Collisional Processes (Chapter 2, App. H, I)

$$A + B \rightarrow \text{collision}$$

reaction rate per unit volume

$$= n_A n_B \langle \sigma_V \rangle_{AB}$$

$$\langle \sigma v \rangle_{AB} \equiv \int_0^\infty \sigma_{AB}(v)\, v\, f(v)\, dv \qquad cm^3 s^{-1}$$

v is the relative velocity.

$$\mu = \frac{m_A m_B}{m_A + m_B} \qquad \text{Energy in center-of-mass} = \frac{1}{2}\mu v^2$$

$$f_v = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\mu v^2/2kT}\, v^2\, dv$$

Using energy as the parameter

$$\langle \sigma v \rangle_{AB} = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E)\, \frac{E}{kT}\, e^{-E/kT}\, \frac{dE}{kT}$$

In most instances, for ISM, we will only consider 2-body interactions.

Four types of collisions

- Long-range Coulomb interaction

$$\phi \propto \frac{1}{r} \quad \text{(potential)}$$

ex. ions-ions, ions-electrons, electrons-electrons

- intermediate range Induced-dipole reactions

$$\phi \propto \frac{1}{r^4}$$

ex. ions and neutral.

- electron and neutrals

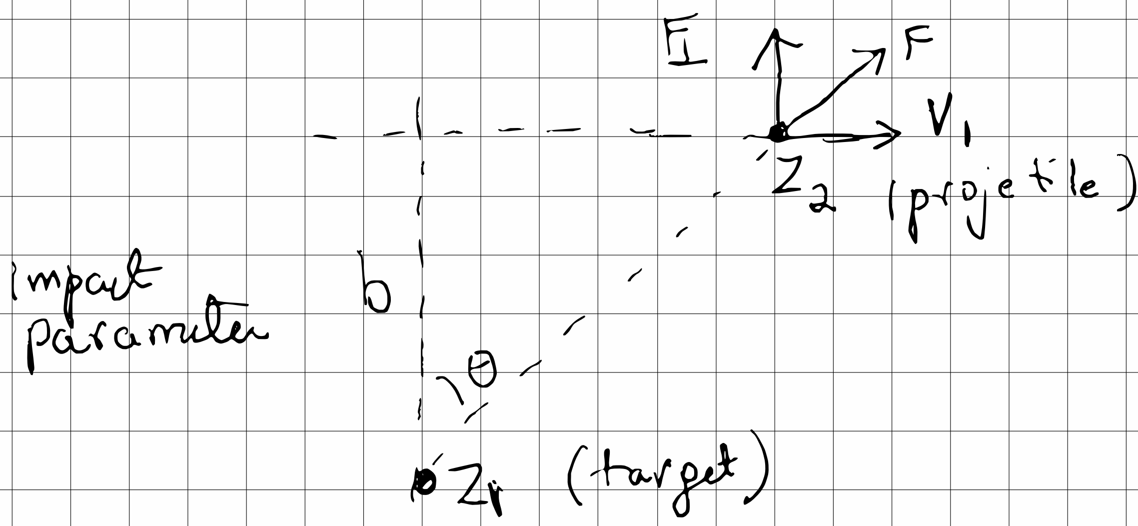
use experimental data

- short range (neutral, neutral)

van der Waals forces

approximated by "hard spheres"

Inverse square law: scattering & ionization



Impact approximation:

Assume projectile & target velocities remain constant throughout encounter

$$F_\perp = \frac{Z_1 Z_2 e^2}{(b/\cos\theta)^2}$$

$$\Delta p_\perp = \int_{-\infty}^{\infty} F_\perp \, dt = \frac{Z_1 Z_2 e^2}{b v_1} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= 2 \frac{Z_1 Z_2 e^2}{b v_1}$$

Collisional Ionization:

Consider the case of an electron moving much faster than the speed of a bound electron. In this approximation the assumptions are satisfied.

$$v \gg \left(\frac{2I}{m_e}\right)^{1/2} \qquad I = \text{ionization potential}$$

For ionization

$$\frac{1}{2m_e}\left(\Delta p_\perp\right)^2 > I$$

$$\therefore \quad \sigma(v) \simeq \pi b^2 \simeq \frac{2\pi Z_p^2 e^4}{m_e v^2 I}$$

Only electrons with $v > v_{min} = \left(\frac{2I}{m_e}\right)^{1/2}$ can ionize (have sufficient energy)

$$\langle \sigma v \rangle = \int_{v_{min}}^{\infty} \sigma(v)\, v\, dv$$

$$= Z_p^2 \left(\frac{8\pi}{m_e kT}\right)^{1/2} \frac{e^4}{I}\, e^{-I/kT}$$

The "size" of atoms lie between 0.5 Å ~ 3 Å. However, for the field of Radio Recombination Lines (RRL) the size can approach a mm!

These are H atoms in high state.

ex. H92α

The interaction cross-section ~~varies~~ is large for such puffy HI atoms.

Scattering and Gradual Losses:

Projective: $Z_1 e$; velocity, v_1

Targets: $Z_2 e$, stationary

We use the impulse approximation. The projectile receives $\Delta p_\perp^2$ on each flyby.

$$\left\langle \frac{d}{dt}\left[(\Delta p_\perp)^2\right]\right\rangle = \underbrace{\int_{b_{min}}^{b_{max}} 2\pi b\, db\; n_2 v_1}_{\text{event rate}} \underbrace{\left[\frac{2 Z_1 Z_2 e^2}{b v_1}\right]^2}_{(\Delta p_\perp)^2}$$

$$= \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \int_{b_{min}}^{b_{max}} \frac{db}{b}$$

Clearly b_{min} is given by

$$\frac{Z_1 Z_2 e^2}{b_{min}} = E = \frac{1}{2} m_1 v_1^2$$

For b_{max} we use Debye length

$$L_D = \left(\frac{kT}{4\pi n_e e^2}\right)^{1/2} = 690\,\text{cm} \left(\frac{n_e}{\text{cm}^{-3}}\right)^{-1/2} \left(\frac{T}{10^4\,\text{K}}\right)^{1/2}$$

Thus

$$\left\langle \frac{d}{dt} (\Delta p_\perp)^2 \right\rangle = \frac{8\pi n_2 Z_1^2 Z_2^2 e^4}{v_1} \ln \Lambda$$

$$\Lambda = \frac{b_{max}}{b_{min}} = 4\times10^9 \left(\frac{E}{kT}\right)\left(\frac{T}{10^4 K}\right)^{3/2} \left(\frac{cm^{-3}}{n_e}\right)^{1/2}$$

$\ln\Lambda \simeq 20-35$ for ISM conditions

The weak distant encounters appear to dominate.

The timescale for energy loss for the electron is

$$t_{loss} = -\frac{E}{\langle dE/dt \rangle}$$

$$= \frac{m_1 v_1^2}{\left\langle \frac{d}{dt} \Delta p_\perp^2 \right\rangle / m_2} = \frac{m_2 m_1 v_1^2}{\langle d/dt (\Delta p_\perp)^2 \rangle}$$

$$t_{loss}(e^- \text{ to } p^+) = 1.4\times10^7 s \; T_4^{3/2} \left(\frac{cm^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

The time scale for deflection by 90°

$$t_{defl} = \frac{(m_1 v_1)^2}{\left(\frac{d}{dt} \Delta p_\perp^2\right)}$$

$$t(e^- \text{ by } p^+) = 7.6\times10^3 s \; T_4^{3/2} \left(\frac{cm^{-3}}{n_e}\right)\left(\frac{25}{\ln\Lambda}\right)$$

These two timescales define how rapidly electrons isotropize and thermalize.

This is a big issue for young shocks at low densities..

~~At~~ In this limit of distant encounters dominating over nearby encounters the assumptions we made are equally good.

Neutral-neutral collisions

van der Waals forces are dipole-dipole interactions

$$\phi \propto r^{-6}$$

The "size" of atoms varies from $0.5\,\mathring{A}$ (HI) to a few $\mathring{A}$ (high Z elements)

Hard sphere collisions require $b < R_1 + R_2$

$$\therefore \quad \sigma = \pi (R_1 + R_2)^2 \simeq 1.2 \times 10^{-15}\ \mathrm{cm}^2$$

$$\langle \sigma v \rangle = \left(\frac{8kT}{\mu}\right)^{1/2} \pi (R_1 + R_2)^2$$

$$= 1.8 \times 10^{-10}\, T_2^{1/2} \left(\frac{m_H}{\mu}\right)^{1/2} \left(\frac{R_1 + R_2}{\mathring{A}}\right)^2 \mathrm{cm}^3\,\mathrm{s}^{-1}$$

At low temperatures (CNM, GMC.) the neutral-neutral interactions are $\gtrsim$ order of magnitude smaller than ion-neutral interactions.

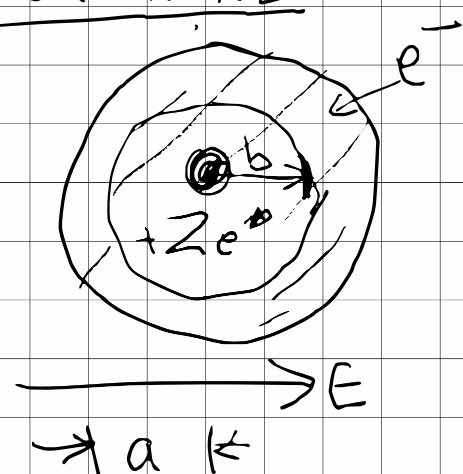
ion-neutral collisions:

The Coulomb interactions are ~~drive~~ dominated by distant encounters.
The neutral-neutral interactions are dominated by geometrical sizes.

The ion-neutral collisions are in between

Essentially, the electric field of the ion induces a dipole in the neutral atom. The resulting dipole interacts with electric field and thereby increases the cross-section of the interaction.

BACKGROUND:



Electric field at radius b

$$= e\left(\frac{b^3}{a^3}\right)\frac{1}{b^2}$$

$$= \frac{e\,b}{a^3}$$

This is matched by external field

$$E = \frac{eb}{a^3} \Rightarrow b = \frac{a^3 E}{e}$$

~~De~~ The dipole moment is

$$p = \cancel{e}b = a^3 E$$

$$\therefore \quad \vec{p} = \alpha_N \vec{E}$$

atomic polarizability

For Hydrogen $\alpha_N = a_0^3$. An exact QM calculation shows $\alpha_N = \left(9/2\right) a_0^3$.

Element	H	He	Li	Be	C	Ne	Na	A	K
α_N $(10^{-24}\,cm^3)$	0.67	0.21	12	9.3	1.5	0.4	27	1.6	34

Recapitulation

For a dipole

$$\phi = \frac{\hat{r}\cdot\vec{p}}{r^2} \qquad \text{potential}$$

Dipoles only feel gradients in electric field

$$\vec{F} = \vec{p}\cdot\nabla E$$

Consider an ion of charge Z. Let it induce dipole moment $\vec{p}$

$$F_r = p \cdot \frac{dE}{dr} = -2\alpha_N \frac{Z^2 e^2}{r^5}$$

The corresponding potential is

$$U(r) = -\frac{1}{2}\alpha_N \frac{Z^2 e^2}{r^4}$$

Define:

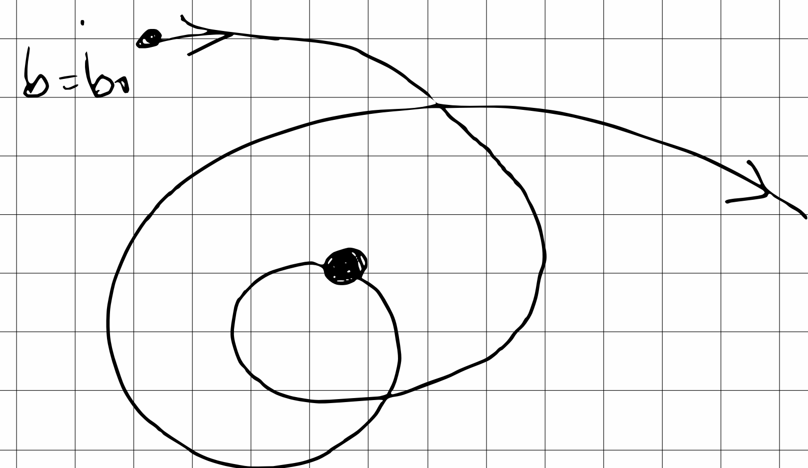
$$b_0 \equiv \left(\frac{2\alpha_N Z^2 e^2}{E_{cm}}\right)^{\frac{1}{4}}$$

where $E_{cm} = \frac{1}{2}\mu v^2$ is the center-of-mass energy

We find

$$U(b_0) = -\frac{E_{cm}}{4}$$

For $b = b_0$ the projectile smashes into the target. (potential overcomes the ~~pa~~ projectile)



$$\sigma_{orb} = \pi b_0^2 = 2\pi Ze \left(\frac{\alpha_N}{\mu}\right)^{1/2} \frac{1}{v}$$

The rate coefficient is

$$\langle \sigma v \rangle_{orb} = 9 \times 10^{-10} Z \left(\frac{\alpha_N}{a_0^3}\right)^{1/2} \left(\frac{m_H}{\mu}\right)^{1/2} \mathrm{cm}^3 \mathrm{s}^{-1}$$

Note the lack of dependence on temperature, T.
If the resulting reaction is exo-thermic then the reaction can proceed at very low temperatures.

The ion-neutral reactions are critical to organic chemistry in the ISM.

$$O^+ + H \longrightarrow O(^3P_0) + H^+ \quad \text{"charge exchange"}$$

$$H_2^+ + H \longrightarrow H_3^+ + H \quad \text{"exchange"}$$

$$CH^+ + H_2 \rightarrow CH_2^+ + H \quad \text{"exchange"}$$

The above reactions are exothermic

Electron-neutral collisions

These are measured experimentally. Only in the simplest cases is there a QM calculation.

Electron-ion inelastic collisions

(Draine Appendix I)

1. Consider ion in excited state.
2. Consider low velocity electron incident on it
3. Electron attracted by ion
4. Approaches orbital speed at close encounter

A. $$\frac{1}{2} m_e v_{max}^2 = \frac{1}{2} m_e v^2 + \frac{Ze^2}{r_{min}}$$

B. $$v_{max} r_{min} = v b \quad \text{(angular momentum)}$$

Thus impact parameter is

$$b = r_{min} \left[1 + \frac{Ze^2/r_{min}}{m_e v^2/2} \right]^{1/2}$$

<u>Model:</u> If $r_{min} < W a_0$ then ion is de-excited

so critical impact parameter is

$$b_{crit}(v) = W a_0 \left[1 + \frac{Ze^2/W a_0}{m_e v^2/2} \right]$$

$$\sigma_{ul} = \pi b_{crit}^2$$

$$\langle \sigma v \rangle_{u \to l} = \int_0^{\infty} 4\pi \left(\frac{\mu}{2\pi kT} \right)^{3/2} e^{-\mu v^2/2kT} \, v^2 \, \sigma_{ul} \, dv$$

$$= \pi W^2 a_0^2 \left(\frac{8kT}{\pi m_e} \right)^{1/2} \left[1 + \frac{Ze^2}{W a_0 kT} \right]$$

$$\frac{Ze^2/a_0}{kT} = \frac{15.8 Z}{T_4}$$

For $T_4 \lesssim 1$

$$\langle \sigma v \rangle_{u \to l} = \frac{h^2}{(2\pi m_e)^{3/2}} \frac{1}{(kT)^{1/2}} \underbrace{2WZ}_{\Omega_{ul}/g_u}$$

Ω_{ul} = "collisional strength"

$\Omega_{ul} \simeq 1$ for $Z = 1$