Some general points: §6.3

$$f_{lu} \equiv \frac{m_e c}{\pi e^2} \int \sigma_{lu}(\nu)\, d\nu$$

$\uparrow$

oscillator strength. (dimensional-less)

$$A_{ul} = \frac{8\pi^2 e^2 \nu_{lu}^2}{m_e c^3} \frac{g_l}{g_u} f_{lu}$$

$$= \frac{0.667 \text{ cm}^2 \text{s}^{-1}}{\lambda_{lu}^2} \frac{g_l}{g_u} f_{lu}$$

unit of A_{ul} = second^{-1}.

ex:

A_{21} (Lyα)	0.4162
A_{31} (Lyβ)	0.079
A_{41} (Lyδ)	0.029
A_{32} (Hα)	0.6408
A_{42} (Hβ)	0.1193

Lyman-α: large cross-section
most number of atoms (H!)

$$\tau_\nu = \frac{\pi e^2}{m_e c} f_{lu} N_l \phi_\nu \left[1 - \frac{N_u/g_u}{N_l/g_l}\right]$$

f_{lu} ... oscillator strength

ϕ_ν ... velocity profile

Example: Lyman-α

τ_0 ... optical depth in the center of line

$$\tau_0 = 0.76 \left(\frac{N_H}{10^{13}\,\mathrm{cm}^{-2}}\right)\left(\frac{f_{lu}}{0.4164}\right)\left(\frac{\lambda_{lu}}{1216\,\mathrm{\AA}}\right) \frac{10\,\mathrm{km\,s^{-1}}}{b}$$

$$\tau_\nu = \tau_0 \exp\left(-\frac{u^2}{b^2}\right)$$

$$b = \sqrt{2}\,\sigma_v$$

$\uparrow$ RMS velocity

$$\tau_0 = 0.76\left(\frac{N_H}{10^{13}\,\mathrm{cm}}\right)\left(\frac{f_{lu}}{0.4164}\right)\left(\frac{\lambda_{lu}}{1216\,\text{Å}}\right)\frac{10\,\mathrm{kms}}{b}$$

Lyα: $A = 5\times10^{8}\,\mathrm{s}^{-1}$ $\therefore\ \Delta\nu = 5\times10^{8}\,\mathrm{Hz}$

$$\therefore\ \frac{\Delta\nu}{\nu_0} \simeq \frac{5\times10^{8}\,\mathrm{Hz}}{c/1216\,\text{Å}}$$

$$\Rightarrow\ \sigma_v\,(\text{intrinsic}) \simeq 0.06\,\mathrm{km/s}$$

$$\tau_0 \simeq 1.2\left(\frac{N_H}{10^{11}\,\mathrm{cm}^{-2}}\right)$$

Say $\sigma = \lambda_{lu}^2 \simeq 1.5\times10^{-10}\,\mathrm{cm}^2$

Allowed (strong) lines have

$\sigma \simeq \lambda^2$ at line-center.

Photo-electric absorption:

Hydrogen:

$$\sigma_{pi} \simeq \sigma_0 \left(\frac{h\nu}{Z^2 I_H} \right)^{-3}$$

Z .. atomic charge of nucleus

I_H .. ionization potential of Hydrogen

$\sigma_0 \simeq 6.3 \times 10^{-18}\ \mathrm{cm}^2$

Compare to Thompson cross-section:

"Lyman-thick"

$$\Longrightarrow \quad N_H \gtrsim 10^{18}\ \mathrm{cm}^{-2}$$

If a region is Lyman thick then

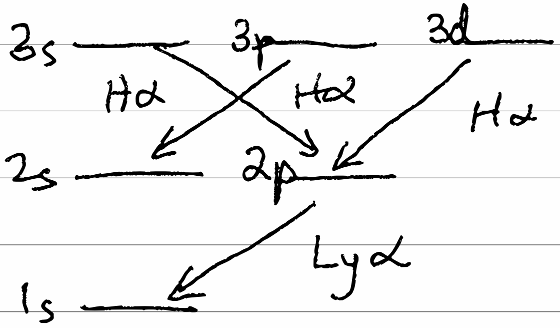
$Ly\alpha$, $Ly\beta$, .. will rattle around!

...destruction by dust!

Q:

Hydrogen:

§14



Case A: Lyα escapes from region
ex. fast shocks, low column density
high temperature

Recombinations to all ~~states~~ levels

$$\alpha_A = \sum_{n=1}^{\infty} \alpha_{nl}(T)$$

Recomb per unit volume per unit second

$$\alpha_A n_e n_p$$

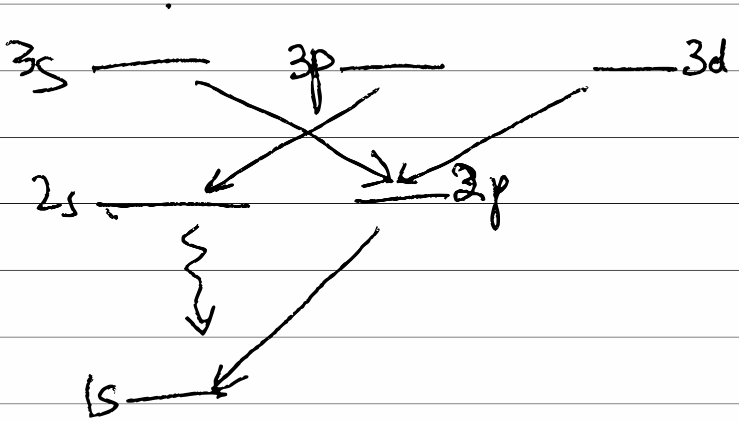
case B:

Lyα rattles around

"on the spot absorption"

$$\alpha_B = \sum_{n=2}^{\infty} \alpha_{n\ell}(T)$$

Two-Photon Decay



$A(2s \to 1s) \simeq 8\,s^{-1}$.

So can have collisions and push H into 2p state.

$$n_{e,\mathrm{critical}} = \frac{A_{2s\to 1s}}{q_{p\,2s\to 2p} + q_{e\,2s\to 2p}}$$

$q_{p\,2s\to 2p}$: proton collision

$q_{e\,2s\to 2p}$: electron collision

$$j_\nu = \frac{n_e n_p \alpha_{2s}}{\left(1 + \frac{n_e}{n_{\mathrm{crit}}}\right)} \cdot h\nu \cdot \frac{1}{4\pi} \underset{\uparrow}{P_\nu}$$

two-photon profile

Heliom:

$$He^{++} + e^- \rightarrow He^+$$

This is hydrogenic

"Lyα" = $\frac{1216}{4}$ Å

Lyman edge = $\frac{912}{4}$ Å

- Helium Lyman series can ionize H
- However photoelectric cross-section is much smaller than resonance line transitions
- So you see line spectrum but 4 times higher in frequency relative to H
- $2s \rightarrow 1s$ $A = 527\ s^{-1}$ (Z^6!)
  $\Rightarrow$ ionize H atoms

Locations: Planetary nebula, Wolf Rayet stars

$He^+ + e^- \rightarrow He^0$

$\alpha_{1s^2}(He)$ ... recombination ~~to~~ directly to ground state

This will generate >24.6 eV which can ionize either HI or HeI.

$\alpha_B(He)$ ... recombination to any state but $1s^2$

$\alpha_{1s_2} \simeq 1.54 \times 10^{-13}\ cm^3 s^{-1}$ $T = 10^4 K$

$\alpha_B \simeq 2.7 \times 10^{-13}\ cm^3 s^{-1}$ $T = 10^4 K$

If case B holds then recombinations to ground state can ionize HeI

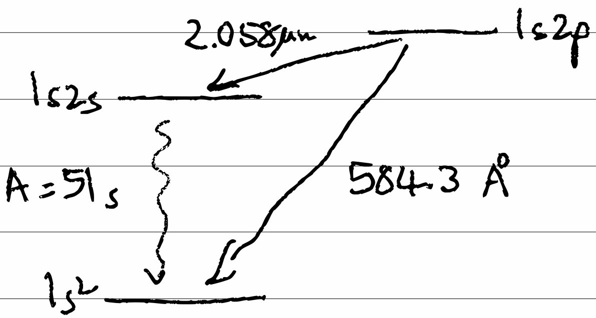
$$\alpha_{eff}(He) = \alpha_B(He) + y\,\alpha_{1s_2}(He)$$

↑

fraction of H ionization

Recombination to levels other than $1s^2$

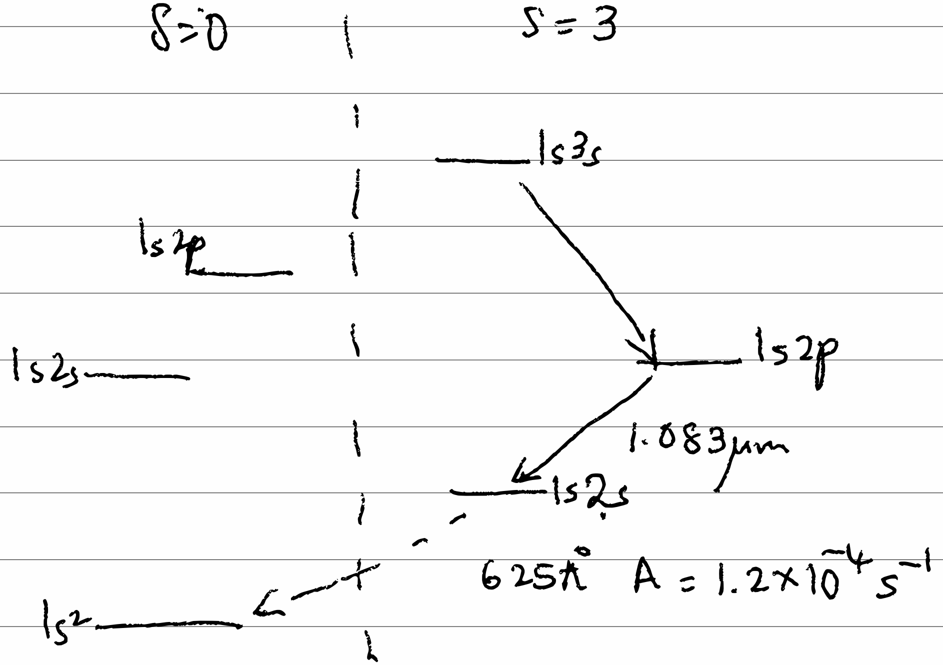
25% of recombination is to singlet
75% of recombination is to triplet



$A(1s\,2p \rightarrow 1s2s) = 1.97\times10^6\ s^{-1}$
$A(1s\,2p \rightarrow 1s^2) = 1.8\times10^9\ s^{-1}$

The λ584.3 Å is absorbed, being resonant, rapidly until it dies by hydrogen ionization or destruction by dust. or by two photon decay

Recombination to triplet series:



The 1s2s S=3 state is meta-stable.

The long duration allows collisional excitations to
1s2p (triplet), 1s3s (triplet)
1s2p (siglet), 1s 2s (siglet), 1s 2p (singlet)

HeI 1.0833 μm ⇒ diagnostic of temperature
& density