Ay126: Master List of Questions for Final Exam

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a) Closed Book.
b) Constants are supplied (though you should make an attempt to memorize all constants listed at the end of this question paper). You are allowed to use calculators ranging from abacus to HP RPN and that on your lap top.
c) Ninety (90) minutes before your Zoom call in time I will email one of the sets of questions to you. Please join the Zoom call at the appointed time. The questions will span the course and will be a mix of short (factual) and long (you need to work through the question).
d) Following that you have 90 minutes to cogitate. I suggest that you write down your answers. Then you have to explain your answer on a piece of paper (which can be photos taken with your camera).
e) Please call in at your appointed time. The duration of the oral exam is 45 minutes. We need to go through all the five questions and so I will keep it moving along.
f) The Spirit: I am of the opinion that the oral exam has great pedagogical value and represents yet another opportunity to learn. It is best to over answer the question rather than stick to the apparent legal scope of the question. Thus when I say "please write down ..." then what I am really asking you is to explain your answer.

Suggestions for preparing for the Exam. The best way to prepare for the exam is to review the 18 lectures, read the notes and associated chapters (i.e. Draine's ISM book). I encourage you to discuss amongst yourselves, set up practice sessions and go through each and every question. This preparation alone will be a learning experience (you will get different perspective from your friends).

Constants: You are expected to know the following physical constants by heart: c, G, h, k_B, m_H (or inversely Avogadro's number), m_p/m_e. All other constants and parameters that you may possibly need are given in the Appendix.

You should be able to convert eV to temperature, wavelength, frequency and ergs. Thus $1\,\mathrm{eV}=1.6\times10^{-12}\,\mathrm{erg}$, $1\,\mathrm{eV}\rightarrow 12345\,\mathrm{Å}$ etc. and 1 eV (in ergs or Joules). It would be helpful to your own career that you have developed your own list of formulae that allow you to make estimates rapidly, e.g. the rms velocity of an H-atom is $\sqrt{T/121}\,\mathrm{km\,s^{-1}}$ and the gyro frequency is $3\,\mathrm{Hz}/\mu\mathrm{G}$, $10\,\mathrm{keV}/10^{12}\,\mathrm{G}$. In the same spirit I note: approximately $3\times10^{7}\,\mathrm{s}$

in a year, 84600 s in a day, 1 parsec=3.1×10^{18} cm, a radian is approximately 2×10^5 arc seconds and there are 40,000 square degrees in the sky.

Should you find yourself in the situation where you are not able to recall the value of a constant and the constant is not included at the end of this list then please make an educated guess (that is a part of research) and note that you made the same during the exam.

Formulae. I have provided what I think are arcane formulae. Review the Appendix before panicking.

Please do not discuss the details your exam (questions asked, in particular) with your colleagues. You are, however, welcome to discuss your experience in general terms. Feel free to mention how kind the examiner was!

Shortly after your oral exam, whilst both of our memories are fresh, I will send you a letter with my analysis of your answers and also the grade for the overall exam.

A. General Knowledge: ISM

1. What are the various phases or constituents of the ISM? Describe typical densities, temperature and global mass of each component.

2. What is the local energy density (ev/cm^3) in gas thermal motions, star light, magnetic field and cosmic rays?

3. What are the six most abundant elements in our Galaxy? What is the approximate ratio of these elements (hydrogen number density set to 1 for normalization)?

4. What are the sources and sinks of ISM in our galaxy? On what timescale do you expect the ISM to be completely consumed?

B. Atomic Spectroscopy

1. Consider D I (atomic deuterium). *Derive* the formula to compute to high precision the wavelength of the Balmer series. Compute that for Balmerα and compare that to Hα.

2. Derive and compute the wavelength of the Lyman alpha line of Fe XXVI in the context of the Bohr model. If you want a higher precision what corrections would you be making? Please comment on the expected splitting of the $n = 2$ state.

3. List the 10 most abundant elements. Write down their electronic configurations (e.g. Helium is $1s^2$). Which of them have first ionization potential higher than that of hydrogen.[1] In regard to this discussion what is so special about oxygen?

4. *Use the trick I mentioned in the class to determine the spectroscopic terms for the ground electronic configuration of O I. You will find three spectroscopic terms. Use Hund's rule to order them in energy.*

C. Radiative Transfer, A & B coefficients, Selection Effects

1. Using the selection rules given in §6.7 of Draine's book to classify the list of lines shown in Figure 1 as (1) *Permitted*, (2) *Inter-combination*, or *Forbidden* and give your reason.

(a) C III : $1s^22s2p\ ^3\mathrm{P}^o_1 \rightarrow 1s^22s^2\ ^1\mathrm{S}_0$ 1908.7 Å
(b) O III : $1s^22s^22p^2\ ^1\mathrm{D}_2 \rightarrow 1s^22s^22p^2\ ^3\mathrm{P}_2$ 5008.2 Å
(c) O III : $1s^22s^22p^2\ ^1\mathrm{S}_0 \rightarrow 1s^22s^22p^2\ ^1\mathrm{D}_2$ 4364.4 Å
(d) O III : $1s^22s2p^3\ ^5\mathrm{S}^o_2 \rightarrow 1s^22s^22p^2\ ^3\mathrm{P}_1$ 1660.8 Å
(e) O III : $1s^22s^22p^2\ ^3\mathrm{P}_1 \rightarrow 1s^22s^22p^2\ ^3\mathrm{P}_0$ 88.36 μm
(f) C IV : $1s^22p\ ^2\mathrm{P}^o_{3/2} \rightarrow 1s^22s\ ^2\mathrm{S}_{1/2}$ 1550.8 Å
(g) Ne II : $1s^22s^22p^5\ ^2\mathrm{P}^o_{1/2} \rightarrow 1s^22s^22p^5\ ^2\mathrm{P}^o_{3/2}$ 12.814 μm
(h) O I : $1s^22s^22p^33s\ ^3\mathrm{S}^o_1 \rightarrow 1s^22s^22p^4\ ^3\mathrm{P}_2$ 1302.2 Å

Figure 1: List of Lines.

2. *Explain the origin of the 1.4 GHz hyperfine line of H I. Compute the integrated absorption coefficient for density of 1 atom* cm^{-3} *and at temperature,* T*. The* A_{10} *coefficient for this line is* $2.88 \times 10^{-15}\,\mathrm{s}^{-1}$.

3. From which abundant atoms or ions do you expect to see hyperfine lines?

D. Warm Ionized Medium

1. Describe various observational probes of the Warm Ionized Medium.

2. *The sweep of the brightest FRB to date (Bochenek's STARE2 burst) in the frequency-time plane is displayed Figure 2. Assuming that the sweep is due dispersion in cold*

[1] group elements as follows: He, C, O, N (1:10^4) and group 2 (1:10^5).

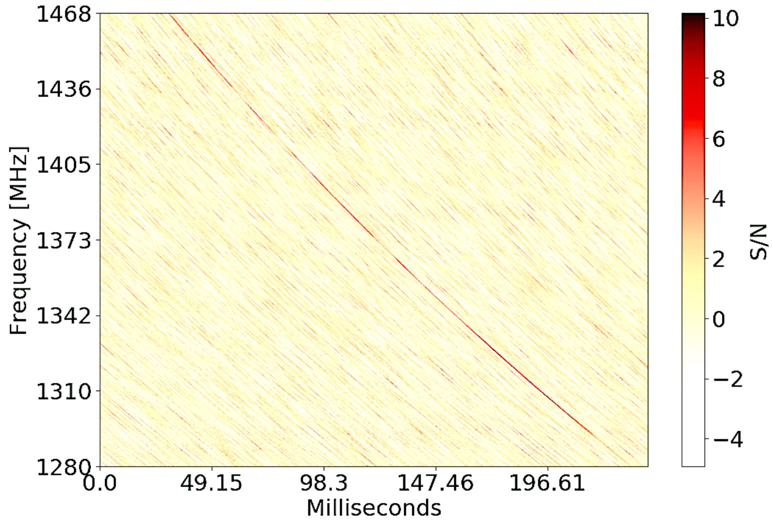

Figure 2: Sweep of Bochenek's STARE2 burst in the time-frequency plane.

plasma analytically derive the time difference between two frequencies. Apply your result to the Bochenek burst and derive the dispersion measure.

3. Consider a WIM sheet with the following physical parameters: temperature of 7500 K, hydrogen density, $n_H = 0.5\,\mathrm{cm}^{-3}$, essentially fully ionized. The sheet is perpendicular to the line of sight and has a thickness of 4 pc. Compute the surface brightness[2] for the resulting Hα and Hβ emission (units: $\mathrm{erg\,cm^{-2}\,s^{-1} steradian^{-1}}$).[3]

E. CNM and WNM

1. State (and be prepared to explain and as many as possible) methods by which the CNM and/or WNM can be studied.

2. Rydberg states refer to levels of an atom with energy quantum number greater than say 100. The transition of $n+1 \to n$ is called as "$n\alpha$". The HI 166α is a well known tracer. Compute the frequency for this transition. Next, the lowest frequency radio recombination line observed to date[4] is CI 843α. Can you comment on the density

[2] The usual unit for surface brightness is Rayleigh which is surface brightness but in units of $\mathrm{photons\,cm^{-2}\,s^{-1}\,steradian^{-1}}$. However, in this sub-field the unit Rayleigh which is $4\pi \times 10^{-6} I\,\mathrm{phot\,cm^{-2}\,s^{-1}}$ is frequently quoted.

[3] Necessary recombination coefficients are given in the Appendix. I have deliberately not given the recombination coefficient for Hβ.

[4] https://arxiv.org/pdf/1701.08802.pdf

of the region in which this line arises? [Bonus: What other physical effects may be important?]

3. *The 157 μm line is the fine structure line arising in the ground spectroscopic term of the CII ion. Write down the spectroscopic designations for the two levels. We will restrict to excitation by collisions with H (or He). For convenience we simplify and use the "hard sphere" approximation (!) and derive the de-excitation rate, k_{10}. Then derive the expression for n_1/n_0 where n_1 is the number density of C+ atoms in the excited state and n_0 in the lower state. Apply your formula for $T = 100\,K$ with $n_H = 100\,\mathrm{cm}^{-3}$ and $n_H = 10^3\,\mathrm{cm}^{-3}$.*
   You will find that the critical density is higher than that given in Draine's book. Explain why this is the case. Data: The A-coefficient for the line is $2.4 \times 10^{-6}\,\mathrm{s}^{-1}$. [See Appendix for helpful formulae].

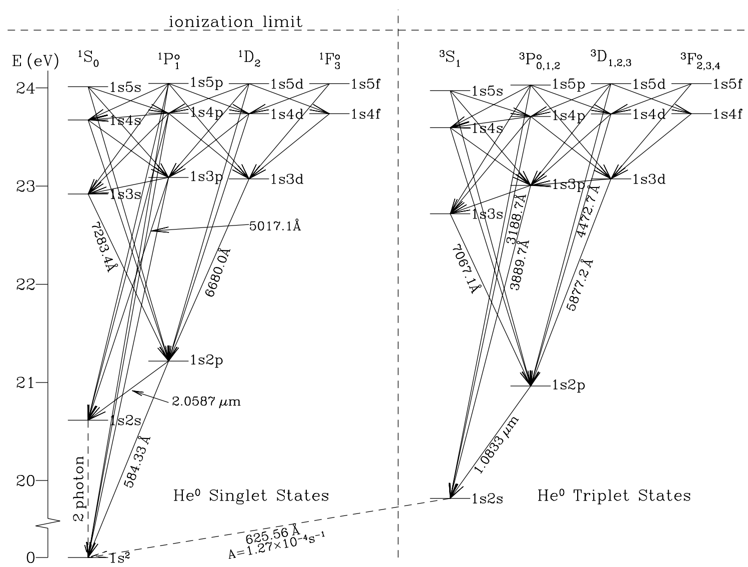

Figure 3: Grotrian diagram for Helium.

F. HII regions

1. *What are the primary features of the spectrum of an HII region (UV, optical, IR, radio). Explain which processes "cool" HII regions.* [This is an open ended question and so be prepared to go into some details with some specificity. Waxing eloquent should be avoided].

2. The peak emission measure of Messier 42 (aka The Orion Nebula) is $\mathrm{EM} = 5 \times 10^6\,\mathrm{cm}^{-6}\,\mathrm{pc}$. If M42 is approximated as a uniform density sphere of diameter 0.5 pc calculate the rate of H recombinations. At what (radio) frequency will M42 become optically thick?

3. *The Grotrian diagram for Helium is shown in Figure 3. Describe transitions into and out of* $1s2s^3S_1$ *state. Why is the* $1.0833\,mum$ *line so bright even at low densities and why does the line become brighter with increasing density and temperature.*

4. With reference to Figures 4 and 5 discuss how you can use the forbidden lines to infer the density and temperature of HII regions.

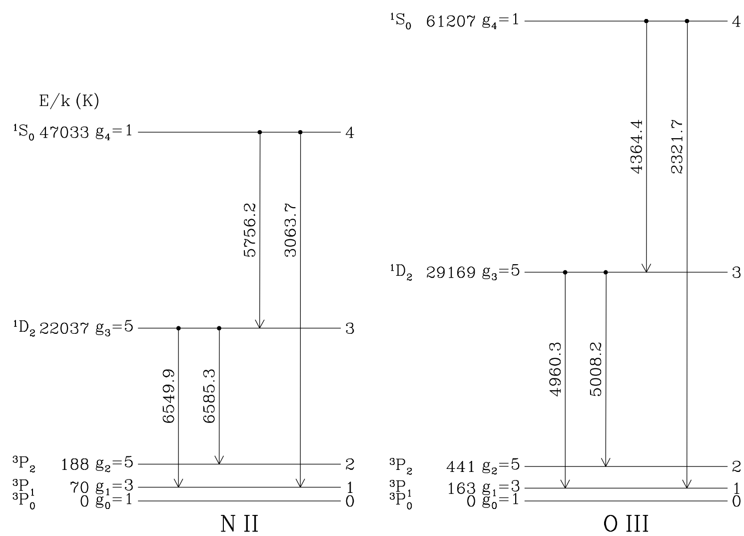

Figure 4: Grotrian diagram for NII and OIII.

G. BPT Diagram, Global Model, Local ISM

1. Explain the BPT diagram.

2. Review the cooling curve shown in Figure 7. Be prepared to walk me through the primary features of the curve (which cooling process/species dominate at various temperatures).

3. *Draw a reasonably quantitive diagram of the two phase model, marking the range of pressure over which the warm and cold phases can co-exist. What are the main cooling channels in this model?*

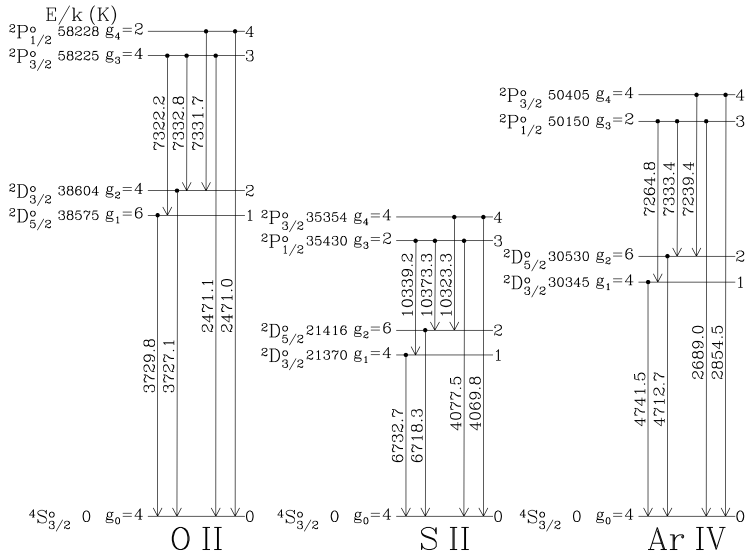

Figure 5: Grotrian diagram OII, SII, ArIV.

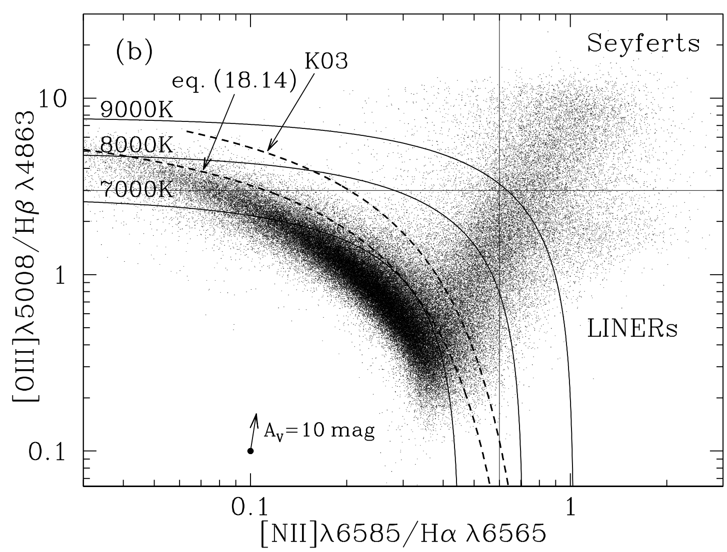

Figure 6: BPT diagram.

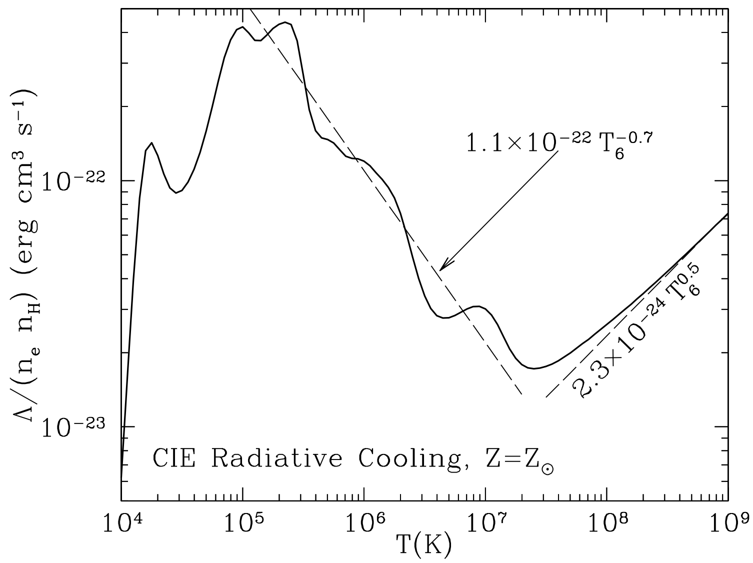

Figure 7: Cooling curve of interstellar plasma (from §34.1 of Draine's book).

4. *The solar system is moving, with a velocity of 24 km s^{-1}, into a WNM cloud: T=7000 K, $n_H = 0.22\,\mathrm{cm}^{-}$ and $n_e = 0.11\,\mathrm{cm}^{-3}$. These H atoms are detected by scattered solar Lyman-α photons (see Figure 8). It turns out that for H atoms the attractive gravitational force is almost exactly canceled by radiative force due to Lyα absorption. So the H atoms stream in essentially un-perturbed. At what distance (from the Sun) is optical depth of unity achieved for Lyman-α photons. The Sun also puts out Lyβ photons. What is the outcome of Lyβ photon absorptions? The H atoms are ionized at about 2 AU. What is the recombination time? [All data and equations necessary for this question can be found in the Appendix.]*

H. Molecules & Star-formation

1. List all known (and plausible) methods by which astronomers can probe molecular gas.

2. Why is CO the choice tracer of molecular gas? Why not H_2 or other oxides or hydrides or water?

3. *A popular formula for the line-width, σ_v, and sizes, L, of Giant Molecular Clouds (GMCs) is*

$$\sigma_v = 1.1 L_{\mathrm{pc}}^{\gamma}, \qquad \gamma \approx 0.38 \tag{1}$$

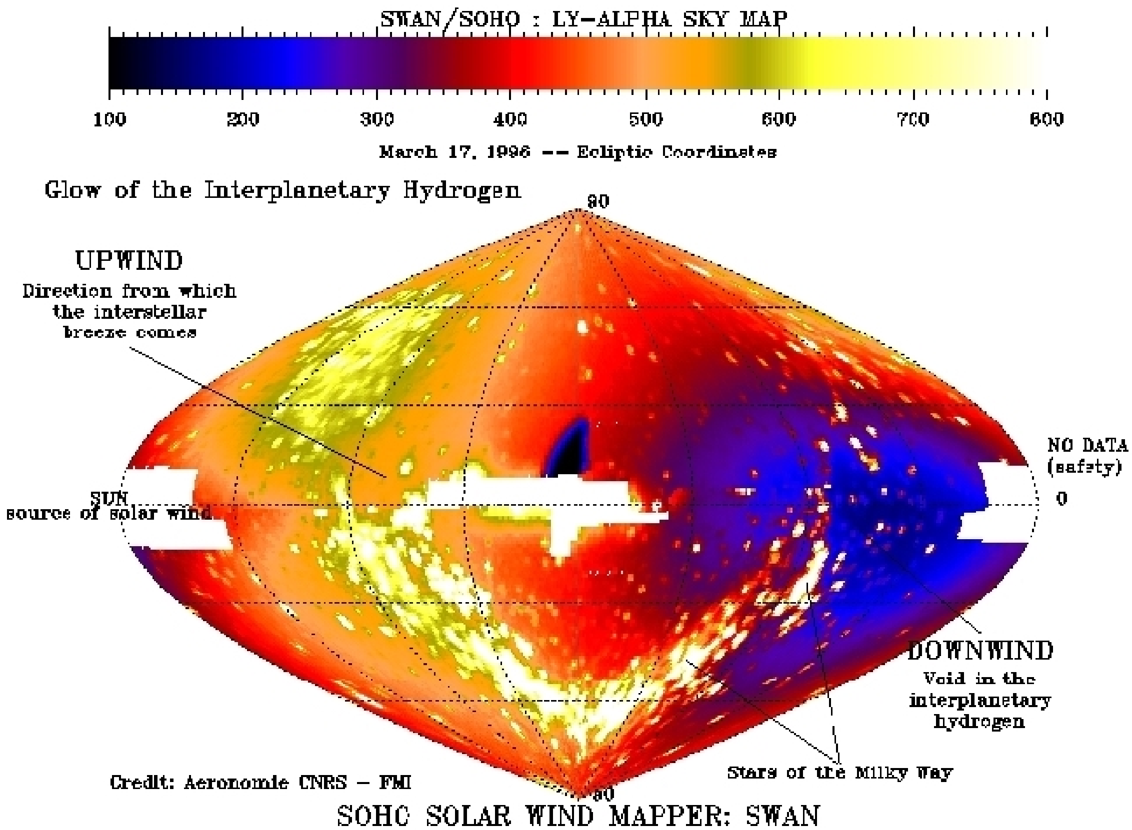

Figure 8: SWAN-SOHO map of solar Lyα photons.

over $0.1 \lesssim L_{\rm pc} \lesssim 10^2$ *(Draine, §32.9). For what* L *does the line width match the isothermal sound speed, assuming that the mean temperature of GMCs is 15 K. What other factors may become dynamically important for large GMCs?*

I. Dust

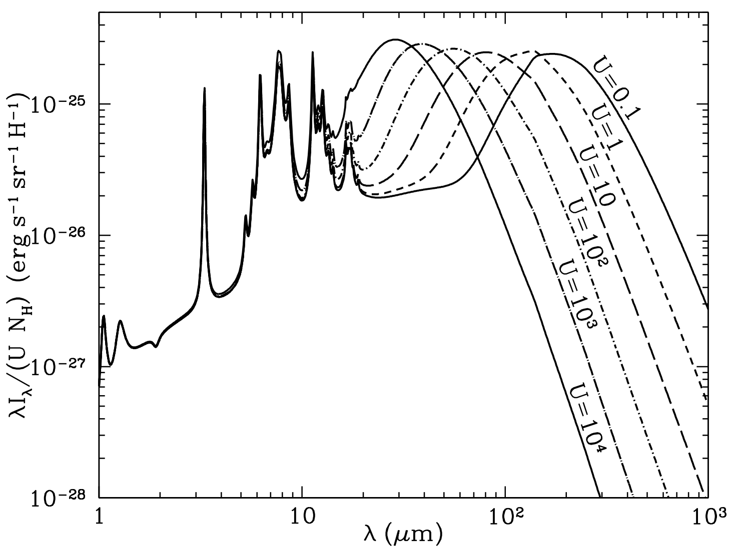

Figure 9: Infrared emission for a model with silicate and graphite+PAH grains in ISRF field. The ISRF intensity scale factor, U, is normalized to 1 for local value ("Habing"). (From Draine, Figure 24.7)

1. *List (and be prepared to discuss) the many diagnostics and tracers of dust.*

2. What clues and observations led astronomers to propose that Polycyclic Aromatic Hydrocarbons.

3. The mean radiation pressure cross-section acting on dust particle from the Interstellar Stellar Radiation Field (ISRF) locally, normalized by H atom, is $2.8 \times 10^{-22}\,{\rm cm}^2\,{\rm H}^{-1}$. Compare this cross-section to other possible channels: Thompson scattering of electrons and radiation acting on atoms and ions.

4. Explain Figure 9

Appendix: Constants ,Formulae & Parameters

$e = 4.8 \times 10^{-10}$ statcoulombs (CGS). In MKS, $e = 1.6 \times 10^{-19}$
$\mathrm{Ry} = R_\infty = 2\pi^2 e^4 m_e/h^3 \approx 13.6\,\mathrm{erg} \to 109,737.316\ \mathrm{cm}^{-1}$ (Rydberg)
$\sigma_T = (8\pi/3)r_e^2 = 0.66\,$barn (1 barn=$10^{-24}\,\mathrm{cm}^{-2}$; r_e is the electron radius)
$a_0 = 0.53\,$Å (Bohr radius, 1Å=$10^{-8}\,$cm)

parsec, $3 \times 10^{18}\,$cm
Mass of Sun, $M_\odot = 2 \times 10^{33}\,$gram
Radius of Sun, $R_\odot = 7 \times 10^{10}\,$cm (or 2 light seconds)
Luminosity of Sun, $L_\odot = 4 \times 10^{33}\,\mathrm{erg\,s^{-1}}$
Astronomical unit is 500 light seconds
Chandrashekar Mass, $1.4\,M_\odot$

The Gaussian 1-D velocity distribution is

$$\phi(v) = \frac{1}{\sigma_u\sqrt{2\pi}}\exp\Big(-\frac{u^2}{2\sigma^2}\Big) \tag{2}$$

The mean speed for a Maxwellian velocity distribution is $\sqrt{8k_BT/\pi\mu}$.
You may need this family of integrals

$$D = \int_0^\infty \frac{x^n dx}{\exp(x)-1}dx. \tag{3}$$

For $n = 3, 4, 5$ we have $D = 6.5, 24.9, 122$.
Einstein A and B coefficients, Oscillator Strength

$$B_{lu} = \frac{g_u}{g_l}B_{ul} \tag{4}$$

$$B_{ul} = \frac{c^3}{8\pi\nu^3}A_{ul} \tag{5}$$

$$\sigma_{lu}(\nu) = \frac{g_u}{g_l}\frac{c^2}{8\pi\nu_{lu}^2}A_{ul}\phi(\nu) \tag{6}$$

$$= \frac{e^2}{m_e c}f_{lu}\phi(\nu) \tag{7}$$

Here, $\sigma_{lu}(\nu)$ is the cross-section for photon of frequency ν to $\nu + d\nu$ to be absorbed by an atom with the resulting $l \to u$ transition, $\phi(\nu)$ is the normalized as $\int \phi(\nu)d\nu = 1$ and f_{lu} is the oscillator strength.

For a column with Gaussian velocity distribution, σ_v, the optical depth at the line center is

$$\begin{aligned} \tau_0 &= \sqrt{\pi}\frac{e^2}{m_e c}\frac{N_l f_{lu}\lambda_{lu}}{b} \\ &= 1.497 \times 10^{-2}\frac{\mathrm{cm}^2}{\mathrm{s}}\frac{N_l f_{lu}\lambda_{lu}}{b} \end{aligned} \tag{8}$$

where N_l is the column density and $b = \sqrt{2}\sigma_v$.

Photoelectric absorption

$$\sigma_{pi} \approx \sigma_0 \Big(\frac{h\nu}{Z^2 I_H}\Big)^{-3} \tag{9}$$

where $I_H = 13.6\,\mathrm{eV}$ is the ionization potential of Hydrogen and $\sigma_0 = 6.3 \times 10^{-18} Z^{-2}\,\mathrm{cm}^2$.

Recombination Coefficients: For temperature $[5, 10, 20] \times 10^3\,\mathrm{K}$:
$\alpha_A = [6.82, 4.18, 2.51] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case A (all levels)
$\alpha_B = [4.54, 2.59, 1.43] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case B (all levels)
$\alpha_{H\alpha} = [2.2, 1.17, 0.6] \times 10^{-13}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ Case B Recombination Hα Coefficient

Free-free absorption:

$$\frac{\kappa_{\mathrm{ff}}}{n_i n_e} = 1.1 \times 10^{-25} Z_i^{1.882} T_4^{-1.323} \nu_9^{-2.118}\,\mathrm{cm}^5 \tag{10}$$

Plasma Frequency:

$$\omega_p^2 = \frac{4\pi n_e e^2}{m_e} \tag{11}$$

Table 1: A-coefficients for $n = 2, 3$ lines of H

lower (l)	E_l cm^{-1}	upper, u	E_u cm^{-1}	A_{ul} (s^{-1})	trans
1s $^2S_{1/2}$	0	2p $^2P_{1/2}$	82,258.919	6.26×10^8	
1s $^2S_{1/2}$		2s $^2S_{1/2}$	8.954	2.5×10^{-6}	M1
1s $^2S_{1/2}$		2p $^2P_{3/2}$	9.285	6.26×10^8	
1s $^2S_{1/2}$		3p $^2P_{1/2}$	97,492.211	1.67×10^8	
1s $^2S_{1/2}$		3s $^2S_{1/2}$	2.221	1.11×10^{-6}	M1
1s $^2S_{1/2}$		3p $^2D_{3/2}$	2.319	5.94×10^2	E2
1s $^2S_{1/2}$		3p $^2P_{3/2}$	2.320	1.67×10^8	
1s $^2S_{1/2}$		3d $^2D_{5/2}$	2.356	5.94×10^2	E2
2p $^2P_{1/2}$	82,258.919	3d $^2D_{3/2}$	2.319	5.39×10^7	
2s $^2S_{1/2}$	8.954	3p $^2P_{3/2}$	2.320	2.25×10^7	
2p $^2P_{1/2}$	8.919	3s $^2S_{1/2}$	2.221	2.10×10^6	
2s $^2S_{1/2}$	8.954	3p $^2P_{1/2}$	2.211	2.25×10^7	
2p $^2P_{3/2}$	9.285	3d $^2D_{5/2}$	2.356	6.46×10^7	
2p $^2P_{3/2}$	9.285	3d $^2D_{3/2}$	2.319	1.08×10^7	
2p $^2P_{3/2}$	9.285	3s $^2S_{1/2}$	2.221	4.21×10^6	