Ay123

Fall 2013

STELLAR STRUCTURE AND EVOLUTION

Problem Set 4

Solutions due Monday November 18th 2013

1. 500 kms above the visible surface of the Sun, the temperature of the atmosphere reaches a minimum of 4170 K. Assuming μ=1.26 atomic mass units, find the scale height H and show that the critical frequency, ω_c, for pressure waves corresponds to a period of oscillation of about 3 minutes.

2. a. Show, from first principles, that the restoring force from buoyancy considerations for an element displaced upwards in stellar atmosphere is given by:

$$F = -\omega_{BV}^2 \, \Delta x = -\frac{g}{\rho}[\frac{\Delta\rho}{\Delta x} - \frac{\delta\rho}{\Delta x}]$$

where $\Delta\rho$ refers to the atmosphere and $\delta\rho$ to gas in the element, and ω_{BV} is the Brunt-Vaisala frequency.
b. Show that, for adiabatic motions,

$$\omega_{BV}^2 = -g[\frac{\gamma-1}{\gamma}\frac{1}{P}\frac{dP}{dx} - \frac{1}{T}\frac{dT}{dx}]$$

By considering the convective criterion discussed in class, show that gravity waves due to buoyancy oscillations cannot propagate in a convective region.
c. Estimate ω_{BV} and the corresponding period of oscillations for the terrestrial and Martian atmospheres. Indicate the assumptions being made. In the case of the Earth, estimate also the acoustic critical frequency ω_c and corresponding period. What happens to a terrestrial wave in the frequency interval inbetween ω_c and ω_{BC}.

NB: Most textbooks refer to ω_{BV} as N.

3. Assume the Sun is fully ionized and fully convective all the way to its surface.
a. Show that the sound speed close to the surface is given by $c^2 = (\gamma - 1)\, g\, z$ where $z = R - r << R$ is the distance from the surface.
b. For a chosen horizontal wavelength k_y, find the associated spherical harmonic l in terms of R and, for a given frequency ω, the maximum distance z_{max} an acoustic wave can penetrate into the star.
c. For a standing wave, show that

$$\int_0^{z_{max}} k_r \, dr = n\,\pi$$

Hence find a relation between n, l and ω.

d. Compare your result with the relevant plot in a review of helioseismology (see course web page or astro-ph/0207403). Check that the order of magnitude of your answer is correct and that the scaling with l, n is appropriate. Comment on any discrepancy. Print out the relevant figure and mark the value of n for a few of the curves shown.

4. The mass and mean radius of a typical Cepheid variable are given by log $M/M_\odot = 0.8$ and log $R/R_\odot$=1.4. Show that $\delta r/r_0$ = constant satisfies the equations derived in class for adiabatic stellar pulsations and find the period and surface velocity of the star. What is the range in effective temperature T_{eff} assuming $\delta r / r_0$=0.1?

5. Adopt a homologous model of a star in which radiation pressure is negligible and in which the CNO cycle supplies the energy. Indicating your assumptions, obtain a period - luminosity relation and specify a procedure for finding a value for the constant term.