Pion decays and \( \Re/\mu \)

We want to consider decays of \( \pi^+ \)

\[
\begin{align*}
M_{\pi^+} & \approx 140 \text{ MeV} \\
\tau & = 1 \times 10^{-8} \text{ s} \\
\Gamma(\pi^+ \to \text{anything}) & = 3 \times 10^{-8}
\end{align*}
\]

two main decay channels

\[
\begin{align*}
\pi^+ & \to e^+ \nu_e \\
\pi^+ & \to \mu^+ \nu_\mu
\end{align*}
\]

(\( m_e \approx 0.511 \text{ MeV} \))

(\( m_\mu \approx 107 \text{ MeV} \))

(\( \text{assume } m_\nu_e = m_\nu_\mu = 0 \))

\[
\text{define } \Re/\mu = \frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)}
\]

plan:

(i) calculate \( \Re/\mu \) in SM as accurately as possible

(ii) measure \( \Re/\mu \)

(iii) look for deviations between \( \Re^{\text{SM}}/\mu \) and \( \Re^{\text{exp}}/\mu \)

calculate deviations due to your favorite model of "Beyond-the-SM" new physics (e.g. SUSY)

i. \( \Re/\mu \) in the SM

First of all, why consider a ratio \( \Re/\mu \)?

Suppose we just wanted the decay rate for \( \pi^+ \to l^+ \nu_\ell \) (\( l = e, \mu \))

\[
\Gamma(\pi^+ \to l^+ \nu_\ell) = \Gamma^{\text{tree-level}} \left( \frac{\alpha}{16\pi^2} \right) \left( \frac{M_{\pi^+}}{M_W} \right)^3 \left( \frac{m_\ell}{M_W} \right)^2 + \Gamma^{\text{radiative corrections}}
\]

\[
\Gamma^{\text{tree-level}} \quad \Gamma^{\text{radiative corrections}}
\]

theoretical difficulties

QCD

pion decay const. \( F_\pi \approx 94 \text{ MeV} \) (3% from lattice)

also includes pion-structure dependent effects, mostly independent of \( l \)
These QCD effects bring large theoretical uncertainties to the calculation of $\Gamma (\pi^+ \rightarrow \ell^+ \nu_\ell)$ since theorists can't accurately calculate QCD effects at low energies.

However, these effects are mostly independent of $\ell$ (lepton flavor) and cancel out of the ratio $\Re \ell / \mu$ even though $\Gamma (\pi^+ \rightarrow e^+ \nu_e)$ and $\Gamma (\pi^+ \rightarrow \mu^+ \nu_\mu)$ can't be calculated accurately; $\Re \ell / \mu$ can!

\[
\frac{\Re \ell}{\mu} = \frac{\sqrt{m_\ell^2 - m_e^2}}{m_\mu} \frac{m_\ell^2 - m_e^2}{m_\mu^2} (1 + \text{small radiative corrections}) = (1.2352 \pm 0.0005) \times 10^{-4}
\]

uncertainty comes from lepton-flavor dependent pion structure effects (which are unknown & estimated)

$\pi^+$ only couple to left-handed particles (but RH antiparticles)

but this violates angular momentum conservation: need $\pi^+ \rightarrow (\nu_e)_L (e^+)_L$

mass term mixes $(e^+)_L$ and $(e^+)_R$, so decay can proceed for $m_e \neq 0$, but at the cost of picking up a factor of $m_e^2$.
ii. Measurement of $R_{e\mu}$

$R_{e\mu}$ was measured in early 1990s by two labs: PSI (Paul Scherrer Institute), Switzerland
TRIUMF (Tri-University Meson Facility), British Columbia

places are huge national labs with collaborators from
d all over the world. Measurement of $R_{e\mu}$ was just one
experiment out of many.

measurements had comparable precision and agree within errors

Average: \[ R_{e\mu}^{\text{exp}} = (1.230 \pm 0.004) \times 10^{-4} \]

Although $R_{e\mu}^{\text{exp}}$ and $R_{e\mu}^{\text{SM}}$ agree, clearly there is much room

for better experiments to make a more precise test of SM

Hence

Upcoming experiments at PSI & TRIUMF will remeasure $R_{e\mu}$
with precision increased by a factor of 10 — comparable
to theoretical uncertainty in $R_{e\mu}^{\text{SM}}$
Beyond the SM

New, heavy particles may contribute to $\rho / \mu$ through virtual effects
(just as $\pi^+$ decays through virtual $W^+$)
even though available energy in decay is $m_{\pi} < 140$ MeV $\ll M_{\text{new}}$

**Example:** SUSY

1. charged Higgs $H^+$

   $H \to h^0$

   recall: SM - 1 neutral Higgs

   MSSM - 3 neutral Higgs & 1 charged Higgs

   $H_u, H_d \to h, H^0, A^0, H^+$

   pion can decay through $H^+$ exchange at tree level

   $\pi^+ \xrightarrow{u \bar{d}} H^+ \xrightarrow{\ell^+} \nu_\ell$

   because $H^+$ is a scalar ($s = 0$), there is no helicity suppression (good)

   but $H^+ \to \ell^+ \nu_\ell$ interaction has strong strength $g \sim \frac{m_{H^+}}{m_{\ell} \sim 100 GeV}$

   (bad)

   can show:

   $\left( \begin{array}{c} u \\ \bar{d} \end{array} \right) \xrightarrow{H^+} \left( \begin{array}{c} \ell^+ \\ \nu_\ell \end{array} \right) = \frac{m_{H^+}^2 \tan \beta (m_u \cot \beta - m_d \tan \beta)}{(m_u + m_d)^2 M_{H^+}^2} \left( \begin{array}{c} u \\ \bar{d} \end{array} \right) \left( \begin{array}{c} \ell^+ \\ \nu_\ell \end{array} \right)$

   need large $\tan \beta$

   note: independent of $m_\ell$

   $\Rightarrow$ cancels from $\rho / \mu$

in general, SUSY can give strong constraints on new scalars
due to absence of helicity suppression
e.g. leptoquarks from GUT's
new R-parity violating interactions

Recall: R-parity is a postulated symmetry of MSSM
quarks, leptons, gauge & Higgs bosons have \( p_R = + \)
superpartners have \( p_R = -1 \)
purpose: keeps proton from decaying
e.g., \( \tau(\rho \rightarrow e^+\pi^0) > 10^{33} \text{ yrs} \)
allows 1 stable superpartner (DM candidate)
it is possible to get rid of R-parity without allowing
\( p \)-decay (but now no stable DM)

with RPV interaction, \( \pi^+ \) can decay \( \rightarrow e^+ \) through intermediate

virtual down quark \( d \):

\[
\pi^+ \rightarrow e^+ + d \quad \text{virtual down quark} \quad d
\]

measurement of Relu provides limits on magnitude of
RPV interaction strength \( |\lambda|^2 \).
(3) 1-loop contributions to Rell in MSSM (w R-parity)

\[ \begin{align*}
&\text{e.g., } \begin{array}{c}
\tilde{\chi}^+ + \tilde{\chi}^0 + l^+ \\
\tilde{\chi}^0 + \tilde{\chi}^+ + l^- \\
\tilde{\chi}^- + \tilde{\chi}^0 + l^-
\end{array}
\end{align*} \]

- vertex correction
- (penguin)
- box graphs

expectations:

1. all MSSM contributions vanish if Wino is very heavy
   \[ \Rightarrow \text{want } m_{\tilde{\chi}^+} \text{ at least one chargino & neutralino must be light} \]
2. MSSM contribution to \( \pi^+ \rightarrow e^+ \nu_e \) same as to \( \pi^+ \rightarrow \mu^+ \nu_\mu \)
   if \( m_{\tilde{\chi}^0} = m_{\tilde{\chi}^+} \)
   \[ \text{i.e., MSSM corrections cancel from Rell} \]
   \[ \Rightarrow \text{want } \frac{m_{\tilde{\chi}^0}}{m_{\tilde{\chi}^+}} \ll 1 \text{ or } \gg 1 \]
Leptonic Pion Decay

Experimental Aspects
The TRIUMF Cyclotron

500 MeV protons
PIENU Experiment

Early 90s:
- measured

Current Experiment: Similar to original experiment but
- More statistics
- Better MC to understand energy dependence of multiple e⁻ scattering
- Narrower π momentum distribution
- Better dE/dx measurements in target

\[ R_{e/\mu}^{exp} = 1.2265 \pm 0.0034(stat) \pm 0.0044(syst) \times 10^{-4} \]
Current Experiment

π beam:

\[ \sim 5 - 10 \times 10^4 \text{ pions/s.} \]

75 MeV/c \( \pi^+ \) beam
Counting $\pi \rightarrow e\nu$ vs $\pi \rightarrow \mu\nu \rightarrow e\nu\nu$  

- All $\pi^+$’s decay to final state $e^+$
- Only the $\pi^+$ and $e^+$ is detected (not the $\mu^+$)
  - So how do we distinguish between the two decay channels?

Solutions:
- Energy Spectrum of $e^+$
  - $\pi^+ \rightarrow e^+\nu\ (T_{e^+} = 69.3$ MeV)
  - $\mu^+ \rightarrow e^+\nu\bar{\nu}\ (T_{e^+} = 0 - 52.3$ MeV)

- Time of $e^+$ shower in detector
  - $\mu$ lifetime is long: $\sim 2\ \mu$s
Counting $\pi \rightarrow e\nu$ vs $\pi \rightarrow \mu\nu \rightarrow e\nu\nu$

Plots from past TRIUMF experiment

Fig. 2 Left: Energy spectrum of $e^+$’s in the early time window. Right: Time spectra for events in the (a) $\pi^+ \rightarrow e^+\nu$ and (b) $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ energy regions.
Beyond the SM Prospects

Possible signatures of SUSY in $R_{e/\mu}$
R-Parity Violation
1-Loop MSSM Contributions
Conclusions

\( R_{e/\mu} \) is an observable which can be calculated very precisely due to cancellation of unknown QCD effects.

A lot of room for current experiments (PSI and TRIUMF) to test SM prediction.

SUSY can cause deviations in \( R_{e/\mu} \):
- RPV may be observable.
- 1-Loop contributions probably not observable at these expts — if detected at future experiments, will provide strong probe of slepton and chargino sectors.
In this note, I address the questions that were asked during the talk.

**What is the quark content of $\pi^+$?**

The valence quarks of $\pi^+$ are $(u\bar{d})$. Of course, there are also sea quark-antiquark pairs and gluons.

**What is $f_\pi$?**

The quantity that enters into the pion decay amplitude at the $\pi^+ - W^+$ vertex is

$$\langle 0 | j_5^{\mu+}(x) | \pi^+ \rangle .$$

This mysterious object $j_5^{\mu+}(x)$ is called the axial current. It is basically the weak analog of the electromagnetic current. The expression above represents the amplitude for the $\pi^+$ to go to the vacuum $|0\rangle$ via this axial current (which is also coupled to the leptons). Because of the difficulties of QCD, it is not possible to calculate this amplitude exactly. However, though some tricks, it is possible to determine it up to an overall constant. This constant is $f_\pi$.

**Why is the $\pi \to W$ vertex not just a matter of the electroweak vertex $u\bar{d} \to W$?**

You need to take into account the wavefunction of the $\pi^+$.

**Do the radiative corrections to the tree-level pion decay diagram increase or reduce the matrix element?**

It could be either, depending on the sign of a particular diagram.

**What are the details of how the pion structure effects cancel out?**

Suppose $t_\ell$ and $r_\ell$ are the tree-level and 1-loop, respectively, amplitudes to the process $\pi^+ \to \ell^+ \nu_\ell$. Then $R_{e/\mu}$ is proportional to

$$R_{e/\mu} \propto \frac{|t_e + r_e|^2}{|t_\mu + r_\mu|^2} \approx \frac{|t_e|^2}{|t_\mu|^2} \left[ 1 + 2\text{Re}(r_e/t_e) - 2\text{Re}(r_\mu/t_\mu) \right],$$

where we have Taylor expanded to leading order in $r_\ell$, assuming $r_\ell \ll t_\ell$. We see that for $r_e/t_e = r_\mu/t_\mu$, the radiative corrections cancel out. This is what happens with charged Higgs exchange in the MSSM, as I discussed at the end of the talk.

**How do uncertainties on e and mu mass affect the overall uncertainty on $R(e/mu)$?**
The fractional uncertainties on the measurements of the $e$, $\mu$, and $\pi^+$ masses are:

\[
\delta(m_e) = 8 \times 10^{-8} \quad (3)
\]
\[
\delta(m_\mu) = 9 \times 10^{-8} \quad (4)
\]
\[
\delta(m_{\pi^+}) = 3 \times 10^{-6} \quad (5)
\]

The current theoretical and target experimental uncertainty is about $4 \times 10^{-4}$. The uncertainties in the particle masses are irrelevant for $R_{e/\mu}$ in the foreseeable future.

**What are the veto counters for – i.e., what do they veto?**

Veto counters are placed after the target. They reject particles that penetrate through the target, rather than being stopped.

**How does the scintillator measure the total energy?**

The incident positron causes an electromagnetic shower of gamma rays and electron-positron pairs (whose annihilations result in more gamma rays) inside the scintillator crystal. The scintillator absorbs the energy of the electromagnetic shower and re-emits this energy as lower energy (visible) photons. A photomultiplier tube counts the photons; the number of photons is (to first approximation) proportional to the initial energy of the positron.

**How does one get the $e^+$ direction?**

The $e^+$ direction is obtained by a wire chamber located between the $\pi^+$-stopping target and the scintillator crystal. This helps with knowing if the positron was aimed close to the edge of the scintillator so that some of the electromagnetic energy might have escaped.

**Why is there a 511 keV peak in the mu event spectrum?**

This peak is due to low energy positrons which activate (by annihilating with a surface electron) but do not penetrate the scintillator crystal.

**What is the background in the $\pi \rightarrow e\nu$ time spectrum?**

This background is due to pile-up of muons in the target. Suppose there is a $\mu$ still in the target when another $\pi \rightarrow \mu \rightarrow e$ decay comes along, and suppose both the first $\mu$ and the second $\mu$ happen to decay at about the same time. The result is that the scintillator will detect an event of higher energy, which can get misinterpreted as a $\pi \rightarrow e\nu$ event. To avoid this, the experiments implement a cut to avoid pile-up: throw out $\pi \rightarrow e\nu$ events which came within 6 $\mu$s of a previous $\pi^+$ stopped in the detector.

**Why does the $\pi \rightarrow \mu \rightarrow e$ time spectrum have that turn-on shape?**

The turn on shape occurs because this decay is a two step process. Solving the equations describing the two-step decay process, we find that

\[
\frac{N_e}{dt} = N_\pi(0) \frac{\Gamma_\mu \Gamma_\pi}{\Gamma_\pi - \Gamma_\mu} \left( e^{-\Gamma_\mu t} - e^{-\Gamma_\pi t} \right), \quad (6)
\]

where $N_e$ and $N_\pi$ are the number of positrons and pions, respectively. The detector looks for the positron up to $t = 300$ ns. Plotting this, we get
which looks the same as the time-plot I showed during the talk. If we plot to a later time (e.g. \( t = 3000 \) ns), we get

where we can now see the turn-on behavior and the late-time exponential decay.

*How does one in detail get a handle on the \( \pi^- > e\nu\gamma \) background?*

Actually, the \( \pi^- > e\nu\gamma \) background is not the dominant source of the long tail that extends below the energy cut. (The \( \gamma \)'s are forward-peaked — i.e. mostly going in the same direction as the \( e^+ \), and into the scintillator crystal.) The tail is due the response function of the scintillator — which, as I understand it, is the small likelihood that the scintillator will scintillate less than it should, leading to a measured energy lower than the true energy of the incident positron.

The \( \pi^- > e\nu \) tail was determined in two ways. (1) Using the timing information, as I discussed. (2) By measuring the total momentum deposited in the target. (Here is where it is important to get a narrow momentum distribution for the \( \pi^+ \) beam.) In a \( \pi^- > e\nu \) decay, the momentum deposited in the target is equal to the initial momentum of the pions in the beam, which is about 80 MeV/c. For the \( \pi^- > \mu^- > e \) decay, the total momentum is equal to this 80 MeV/c plus an additional 4 MeV/c from the kinetic
energy of the $\mu$, which is also stopped within the target.

**Is R-parity conservation necessary?**

R-parity protects against proton decay and gives a stable SUSY dark matter particle. There are two sets of R-parity violating interactions: those that violate $B$ (baryon number) and those that violate $L$ (lepton number). So R-parity implies conservation of both $B$ and $L$. If we throw out R-parity, but keep *either* $B$ or $L$ as a symmetry of the MSSM (Minimal Supersymmetric Standard Model), then the proton is still stable, although the SUSY DM particle is not.

**What are the reasons to have/not to have R-parity violating theories?**

R-parity is an ad hoc assumption of the MSSM, introduced to prohibit a variety of new interactions which allow for proton decay. R-parity also allows for a stable SUSY dark matter particle. It is possible to prohibit proton decay through a weaker ad hoc assumption, which is *either* $B$ (baryon number) conservation or $L$ (lepton number) conservation. One of those assumptions is sufficient to prohibit proton decay. (R-parity includes both $B$ and $L$ conservation.) However, if either $B$ or $L$ is violated, there is no stable SUSY DM particle.

In a sense, the postulate of R-parity or $B$ or $L$ conservation is a step backwards from the usual Standard Model, in which $B$ and $L$ conservation arise naturally, without added assumptions. (This is just one of the theoretically hurdles toward building a viable model of SUSY.)