Ay126: Homework 1

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Spirit: A student can make a homework educational and interesting or tedious. Your choice. An ideal homework has three goals: (1) Pedagogy (application of the concepts taught in the class to data with the expectation that by completing the problem set the student gains confidence). (2) Whet the the student's curiosity in matters beyond that discussed in the class. (3) In the process of devising the homework the instructor acquires deeper understanding of the subject.[1]

Due: COB, April 16, 2021

I. Fraunhofer lines.[2] Please browse around for a pictorial spectrum obtained by Joseph von Fraunhofer. Research and link each line to a specific transition(s) .e.g. D_1, D_2 would be due to resonance line of neutral sodium, $1s^2 2s^2 2p^6 (3s\ ^2S_{1/2} \rightarrow 3p\ ^2P_{1/2,3/2})$. [10 points]

II. Trick but Educational Question. The recent CODATA[3] for the Rydberg constant, $R_\infty = 10\ 973\ 731.568\ 160(21)\,\mathrm{m}^{-1}$. Given this value, compute the expected wavelength of the Hα and Hβ lines. Astronomers routinely[4] quote 6563 Å (Hα), 4861 Å (Hβ), 4341 Å (Hγ), and 4102 Å (Hδ); for example, see Table 3 of F. Sidoli et al. (2006). Please reconcile your *ab initio* calculation with the routine quoted values. [10 points]

III. Zeeman splitting. From basic physics derive the formula for splitting of the $2p$ electron of a H-like atom under the influence of an external magnetic field (in Hz). Compute the expected splitting of the Balmer Hα absorption line ($2s \rightarrow 3p$), assuming $B = 5\,\mathrm{kG}$ which is typical of the magnetic field strength of so-called Ap stars. [10+5 points]

[1] which is why, as Feynman remarks in his biography, better research is done at teaching+research institutions as opposed to purely research enterprises.

[2] If you wish to be a dedicated researcher then you need to have wide knowledge.

[3] Every four years, the Committee on Data for Science and Technology (CODATA) issues recommended values of the fundamental physical constants. The values are determined by a least-squares adjustment, based on all the available theoretical and experimental information.

[4] For example, it is common in several observatories to use 6563 for number locks.

IV Dirac Solution for Hydrogen Atom. The exact eigenvalues[5] for the Hydrogen-like atoms of the Dirac equation are given by

$$E_{njlm} = \mu c^2 \left[1 + \left(\frac{Z\alpha}{n - (j+1/2) + \sqrt{(j+1/2)^2 - (Z\alpha)^2}} \right)^2 \right] \quad (1)$$

where mu is the reduced mass, $n = 1, 2, ..$ is the principal energy quantum number and $j + 1/2 \leq n$.

a) Undertake an expansion in $Z\alpha$ and connect the resulting terms to the Bohr values, the relativistic and spin-orbit coupling terms. [10 points]

b) Estimate the relativistic and spin-orbit coupling energy terms (in eV), relative to the Bohr values, for FeXXVI. [10 points]

V. Radio Recombination Lines (RRL).[7] LOFAR is a new facility in radio astronomy operating in the 10–240 MHz. RRL arise when a free electron combines with an ion. The RRL designation is as follows: H109α is the line that results when an electron combines with a proton and cascades from Rydberg energy level $n = 110$ to $n = 109$ whereas H137β is 139→137 (and so on).

Asgekar et al. (2013), using LOFAR, report the following lines Carbon RRL towards Cas A: C548α and C518α (amongst other lines). Compute the rest frequencies of these lines (accurately). Next, why is that only Carbon RRL lines are detected from the cold/warm netural medium (CNM/WNM) but not that from H, He, O and N? [5+5 points]

References

Sidoli, F. et al., The massive star population in the giant HII region Tol89 in NGC5398, MNRAS 370, 799 (2006)

A. Asgekar et al., LOFAR detections of low-frequency radio recombination lines towards Cassiopeia A, A & A 551, L11 (2013)

D. Morton, Atomic Data for Resonance Absorption Lines. ApJ Suppl. 77, 119 (1991)

[5]ps. (1) Notice that the solution fails when $Z\alpha > 1$. Fortunately, $Z = 118$ for Oganesson.[6] (2) The Dirac energy levels only depend on j. However, there is a small energy difference between $2s_{1/2}$ and $2p_{1/2}$ (both of which have the same j value). This shift was first measured by Lamb and Retherford and can only be explained when the electric field is quantized (Quantum Electrodynamics).

[7]IMNHO the field of RRL is enjoying a renaissance. Lots of old issues can be revisited with LOFAR and MWA.