Phys 135

EXTRA DIMENSIONS

An analogy

- Picture a piece of paper, rolled up so tightly that from only motion along one dimension is permissible
- There are still two (arguably three) dimensions to the piece of paper, but for those living on the outermost surface only one is really perceptible
- This is an analogy that drives the consideration of extra dimensions

A Little Bit of History

- The history of extra dimensions goes back to the 1920's, with the work of Kaluza and Klein
- They hypothesized the existence of a fifth dimension to unify gravity and electromagnetism
- The theory had problems due to self-consistency, and was abandoned until the 1970's.

A Little Bit of History, ctd.

- In the 1970's, the concept of supergravity brought back interest in extra dimensions, and brought it back into vogue as a possible explanation for aspects of cosmology

Why are Extra Dimensions Considered Viable?

- String Theory (M-Theory requires 11 dimensions)
- Change cut-off scale of Standard Model--may solve hierarchy problem by pulling down Planck scale to ~TeV.
- Certain theories (Universal Extra Dimensions) may explain abnormalities in the SM such as the lack of proton decay.

Extra-dimensional Theories

- There are three main theories of extra dimensions
  - Universal Extra Dimensions
  - Large Extra Dimensions
  - Warped Extra Dimensions
- Technically, UED appeared first, followed by Warped and then Large, but serious new research into UED did not begin until after the advent of the other two

Universal Extra Dimensions

- Consider a flat metric with compact extra dimensions. Allow both gravity and SM fields to propagate in all dimensions.
- So far most work has considered a single extra dimension, although additional ones also have theoretical motivation.

Motivations for UED

- UED suppresses higher dimensional operators that violate baryon and lepton number
  - Consider 6-dimensional UED theory
  - Lowest proton decay dimension is 9: p -> $e\pi^+\pi^-\nu\nu$, with lifetime

$$\tau_p \sim 10^{33}\,\text{years}\left(\frac{1/R}{500\,\text{GeV}}\right)^{12}\left(\frac{\Lambda R}{5}\right)^{22}$$

  - R is size of the ED, Λ is the theory cutoff.
- This can prevent proton decay, neutron/antineutron oscillations, lepton-number violation, large Majorana masses, and other effects due to higher-dimensional operators.

KK Patricles in Universal ED

- Decompose SM fields into Kaluza-Klein components.

$$(H, A_\mu)(x^\mu, y) = \frac{1}{\sqrt{\pi R}}\left[(H_0, A_{\mu,0})(x_\mu) + \sqrt{2}\sum_{n=1}^{\infty} H_n, A_{\mu,n})(x_\mu)\cos\left(\frac{ny}{R}\right)\right]$$

- Here *n* is called the KK number and gives the momentum of the particle in the extra dimension.
- KK parity $P = (-1)^n$ is generally conserved, causing the lightest KK particle (LKP) to be stable, analogous to the LSP in supersymmetry -> Possible dark matter candidate

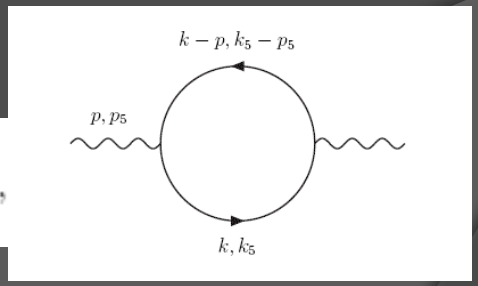
KK Particle Spectrum in UED

- At tree-level, the masses of the KK excitations for different particles are nearly degenerate, as $R^{-1} >> m$

$$m^2_{X^{(n)}} = \frac{n^2}{R^2} + m^2_{X^{(0)}}$$

- Higher order corrections break the degeneracy, giving a mass range for 1st KK excitations of 500-650 GeV.
- The KK photon has a small negative radiative correction, making it likely the LKP.
- This correction is due to the electron loop in the vacuum polarization:

$$\delta m^2_{KK} = -\frac{e^2}{2\pi R^2}\sum_{n\neq 0}\frac{2}{|2\pi n|^3} = -\frac{e^2}{4\pi^4 R^2}\sum_{n=1}^{\infty}\frac{1}{n^3} = -\frac{e^2\zeta(3)}{4\pi^4 R^2},$$



Accelerator Searches in UED

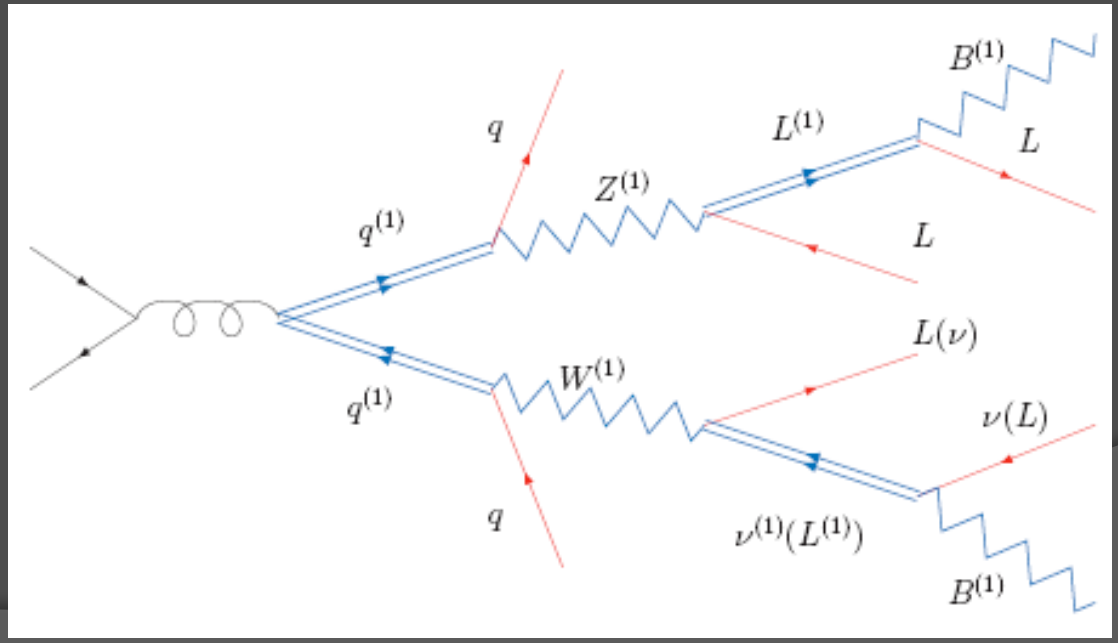
- The KK states in UED can be produced in high energy colliders (Tevatron, LHC)
- In particular, first level KK modes are pair produced in large quantities at hadron colliders like the LHC.
- The decay chains of the colored KK modes then can be observed for the missing energy, as well as the expected $l^+l^-l^\pm$ and the two jets.

Challenges with UED Detection

- The jets are of low energy, and thus are difficult to distinguish from the background
- The same is true for the leptons to a lesser degree

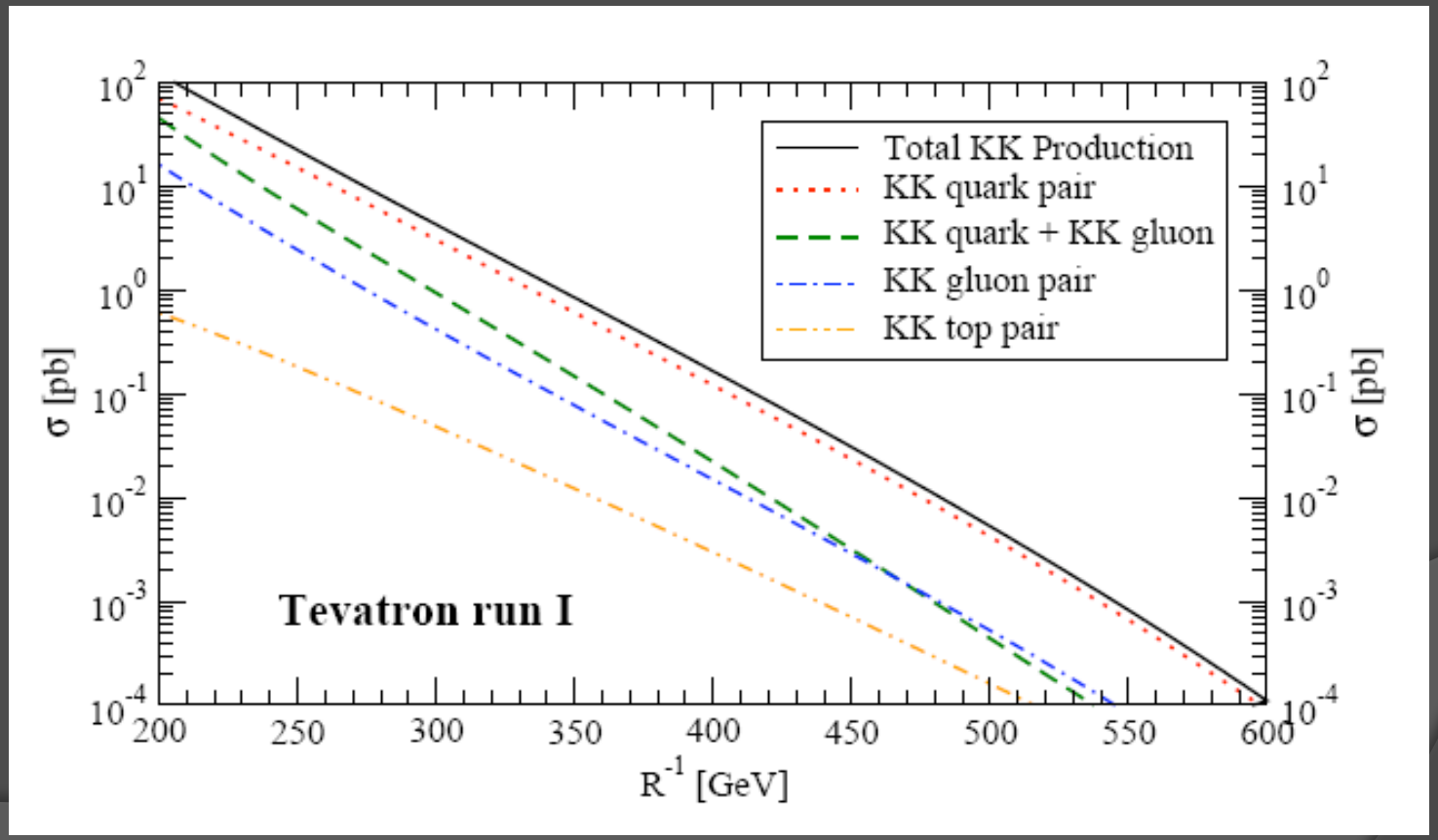
Looking for UED at the Tevatron

- Though largest signature is due to jets plus missing energy, SM background obscures this.
- Instead look at decays where quarks decay into Z bosons decay into multiple leptons:



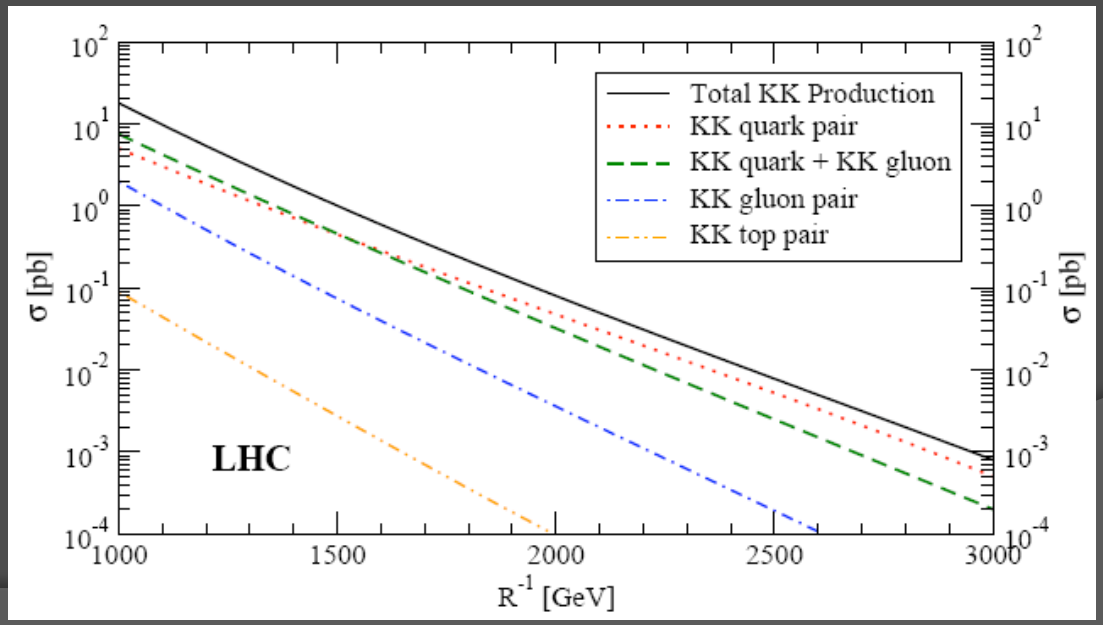
Results from Tevatron Search

- Limit for KK production cross section was 3.3 pb at 95%, giving upper limit on ED radius *1/R* > 270 GeV



UED at the LHC

- The signature of jets + missing energy is more pronounced over the background, giving a reach of *1/R* < 1.2 TeV
- Multilepton states still allow a slightly greater reach, *1/R* < 1.5 TeV

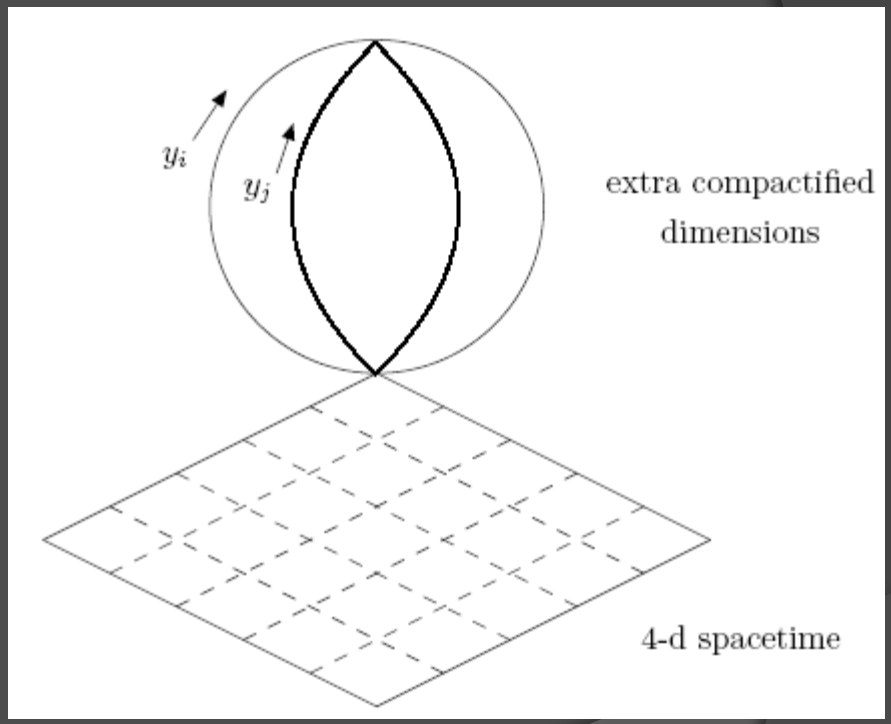


Discriminating Between UED and SUSY

- Signals appear similar to supersymmetry: both have stable neutral LKP/LSP, same couplings for SM particles and heavier partners, etc.
- However, UED produces a tower of KK states: observation of higher KK modes would distinguish between the two

Large Extra Dimensions

- ADD Proposal: Multiple compactified extra dimensions with SM fields localized to 3-brane.
- Started "renaissance" of ED theories in 1998



ADD Proposal for Large ED

- Compacts extra dimensions with radius r, forming a torus of volume $V_n = (2\pi r)^n$.
- Unlike UED, SM fields live only in the 3-brane of normal experience; only gravity propagates in all dimensions.
- Spacetime is assumed to be flat with fluctuations.
- Fluctuations of SM brane into higher dimensions assumed to be negligible.

Effective Planck Scale in LED

- Einstein action in 4+*n* dimensions, with M_* the "true" Planck scale in 4+*n* dimensions (cut-off of SM)

$$S_{\rm bulk} = -\frac{1}{2}\int d^{4+n}x\sqrt{-g^{(4+n)}}M_\star^{n+2}R^{(4+n)}$$

- Assume extra dimensions form a torus, giving metric

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu - r^2 d\Omega^2_{(n)}$$

- Plugging this into the above gives for the action

$$-\frac{1}{2}M_\star^{n+2}(2\pi r)^n \int d^4x\sqrt{-g^{(4)}}R^{(4)}$$

- We thus identify the effective Planck scale of gravity

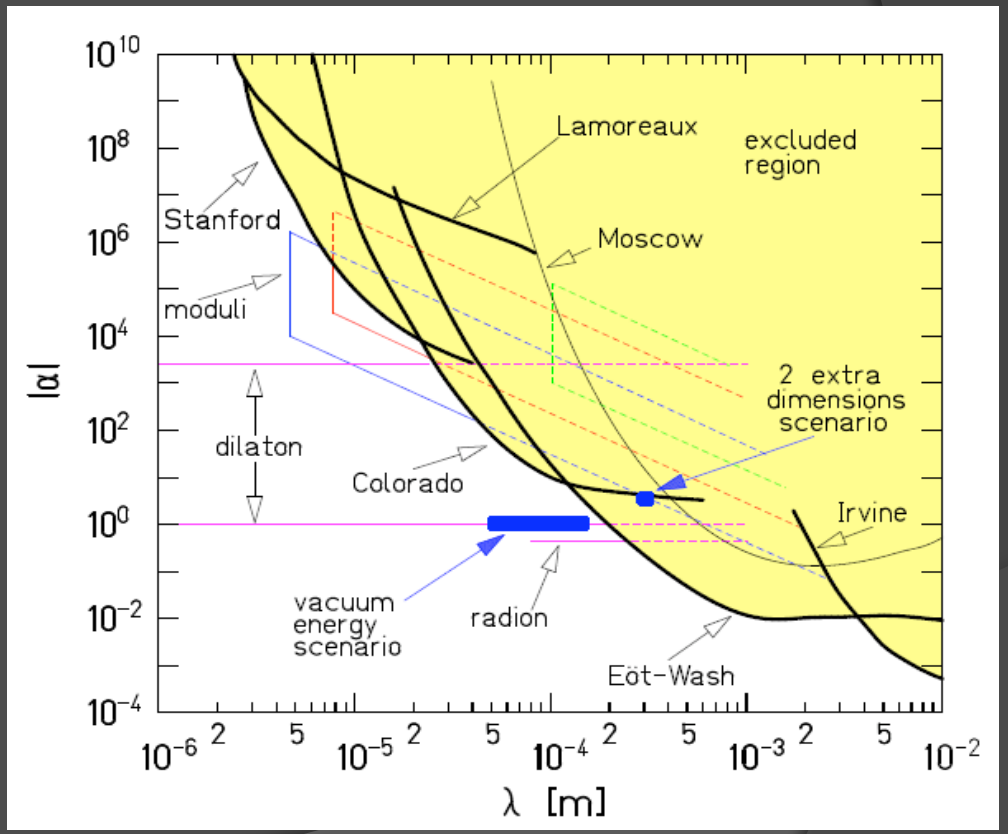
$$M_{\rm Pl}^2 = M_\star^{n+2}(2\pi r)^n$$

M_* ~ TeV to solve hierarchy problem

Precision Gravity Tests of LED

- Recall from an earlier talk that LED modifies gravity from $1/r^2$ to $1/r^{2+n}$ for distances comparable to the size of the extra dimensions.

- Experiment has ruled out $n = 1$ and $n = 2$ for $M_* = 1$ TeV



KK Gravitons in Large ED

- As with SM particles in UED, the graviton in LED can be expanded in KK modes in each extra dimension:

$$h_{AB}(x;y) = \sum_{m_1=-\infty}^{\infty} \cdots \sum_{m_n=-\infty}^{\infty} \frac{h_{AB}^{(m)}(x)}{\sqrt{V_n}} e^{i\frac{m_j y_j}{r}}$$

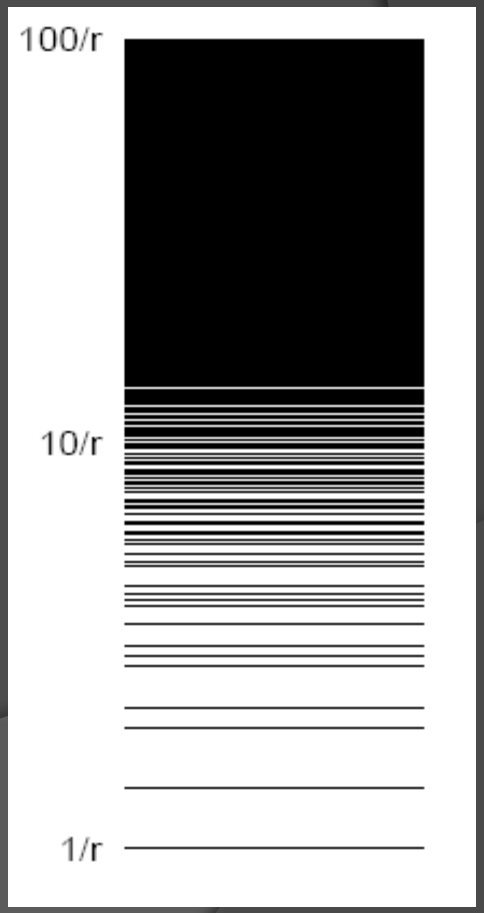
- The KK gravitons have mass

$$m^2 = \sum_i^n \frac{|k_i^2|}{r^2}$$

- Density of KK states is thus

$$dN = S_{n-1}|k|^{n-1}dk$$

$$dN = S_{n-1} r^n m^{n-1} dm$$

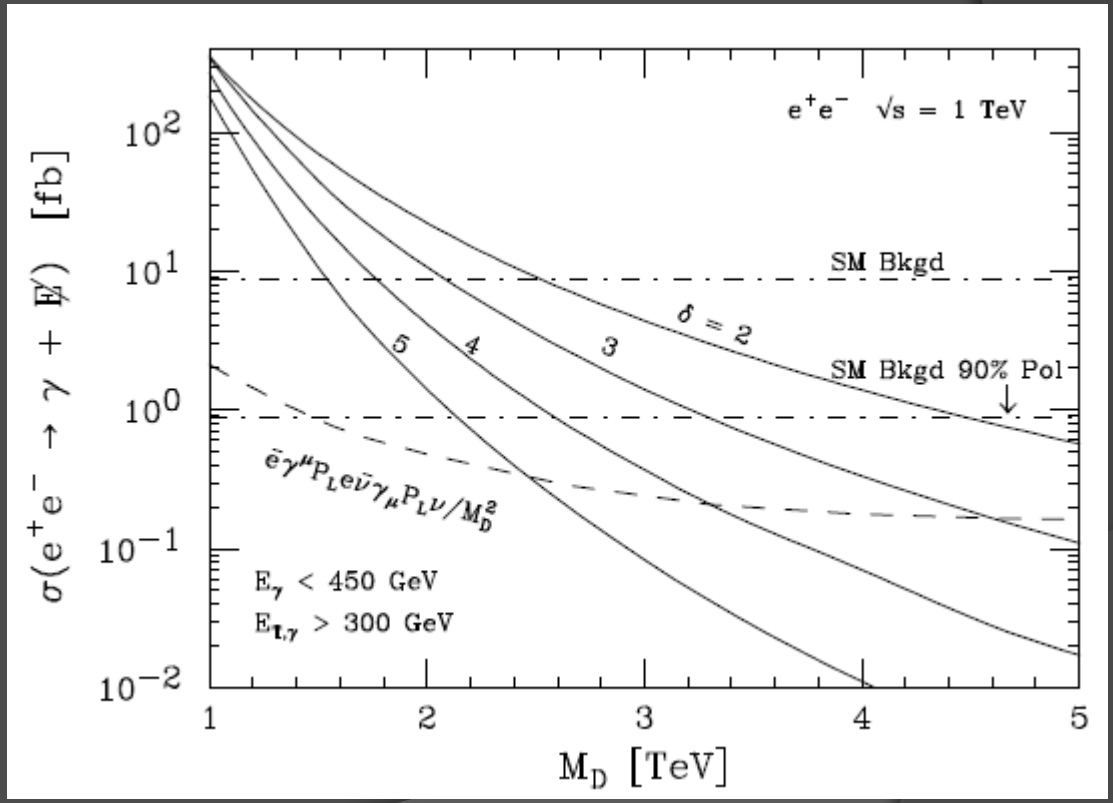


Direct Detection of LED at e^+e^- Colliders

- Look for signal $e^+ + e^-$ → γ + Missing Energy
- Impose cut $E_\gamma < 450$ GeV to eliminate background from

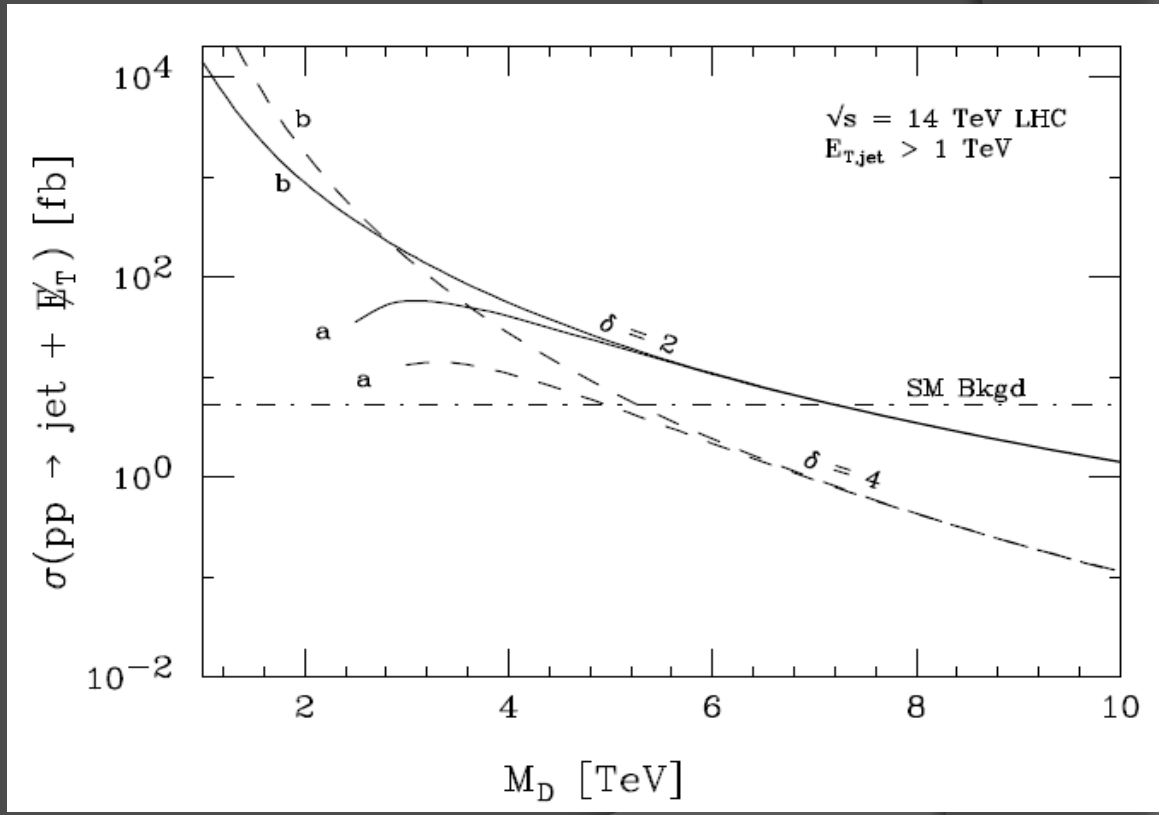
$$e^+ + e^- \rightarrow \gamma + Z \rightarrow \gamma + \nu + \bar{\nu}$$

- Cross-section decreases with number of extra dimensions δ: density of states dN/dm decreases



Direct Detection of LED at Hadron Colliders

- Look for signal p + p → jet + Missing Energy; gravitons produced by q + g → q + G, etc.
- Impose cut again to eliminate background from decay of Z into neutrinos
- Elementary processes q + g → q + G have varying CoM energies, not clear how applicable theory is in UV



Supernovae Constraints on LED

- Astrophysical systems may reach temperatures up to the KK mass range, producing gravitons at rate

$$\frac{1}{M_{\rm Pl}^2}(Tr)^n \sim \frac{T^n}{M_\star^{n+2}}$$

- Supernovae are extremely hot systems that are well understood using SM physics.
- Obtain bound on M_* to prevent excessive graviton production (graviton luminosity less than total SN luminosity):

$$M_D \gtrsim \begin{cases} 50\ \text{TeV} & n=2 \\ 4\ \text{TeV} & n=3 \\ 1\ \text{TeV} & n=4 \end{cases}$$

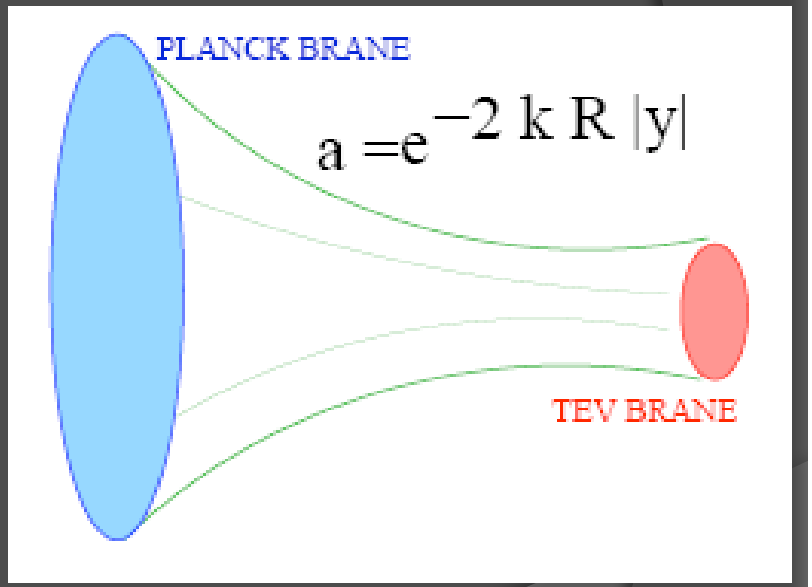
$$M_D = (2\pi)^{n/(n+2)} M_\star$$

Warped Extra Dimensions: R-S Model

- Consider 5-d theory with metric

$$ds^2 = e^{-k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

- The SM brane, or TeV brane, forms one boundary of the spacetime; the “Planck brane” forms the other.
- The exponential *y* dependence is known as the “warp factor,” and essentially acts as an ordinary gravitational redshift between the two branes. The Planck mass is redshifted to the TeV scale on the SM brane



KK Gravitons in Warped ED

- Solve Schrödinger's Equation in extra dimension for graviton, with potential from Einstein equations
- Get one bound state of zero mass Masses of higher modes go as the roots x_j of the Bessel function J_1

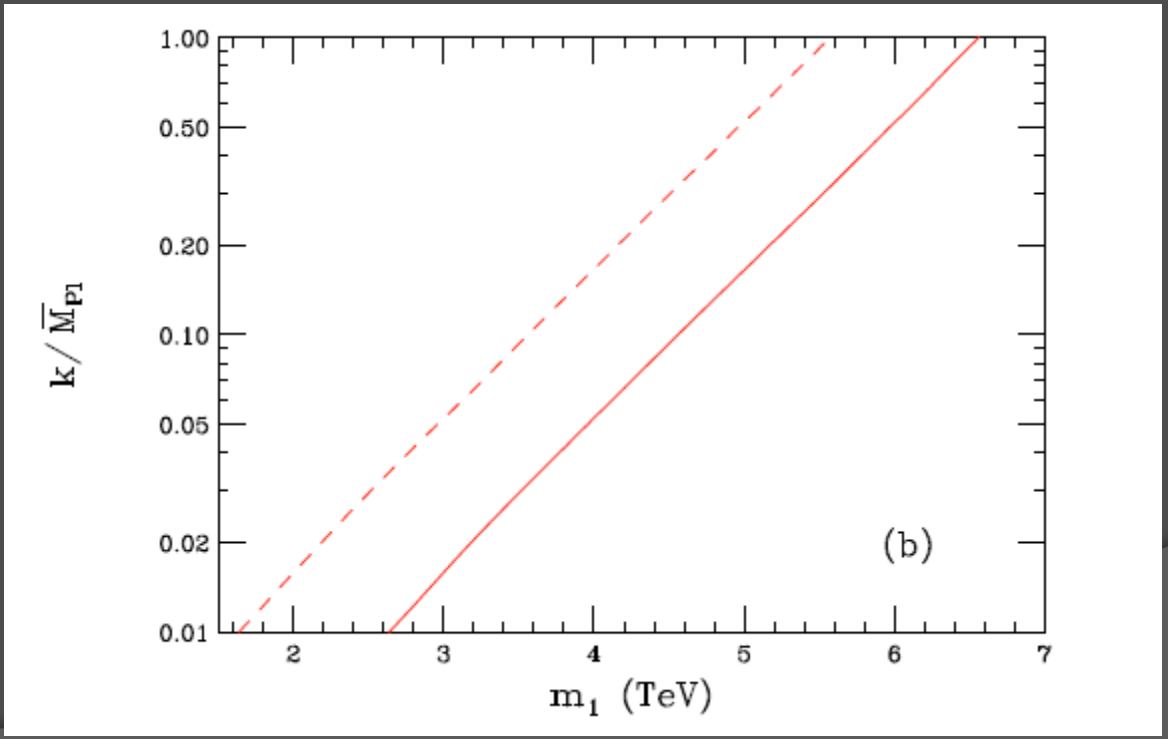
$$m_j = x_j k e^{-kb}$$

$ke^{-kb} \sim$ TeV

- The zero mode graviton wavefunction peaks at the Planck brane and decays exponentially, and thus has $1/M_{pl}$ strength coupling. The higher modes peak near the TeV brane and thus have redshifted 1/TeV couplings.

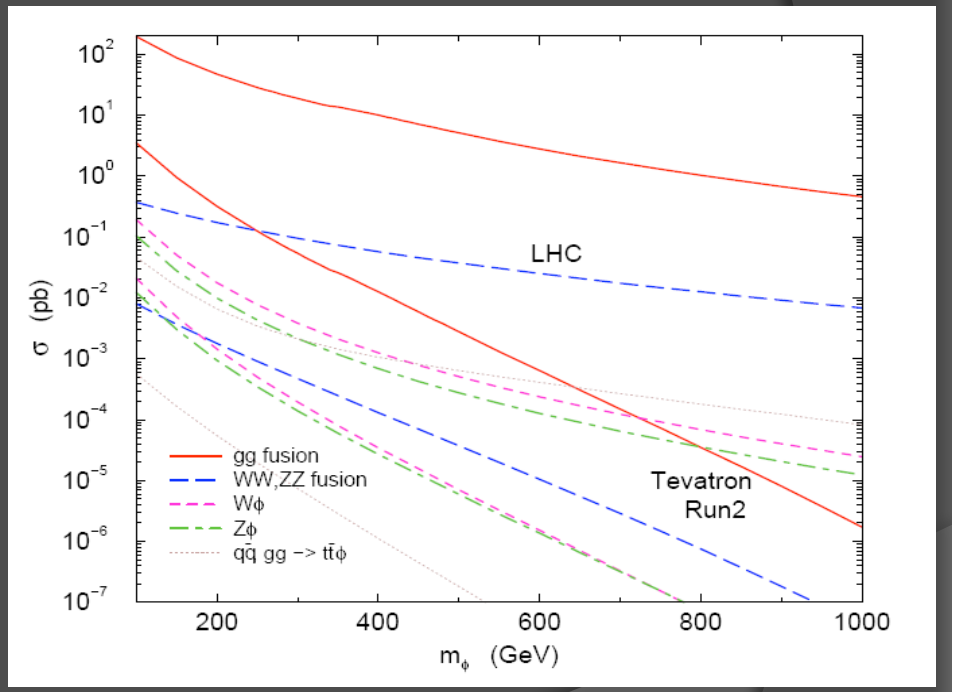
Direct Detection of WED

- First KK graviton mass determines warp factor, and production cross-section depends independently on parameter *k*.



Radion Detection

- Need mechanism to stabilize size of extra dimension (no energy cost to change it)
- This can be achieved through the Goldberger-Wise proposal: Introduce a scalar field with a *y*-dependent vev
- Expect the radion to have mass between .01 and 1 TeV
- Radion couples more strongly to gluons than Higgs and is likely to have a larger production cross-section.



Conclusion

- Extra dimensions may help solve various issues in SM physics such as the hierarchy problem, proton stability, etc.
  - Universal Extra Dimensions: All fields (SM + gravity) propagate in all dimensions
  - Large Extra Dimensions: Toroidal extra dimensions in which only gravity propagates
  - Warped Extra Dimensions: Warp factor suppresses masses
- Accelerator searches and astrophysical/cosmological constraints provide bounds on the number and size of extra dimensions.

QUESTIONS ?