

Ay 1: The Evolving Universe

Assignment 1

Association of TAs to take over the world

May 6, 2010

Stellar Properties

The star ϵ Aurigae has a parallax of 0.31 arcsec and its emission peaks at a wavelength of 604 nm. This star's apparent brightness is 6.73×10^{-13} times that of the sun.

Note: All stars in this problem are purely imaginary, and any resemblance with the real star named ϵ Aurigae in the real sky is purely coincidental.

1) Assuming the radius of the earth's orbit about the sun is 1.5×10^8 km, find the distance to this star.

Solution: The distance to the star is given by the formula,

$$\begin{aligned}d &= \frac{1 \text{ pc}}{\text{parallax (in arcsec)}} \\ &= \frac{1}{0.31} \text{ pc} \\ &= 3.23 \text{ pc} \\ &= 9.97 \times 10^{16} \text{ m}\end{aligned}\tag{1}$$

2) Find the luminosity of this star (in the units of L_{sun}).

Solution: Using the inverse square law, we can calculate the luminosity as follows:

$$\frac{B}{B_{\text{sun}}} = \frac{L}{4\pi d^2} \times \frac{4\pi d_{\text{sun}}^2}{L_{\text{sun}}}\tag{2}$$

where B refers to the brightness as seen at earth (in units W/m^2), L is the total power emitted by the star, and d is the distance between the star and us. We can thus calculate the star's luminosity from given numbers:

$$\begin{aligned}L &= L_{\text{sun}} \times \frac{B}{B_{\text{sun}}} \times \frac{d^2}{d_{\text{sun}}^2} \\ &= L_{\text{sun}} \times 6.73 \times 10^{-13} \times \frac{(9.97 \times 10^{16})^2}{(1.5 \times 10^{11})^2} \\ &= 0.3 L_{\text{sun}}\end{aligned}\tag{3}$$

3) Estimate the surface temperature of the star assuming the emitted spectrum has a blackbody distribution.

Solution: We know that the emission from the star peaks at 604 nm. So we can use Wein's law to calculate the surface temperature of the star:

$$T = \frac{0.0029}{\lambda_{\max}} \quad (4)$$

$$= 4800 \text{ K} \quad (5)$$

4) Estimate the radius of the star relative to the radius of the sun ($R_{\text{sun}} = 6 \times 10^5 \text{ km}$).

Solution: We know that the total luminosity L of the star is related to its radius R and surface temperature T (assuming a perfect blackbody) by:

$$L = 4\pi R^2 \sigma T^4 \quad (6)$$

Comparing with the sun, we get:

$$\frac{L}{L_{\text{sun}}} = \left(\frac{R}{R_{\text{sun}}}\right)^2 \times \left(\frac{T}{T_{\text{sun}}}\right)^4 \quad (7)$$

Hence,

$$R = R_{\text{sun}} \sqrt{\frac{L}{L_{\text{sun}}} \times \left(\frac{T_{\text{sun}}}{T}\right)^4} \quad (8)$$

$$= R_{\text{sun}} \sqrt{0.3 \times \left(\frac{5777}{4800}\right)^4}$$

$$= 0.79 R_{\text{sun}} \quad (9)$$

(Note: The actual radius of the sun is $7 \times 10^8 \text{ m}$, but we had a typo in the problem set and gave the value $6 \times 10^8 \text{ m}$. We have used the same value in the solution set. Once our association takes over the world, we plan to shrink the sun to match this value.)

5) Draw an Hertzsprung-Russel diagram with axis values, main sequence and red giants and label the position of ϵ Aurigae.

Solution: The star sits smack on the main sequence ! (See for example the HR diagram in lecture 6).

Evolution of More Massive Stars

6) The central temperature of the sun is about $12 \times 10^6 \text{ K}$ and the core of the sun is burning H. Helium (He), on the other hand, requires much higher temperatures $\sim 100 \times 10^6 \text{ K}$. Based on the properties of main-sequence stars and the scalings derived in lecture, estimate the mass (in solar units) required for a star to be able to burn helium in its center. Approximately how much more luminous is this star than the sun?

Solution: Lets begin with the expression that relates pressure P at the center of the star to its mass M , radius R and temperature T :

$$P = \frac{GM\rho}{R} = nkT \quad (10)$$

Now, $\rho = n \cdot \mu$ where μ is the mean atomic weight at the center. The core is mostly Helium, so $\mu = 4 \times 1.67 \times 10^{-27}$ Kg. Thus we get,

$$T = \frac{GM\mu}{Rk} \quad (11)$$

Note that as derived in class, we have for main sequence stars:

$$T \propto \frac{M}{R} \quad (12)$$

To eliminate radius from the relation, we use the mass–radius relationship (See lecture 6):

$$\frac{R}{r_{\text{sun}}} = \left(\frac{M}{M_{\text{sun}}} \right)^{0.5} \quad (13)$$

Combining Equations 11 and 13 we get,

$$\frac{T}{T_{\text{sun}}} = \sqrt{\frac{M}{M_{\text{sun}}}} \times \frac{\mu}{\mu_{\text{sun}}} \quad (14)$$

$$\begin{aligned} M &= M_{\text{sun}} \times \left(\frac{T}{T_{\text{sun}}} \right)^2 \times \left(\frac{\mu_{\text{sun}}}{\mu} \right)^2 \\ &= M_{\text{sun}} \times \left(\frac{100 \times 10^6 \text{ K}}{12 \times 10^6 \text{ K}} \right)^2 \times \left(\frac{1}{4} \right)^2 \\ &\sim 4 M_{\text{sun}} \\ &\sim 8.6 \times 10^{30} \text{ Kg} \end{aligned} \quad (15)$$

To calculate luminosity, we use the mass–luminosity relation:

$$L = L_{\text{sun}} \times \left(\frac{M}{M_{\text{sun}}} \right)^{3.5} \quad (16)$$

$$\begin{aligned} &\sim 128 \times L_{\text{sun}} \\ &\sim 5 \times 10^{28} \text{ W} \end{aligned} \quad (17)$$

Note: if assume μ doesn't change:

$$M = 69 M_{\text{sun}} \quad (18)$$

$$L = 2.8 \times 10^6 L_{\text{sun}} \quad (19)$$

Sizes of Earth-Moon-Sun System

7) Use geometry, your understanding of gravity and your impressive creativity to devise techniques to accurately measure one of the quantities below. Most importantly, you should devise a technique which minimizes the errors or uncertainties in the results and estimate the accuracy of the result i.e. simply saying you will measure the size of the earth by walking or sailing around the globe and dividing by 2π is not sufficient.) Also, you should not simply look up standard techniques in a book or on the internet - but instead discuss creative (and hopefully new) schemes with your fellow students. (Your solution should not involve leaving earth or sending a spacecraft from Earth but you can use modern communications and measurement tools on the earth with self-generated, reasonable estimates of accuracy of these tools. Include illustrations as necessary.)

DO ONLY ONE OF a, b, or c !!

a) Describe a technique to measure the diameter of the Earth

Solution: There are many ways to measure the size of the Earth. One of the simplest methods is to assume that the Earth is spherical and measure the surface distance between two points. By then finding the angular distance between those two points, one can use geometry to find the radius of the Earth.

How would one find the angular distance between two points? One easy way is to ensure that your two points are not displaced in the east-west direction. Then the angular difference between two points is simply the latitude distance and can be measured by measuring the positions of astronomical bodies. Preferrably, one will measure the zenith angle of a source as it passes through the meridian.

If we assume we can measure the elevation of the star to 1" and we measure the distance between the two locations on the Earth down to 100 m, then our uncertainty in the angle is $\frac{\Delta\theta}{\theta} \simeq 10^{-3}\%$ and in distance is $\frac{\Delta d}{d} \simeq 1\%$. The radius measurement is then dominated by the distance measurement and will come out as roughly 1%. Note that as we improve our distance measurement, we'll eventually be limited by the asphericity of the globe.

b) Describe a technique to measure the diameter of the Moon

Solution: The Moon's angular size is easily measured. Finding the physical size of the body, however, also requires the distance to the Moon. The distance can be found via a parallax measurement. This requires having two observers at different positions on the Earth's surface. Both observers will measure the position of the Moon on the celestial sphere. These positions will be slightly different due to the parallax of the situation. Knowing the projected separation between the two points and the resulting angular change in position of the Moon allows for a calculation of the distance to the Moon. This is then used with the angular size to find the actual size.

Knowing *a priori* that the Moon is 3.6×10^{10} cm away and the diameter of the Earth is 1.3×10^9 cm, the expected parallax is

$$\pi = \frac{1.3}{36} = 1.5 \text{ deg} \quad (20)$$

Hence if we measure the parallax to one arcsecond, then our measurement is good to 0.02%.

c) Describe a technique to measure the distance to the Sun

Solution: The first method to make this measurement involved using the transit of Venus to measure the scale of the Solar System. The method I will list is slightly more subtle than that.

As the Earth orbits the Sun and rotates about its own axis, a person sitting the equator will feel a centrifugal force. When on the day side, this will add to the Earth's gravity and make the person feel heavier. When on the night side, this will subtract from the gravity and make the person feel heavier. By measuring the gravity very sensitively, one will measure a diurnal modulation in the gravity:

$$\Delta g = 2 \frac{v^2}{a} \quad (21)$$

Now, this measurement does not give the orbital radius directly, but is degenerate with a measurement of the orbital speed. We can break this degeneracy, however, by including the length of the year in our calculation. Doing so gives:

$$T = \frac{2\pi a}{v} \quad (22)$$

so

$$a = \frac{\Delta g T^2}{8\pi^2} \quad (23)$$

As we know the length of the year to fairly high accuracy, it will be the uncertainty in Δg which limits this measurement. Knowing *a priori* that the distance to the Sun is 1.5×10^{11} m, we find $\Delta g = 0.012 \text{ m/s}^2$. This is 0.1% of g , so we would need to measure the weight of objects to a thousandth of a percent to get a one percent error in a .