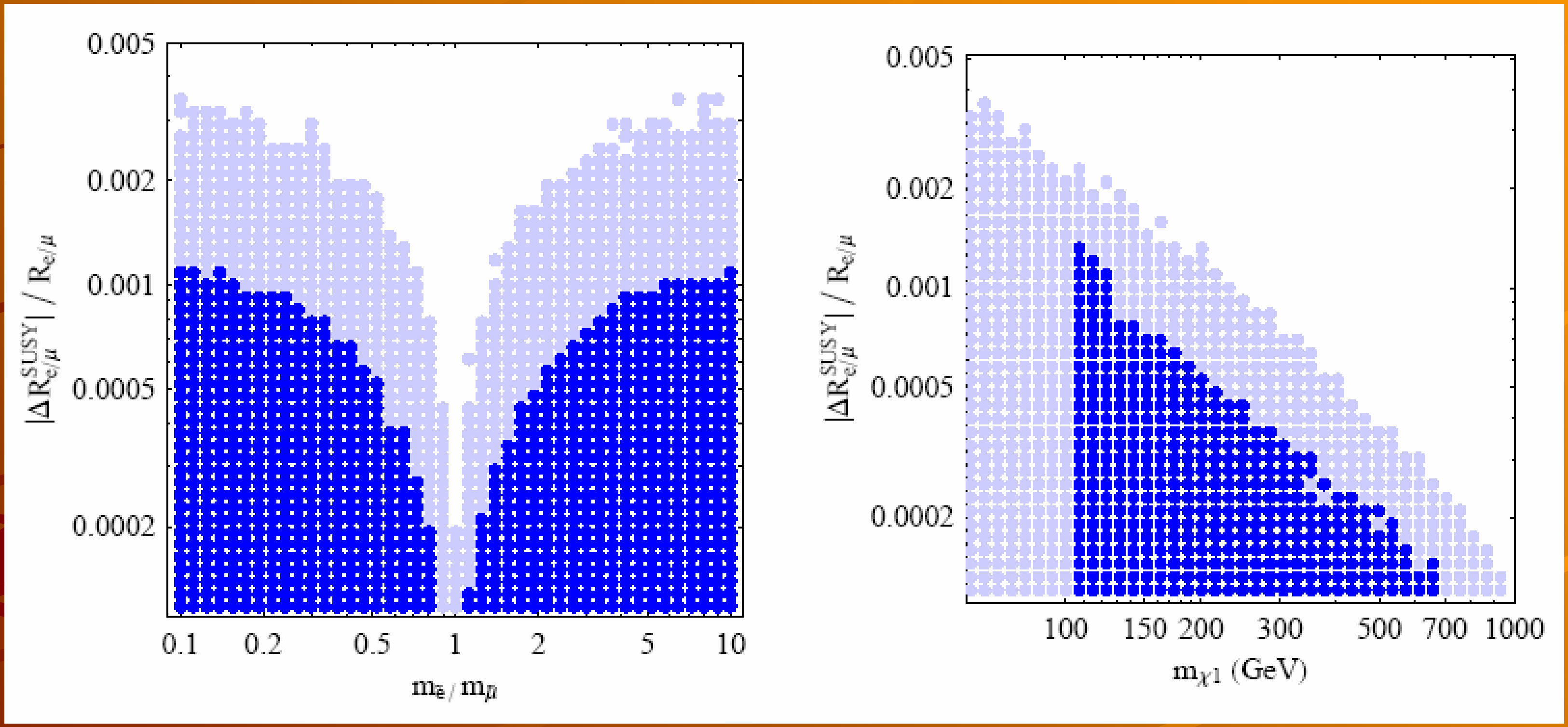
Beyond the SM Prospects

Possible signatures
of SUSY in $R_{e/\mu}$

R-Parity Violation



1-Loop MSSM Contributions



Conclusions

- $R_{e/\mu}$ is an observable which can be calculated very precisely due to cancellation of unknown QCD effects
- A lot of room for current experiments (PSI and TRIUMF) to test SM prediction
- SUSY can cause deviations in $R_{e/\mu}$
  - RPV may be observable
  - 1-Loop contributions probably not observable at these expts — if detected at future experiments, will provide strong probe of sleptons and chargino sectors

Ph135c Talk: Pion Decay and $R_{e/\mu}$

Sean Tulin

April 11, 2007

In this note, I address the questions that were asked during the talk.

What is the quark content of π^+?

The valence quarks of π^+ are $(u\bar{d})$. Of course, there are also sea quark-antiquark pairs and gluons.

What is f_π?

The quantity that enters into the pion decay amplitude at the $\pi^+ - W^+$ vertex is

$$\langle 0 |\, j_5^{\mu +}(x) | \pi^+ \rangle \ . \tag{1}$$

This mysterious object $j_5^{\mu+}(x)$ is called the axial current. It is basically the weak analog of the electromagnetic current. The expression above represents the amplitude for the π^+ to go to the vacuum $|0\rangle$ via this axial current (which is also coupled to the leptons). Because of the difficulties of QCD, it is not possible to calculate this amplitude exactly. However, though some tricks, it is possible to determine it up to an overall constant. This constant is f_π.

Why is the $\pi \to W$ vertex not just a matter of the electroweak vertex $u\bar{d} \to W$?

You need to take into account the wavefunction of the π^+.

Do the radiative corrections to the tree-level pion decay diagram increase or reduce the matrix element?

It could be either, depending on the sign of a particular diagram.

What are the details of how the pion structure effects cancel out?

Suppose t_ℓ and r_ℓ are the tree-level and 1-loop, respectively, amplitudes to the process $\pi^+ \to \ell^+ \nu_\ell$. Then $R_{e/\mu}$ is proportional to

$$R_{e/\mu} \propto \frac{|t_e + r_e|^2}{|t_\mu + r_\mu|^2} \simeq \left| \frac{t_e}{t_\mu} \right|^2 \left[1 + 2Re(r_e/t_e) - 2Re(r_\mu/t_\mu) \right] , \tag{2}$$

where we have Taylor expanded to leading order in r_ℓ, assuming $r_\ell \ll t_\ell$. We see that for $r_e/t_e = r_\mu/t_\mu$, the radiative corrections cancel out. This is what happens with charged Higgs exchange in the MSSM, as I discussed at the end of the talk.

How do uncertainties on e and mu mass affect the overall uncertainty on R(e/mu)?

The fractional uncertainties on the measurements of the e, μ, and π^+ masses are:

$$\delta(m_e) = 8 \times 10^{-8} \quad (3)$$
$$\delta(m_\mu) = 9 \times 10^{-8} \quad (4)$$
$$\delta(m_{\pi^+}) = 3 \times 10^{-6}\ . \quad (5)$$

The current theoretical and target experimental uncertainty is about 4×10^{-4}. The uncertainties in the particles masses are irrelevant for $R_{e/\mu}$ in the forseeable future.

What are the veto counters for – i.e., what do they veto?

Veto counters are placed after the target. They reject particles that penetrate through the target, rather than being stopped.

How does the scintillator measure the total energy?

The incident positron causes an electromagnetic shower of gamma rays and electron-positron pairs (whose annihilations result in more gamma rays) inside the scintillator crystal. The scintillator absorbes the energy of the electromagnetic shower and re-emits this energy as lower energy (visible) photons. A photomultiplier tube counts the photons; the number of photons is (to first approximation) proportional to the initial energy of the positron.

How does one get the e+ direction?

The e^+ direction is obtained by a wire chamber located between the π^+-stopping target and the scintillator crystal. This helps with knowing if the positron was aimed close to the edge of the scintillator so that some of the electromagnetic energy might have escaped.

Why is there a 511 keV peak in the mu event spectrum?

This peak is due to low energy positrons which activate (by annihilating with a surface electron) but do not penetrate the scintillator crystal.

What is the background in the $\pi \to e\nu$ time spectrum?

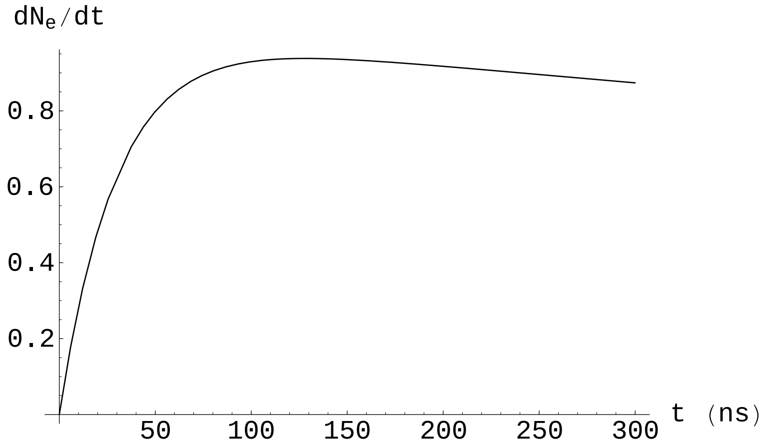
This background is due to pile-up of muons in the target. Suppose there is a μ still in the target when another $\pi \to \mu \to e$ decay comes along, and suppose both the first μ and the second μ happen to decay at about the same time. The result is that the scintillator will detect an event of higher energy, which can get misinterpretted as a $\pi \to e\nu$ event. To avoid this, the experiments implement a cut to avoid pile-up: throw out $\pi \to e\nu$ events which came within 6 μs of a previous π^+ stopped in the detector.

Why does the $\pi \to \mu \to e$ time spectrum have that turn-on shape?

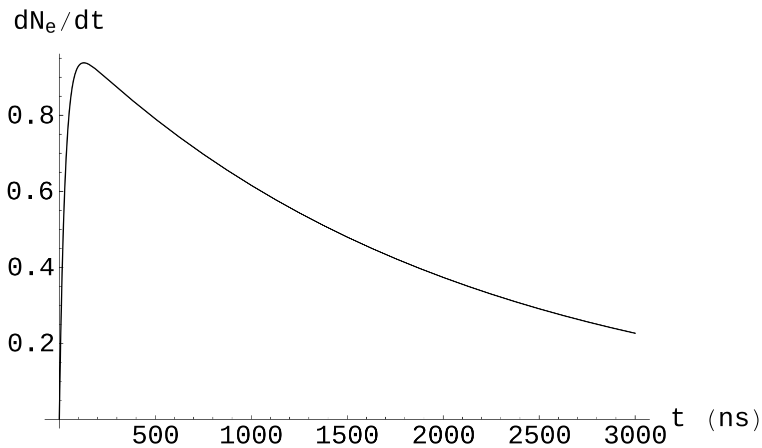
The turn on shape occurs because this decay is a two step process. Solving the equations describing the two-step decay process, we find that

$$\frac{N_e}{dt} = N_\pi(0)\,\frac{\Gamma_\mu \Gamma_\pi}{\Gamma_\pi - \Gamma_\mu}\,\left(e^{-\Gamma_\mu t} - e^{-\Gamma_\pi t}\right)\ , \quad (6)$$

where N_e and N_π are the number of positrons and pions, respectively. The detector looks for the positron up to $t = 300$ ns. Plotting this, we get



which looks the same as the time-plot I showed during the talk. If we plot to a later time (e.g. t=3000 ns), we get



where we can now see the turn-on behavior and the late-time exponential decay.

How does one in detail get a handle on the $\pi- > e\nu\gamma$ background?

Actually, the $\pi \to e\nu\gamma$ background is not the dominant source of the long tail that extends below the energy cut. (The γ's are forward-peaked — i.e. mostly going in the same direction as the e^+, and into the scintillator crystal.) The tail is due the response function of the scintillator — which, as I understand it, is the small likelihood that the scintillator will scintillate less than it should, leading to a measured energy lower than the true energy of the incident positron.

The $\pi \to e\nu$ tail was determined in two ways. (1) Using the timing information, as I discussed. (2) By measuring the total momentum deposited in the target. (Here is where it is important to get a narrow momentum distribution for the π^+ beam.) In a $\pi \to e\nu$ decay, the momentum deposited in the target is equal to the initial momentum of the pions in the beam, which is about 80 MeV/c. For the $\pi \to \mu \to e$ decay, the total momentum is equal to this 80 MeV/c plus an additional 4 MeV/c from the kinetic energy of the μ, which is also stopped within the target.

Is R-parity conservation necessary?

R-parity protects against proton decay and gives a stable SUSY dark matter particle. There are two sets of R-parity violating interactions: those that violate B (baryon number) and those that violate L (lepton number). So R-parity implies conservation of both B and L. If we throw out R-parity, but keep *either* B or L as a symmetry of the MSSM (Minimal Supersymmetric Standard Model), then the proton is still stable, although the SUSY DM particle is not.

What are the reasons to have/not to have R-parity violating theories?

R-parity is an ad hoc assumption of the MSSM, introduced to prohibit a variety of new interactions which allow for proton decay. R-parity also allows for a stable SUSY dark matter particle. It is possible to prohibit proton decay through a weaker ad hoc assumption, which is *either* B (baryon number) conservation or L (lepton number) conservation. One of those assumptions is sufficient to prohibit proton decay. (R-parity includes both B and L conservation.) However, if either B or L is violated, there is no stable SUSY DM particle.

In a sense, the postulate of R-parity or B or L conservation is a step backwards from the usual Standard Model, in which B and L conservation arise naturally, without added assumptions. (This is just one of the theoretically hurdles toward building a viable model of SUSY.)