Ay123

Fall 2013

STELLAR STRUCTURE AND EVOLUTION

Problem Set 2

Solutions due Monday, October 28th 2013

1. Show that when a self-gravitating body of polytropic gas shrinks homologously and adiabatically, its thermal energy scales with radius R as $E_{\rm th} \propto R^{3(1-\gamma)}$. Hence, show that a polytropic star is unstable to gravitational collapse if $\gamma < 4/3$.

2. The material in the envelope of a star is an ideal gas with γ=4/3 and the star is sufficiently centrally concentrated that the mass in the envelope is negligible compared to that in the core, M. If the envelope is just marginally convectively unstable, show that the temperature within the envelope varies with radius as

$$T = \frac{GM\mu m_H}{4k}(\frac{1}{r} - \frac{1}{r_s}) + T_s$$

where r_s and T_s refer to surface values.

3. The aim is to calculate the Rosseland mean opacity in the case of free-free absorption in pure hydrogen. The frequency-dependent opacity is given by the expression:

$$\kappa_\nu \rho = 1.32\,10^{56}\frac{\rho^2 g_{ff}}{\nu^3 T^{1/2}}\,(1\,-\,e^{-h\nu/kT})\;cm^{-1}$$

where g_{ff} is a constant quantum mechanical correction factor called the *Gaunt factor*.

(i) First derive an expression for $\partial\,B_\nu/\partial\,T$

(ii) Next, introduce a dimensionless variable $x = h\nu\,/\,kT$

(iii) Derive an expression for $\frac{1}{\rho\kappa_\nu}\,\partial\,B_\nu/\partial\,T$ and plot the resulting function. Use the plot to argue that the Rosseland mean opacity is largely determined by κ_ν when the frequency ν is a few times kT/h.

(iv) Hence show that the Rosseland mean opacity for free-free absorption obeys Kramer's law where $\kappa \propto \rho T^{-3.5}$.

4. Consider a family of stars in which the opacity is dominated by Thompson scattering by electrons, and in which nuclear energy is generated by the CNO cycle. Use your lecture notes to determine the density and temperature dependencies of opacity and energy generation in such a case. In analogy with the homology relations we derived in class, for this family of stars, find the relation between radius and mass, and between luminosity and mass. Locate this population on the Hertzsprung-Russell diagram.

5. Chandrasekhar mass limit via polytropes:

   Show that an electrically-neutral gas, consisting of positive ions and fully relativistic and completely degenerate electrons, where the positive ions contribute negilgibly to the pressure, obeys a polytropic equation of state with index n=3. Hence, using the equations we derived in class for the mass of a star obeying a polytropic equation of state, derive the mass of a fully-relativistic completely-degenerate white dwarf containing no hydrogen.