Ay123

Fall 2013

STELLAR STRUCTURE AND EVOLUTION

Problem Set 3

Solutions due Monday, November 4th 2013

1. Use the Saha equation for hydrogen to plot the ionization fraction y as function of temperature T for a density of $\rho = 10^{-10}$ g cm^{-3}. Calculate the adiabatic gradient $\Gamma_1 = (d\ln P/d\ln\rho)_{ad}$ as function of temperature for the same value of ρ. Make sure the temperature range of your plot covers both ionized and non-ionized states.

2. State of the Matter

   a. Use your favorite computer program to draw a "State of the Matter" diagram as discussed in class. Make assumptions, where necessary, for the various coefficients and provide on a separate paper (or on the diagram if you can), the equations that define the lines. Your equations should only contain physical constants (i.e. m_e, m_p, $\hbar$, c, k_B etc).

   b. Show that, to within in an order of magnitude, setting the degenerate energy density equal to the electrostatic energy density gives the same density as taking one proton per Bohr radius cube. Both calculations should only contain physical constants (do not substitute numbers).

   c. Look up the density of liquid hydrogen, liquid helium, water and iron. How well do they satisfy the "same volume per atom" statement? What is the representative value of the atomic volume and what is the (cube root of that) typical atomic separation?

3. The cross section for a nuclear reaction is determined by the QM tunneling, and is a function of v (or equivalently of energy E), both evaluated in the center of mass frame of the reacting particles A and B. Then it must be averaged over the Maxwellian velocity distribution.

$$< \sigma v > = 4\pi \; [\frac{m}{2\pi kT}]^{3/2} \int_0^\infty v \; \frac{S(E)}{E} \; exp[-\frac{mv^2}{2kT}] \; exp[-\frac{-2\pi Z_A Z_B e^2}{\hbar v}] \; v^2 dv$$

   Let us write $\sigma \; \propto \; exp(-b/\sqrt{E})$, where

$$b \; = \; \frac{\sqrt{2}\pi Z_A Z_B e^2 \sqrt{m_{AB}}}{\hbar} \; = \; 0.99 \; Z_A Z_B \sqrt{m_{AB}} \; (MeV)^{1/2}.$$

   (a) To do the integral, we replace this complex integral with the integral of a Gaussian. The Gaussian is centered at E_0, with a width Δ and an amplitude C, $g(E) = C \; exp[(E - E_0)^2/(\Delta/2)^2]$. Find the appropriate parameters of this Gaussian, C, E_0 and Δ, and do the integral.

   (b) Then define τ and derive the expression

$$< \sigma v > \propto \frac{1}{Z_A Z_B m} S_0 \, \tau^2 \, e^{-\tau}$$

4. Assume only the main pp chain and the CNO cycle main reactions are operating, and that they are at equilibrium. Use the estimate of the central temperature and central density of a star as a function of its mass obtained from the simple stellar model derived by assuming $\rho(r) = \rho_0(1 - r/R)$ you derived in the first problem of problem set 1.

   (a) Predict the total stellar energy generation rate in the center of the star via nuclear reactions as a function of stellar mass from 0.1 to 100 Solar masses.

   (b) Predict the fraction of energy coming from the pp chain versus that from the CNO cycle in the center of the star as a function of stellar mass.

5. We often desire to have the nuclear generation rate expressed as a power law, e.g. $\epsilon = \epsilon_0 \, \rho^n \, T^\eta$. Consider the reaction $p + p$. Write an expression for ϵ as a function of $< \sigma \, v >$. Use that to determine the value of n in this case. Now use the expression given in Q4 to find η in terms of τ, and hence the solar value.

   Energetic solar neutrinos are produced by the decay of B^8. Show that the rate of the relevant reaction $Be^7(p, \gamma)B^8$ is approximately proportional to T^{14} when the temperature is near 1.5×10^7K. If one attempted to explain the factor of $\simeq$4 discrepancy between the predicted neutrino rate and that observed in Davis' chlorine experiment by postulating an error in the central solar temperature, what change would be required?