Ay123

Fall 2013

STELLAR STRUCTURE AND EVOLUTION

Problem Set 1

Solutions due Monday, October 21st 2013

1. (Counts as two questions) Assume a star obeys a linear density model so that

$$\rho(r) = \rho_c(1 - r\,/\,R),$$

where ρ_c is the central density and R is the radius of the star.

   a. Find an expression for the central density in terms of R and the mass M of the star.

   b. Use the equation of hydrostatic equilibrium and zero boundary conditions to find the pressure as a function of radius. Your answer will be in the form $p(r) = p_c \times f(r/R)$, where $f(x)$ is a function you will determine. What is the dependence of the central pressure p_c in terms of M and R?. Express p_c numerically with M and R in solar units.

   c. Assuming the equation of state for an ideal gas, what is the central temperature T_c?

   d. Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass (expressed in units of $M_\odot$). At what mass does the radiation pressure become comparable to the ideal gas pressure?

   e. Write down an explicit expression for the total gravitational potential energy of this toy star, and verify that the virial theorem is exactly satisfied. Be sure to discuss matter with a general equation of state, not just an ideal monatomic nonrelativistic gas.

2. Use the virial theorem to estimate a typical value for the speed of sound deep inside a star of mass M and radius R. Using this estimate, show that the time required for sound to cross the star is $P \sim (G\bar{\rho})^{-1/2}$. Hence, show that, for the fundamental oscillation mode of a star, the pulsation period takes a value of order 1 hr. Given that the luminosity L of a massive hydrogen-burning star varies with mass roughly as $L \propto M^3$, show that, at fixed effective temperature, the fundamental pulsation period of such stars scales as $P \propto L^{7/12}$.

3. Sketch a plot of the stellar-remnant (i.e., white dwarf, neutron star, black hole) mass as a function of the initial stellar mass. Assuming stars form with a Salpeter initial mass function—i.e., that the number of stars in the mass interval $M \to M + dM$ is $\xi(M)dM$, with $\xi(M) \propto M^{-2.35}$, estimate the fraction of the mass of a stellar population that has been returned to the interstellar medium 10 Gyr after this population was formed. Comment briefly on the result.

4. The equation of state for an isothermal gas sphere satisfies the relation $P \propto \rho$ which is equivalent to a polytrope with γ=1. Derive the associated value of n. Using the equation of hydrostatic equilibrium, show that if $\rho = \rho_c \, e^{-\psi}$ the resulting Lane-Emden equation is:

$$\frac{1}{\xi^2}\frac{d}{d\xi}(\xi^2\frac{d\psi}{d\xi}) = e^{-\psi}$$

By considering the boundary conditions, explain why no star can be usefully represented by an isothermal gas sphere. What do you think might be the likely applications in astrophysics for such an equation of state?