The Fine Structure Constant and Electron (g-2) Factor

Nick Hutzler Physics 135c Spring 2007

Theory Review

- A (spin) magnetic moment μ quantifies how a particle interacts with magnetic fields
- The electron has μ proportional to spin

$$\mu = g\frac{e\hbar}{2m}\frac{S}{\hbar} = g\mu_B\frac{S}{\hbar}$$

- The fine structure constant α characterizes the strength of the EM interaction

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} \approx \frac{1}{137}$$

Theory Review

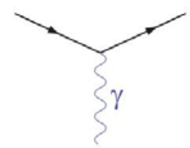
- The (spin) factor g is dimensionless and varies by particle type
  - Classical charged sphere: g = 1
  - Dirac point particle: g = 2
  - Neutron: g = -3.826 085 45(90)
  - Proton: g = 5.585 694 713(46)
- The anomalous magnetic moment is defined as a = (g-2)/2

Theory Review

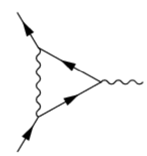
- Why measure g?
  - Probe of possible internal structure
  - One of best tests of QFT/QED, new physics
  - Fine structure constant α determined by g+QED

Theory Review

- Why isn't g = 2?
  - Tree level Feynman diagram gives g = 2



  - Higher order corrections from quantum fluctuations add ~.001



Theory Review

- g and α are closely related
  - *a* has a series expansion in α (remember that α is commonly expanded in perturbation series)

$$a = C_1\left(\tfrac{\alpha}{\pi}\right) + C_2\left(\tfrac{\alpha}{\pi}\right)^2 + C_3\left(\tfrac{\alpha}{\pi}\right)^3 + \cdots + a_{\mu\tau} + a_{hadronic} + a_{weak}$$

  - The C_i are calculated from QED
  - The $a_{\mu\tau,\text{hadronic},\text{weak}}$ terms are the QED muon/tau contribution, and non-QED hadronic and weak contributions (respectively)

Theory Review

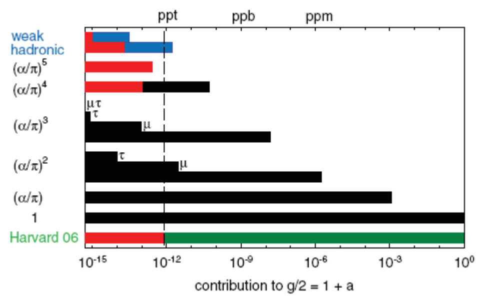


FIG. 2 (color). Contributions to $g/2$ for the experiment (green), terms in the QED series (black), and from short-distance physics (blue). Uncertainties are in red. The μ, τ, and $\mu\tau$ indicate terms dependent on mass ratios m_e/m_μ, m_e/m_τ and the two ratios, m_e/m_μ and m_e/m_τ, respectively.

History

- Earlier α measurements
  - Quantum Hall effect

$$R_K = \frac{\mu_0 c}{2\alpha}$$

  - Atomic spectra

$$R_\infty = \frac{\alpha^2 m_e c}{2h}$$

  - Josephson effect

$$U = \frac{h}{2e}\frac{\partial \phi}{\partial t}$$

  - Recoil measurements, numerology, others

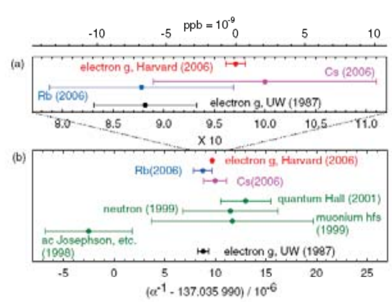


FIG. 1 (color). The least uncertain α determinations [3,5,6] (a), with older determinations [2] on a 10 times larger scale (b). Measured g are converted to α using current QED theory.

Philosophy

- *Why* does α have this value?
  - *Anthropic Principle*: small changes in the value of α would dramatically change the universe
  - *Numerology*: Arthur Eddington calculated α=1/136 using numerology and gave the Eddington Number:
    $N = 2^{256} / \alpha$
  - *Feynman*: "one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man"
  - *James Gilson*: α has a formula:

$$\alpha = \frac{\cos(\pi/137)}{137}\frac{\tan(\pi/(137\cdot 29))}{\pi/(137\cdot 29)} \approx 1/137.0359997867$$

The Experiment

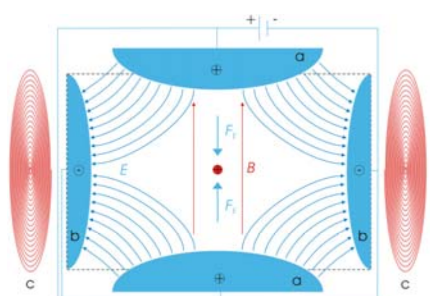
- We shall discuss a recent measurement of *g* and resulting calculation of α
- *New Measurement of the Electron Magnetic Moment Using a One-Electron Quantum Cyclotron*
  - Odom, Hanneke, D'Urso, Gabrielse
  - Harvard Physics Department
  - PRL 2006

The Experiment

- Overview:
  - Put an electron in cyclotron motion
  - Measure jumps between lowest quantum states
  - Calculate *g*
  - Calculate α

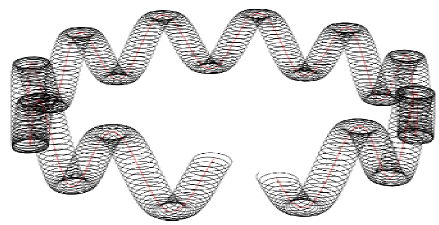
Penning Traps

- Static fields used to trap charged particles
  - Uniform axial B field
  - Quadrupole E field
- *Earnshaw's Theorem*: static electric fields alone cannot confine a charged particle



Penning Traps

- Four motions:
  - Spin (ν_s)
  - Cyclotron ($\nu_c = 2\,\nu_s/g$)
  - Axial (ν_z)
  - Magnetron (ν_m)
- In the present experiment:
  - $\nu_s \approx \nu_{c'} \approx 149$ GHz
  - $\nu_{z'} \approx 200$ MHz
  - $\nu_{m'} \approx 134$ kHz
  - Prime denotes "actual"



Particle Motion

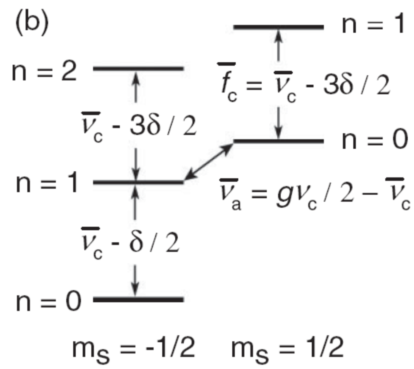
- Spin motion is essentially free
- Cyclotron motion is damped by synchrotron radiation (~0.1 s in free space)
  - However, in this trap the fields are tuned so that cyclotron motion is far from a cavity mode
  - Lifetime extended to several seconds

Energy Levels

- Measure jumps between lowest spin (m_s) and cyclotron (n) levels

$$E(n,m_s)=\frac{g}{2}h\nu_c m_s+(n+1/2)h\nu_{c'}-\frac{1}{2}h\delta(n+1/2+m_s)^2$$

- $\nu_{a'}$ and $f_{c'}$ are anomaly and spin-up frequencies (respectively)
- ν_c is classical cyclotron frequency eB/2πm



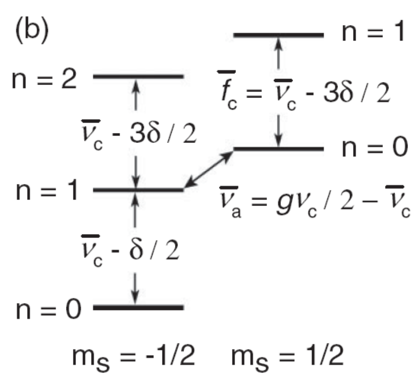
Energy Levels

- ν_c is obtained using the Brown-Gabrielse Invariance Theorem:

$$(\nu_c)^2=(\nu_{c'})^2+(\nu_{z'})^2+(\nu_{m'})^2$$

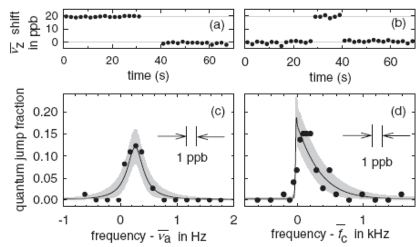
- This allows us to calculate g/2:

$$\frac{g}{2}=\frac{\nu_{a'}+\nu_{c'}}{\nu_c}\cong 1+\frac{\nu_{a'}-\nu_{z'}^2/(2f_{c'})}{f_{c'}+3\delta/2+\nu_{z'}^2/(2f_{c'})}$$



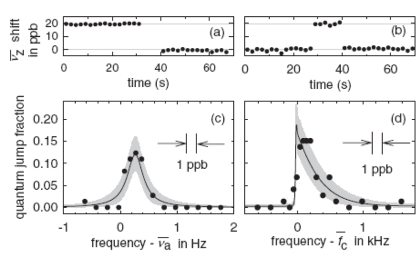
What is *Measured*?

- Spin and cyclotron jumps induce a change in $\nu_{z'}$
- A resonant circuit drives the axial motion at $\nu_{z'}$ (SEO)
- Quantum Jump Spectroscopy



What is *Measured*?

- Anomaly transitions are created by inducing spin-flips with a perpendicular B field
- Cyclotron transitions are induced by microwaves
- Both drives are always on, but tuned so that only one affects the electron



Results

- g/2 = 1.001 159 652 180 85(76)
- That's 76 parts per ***trillion***
- Certainly of the most accurate measurements ever made

Sources of Error

- Magnetic field instabilities
  - After settling for several months, the superconducting solenoid is constant to 10^{-9}/hours
  - Maintain in 0.3K environment
  - Perform measurements at night
- Fitting/statistics
  - ~50 measurements/night
- Cavity radiation modes
- Blackbody Photon Excitation
  - Cool sample to <100mK

Fine Structure Constant

- This g-value translates to

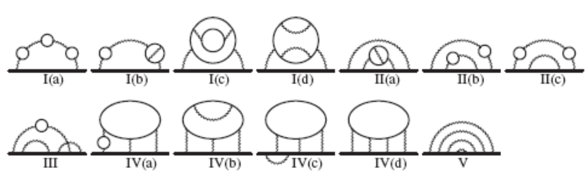
α^{-1} = 137.035 999 710(90)(33)[0.66][0.24]

= 137.035 999 710(96)[0.70]

- The top line errors are experimental, QED respectively (second line gives total error)
- Square brackets give error in ppb

Fine Structure Constant

- C_4,C_6,C_8,C_{10} require the evaluation of 7,72,891,12 672 Feynman diagrams respectively
- C_8 is known fairly well, C_{10} is still being calculated



Important Conclusions

- QED continues to be a highly accurate description of the physical world
- The electron can still be viewed as having no internal structure

Electron Internal Structure

- Internal constituents must have m > 130 GeV
  - Calculations made assume no internal structure
  - Thus corrections must come from very high energy/short distance scales
  - i.e. constituents must be *very* tightly bound
- No QED error: m > 600 GeV

References

- *New Measurement of the Electron Magnetic Moment Using a One-Electron Quantum Cyclotron*
  Odom, Hanneke, D'Urso, Gabrielse. PRL 97, 030801
- *New Determination of the Fine Structure Constant from the Electron g Value and QED*
  Gabrielse, Hanneke, Kinoshita, Hio, Odom. PRL 91 030802
- *Fully Quantum Measurement of the Electron Magnetic Moment*
  Brian Odom, Thesis, Harvard University, 2004
- *A Finer Constant*, Czarnecki, Nature 443 p. 516
- James Gilson, fine-structure-constant.org