Candidacy exam summary

Margaret Pan
Advisor Re'em Sari

1 Introduction

During the past two years I've been working on problems related to the dynamics and population evolution of small bodies in the solar system including, in particular, Kuiper belt objects (KBOs) and comets. These bodies are interesting and important to our understanding of planetary systems for several reasons:

- Small, cold bodies like comets and KBOs are some of the least processed bodies in the solar system: they were never subject to the heat or pressure associated with the accretion and differentiation of the major planets. As such, comets and KBOs are relics of the early solar system; understanding their size distribution and composition may provide clues to early solar system composition and the accretion process.
- While the planets were forming, numerous small bodies provided a mass reservoir for accretion and constrained the protoplanets' velocity evolution. Since small bodies now contain much less mass than the planets and are concentrated in just a few regions, most of the small bodies must have been removed. Small-body reservoirs like the Kuiper belt represent the tail end of the removal process; comets were removed from areas near the giant planets. So the Kuiper belt and Oort cloud may shed light on the final stages of planet formation and the fate of the absent planetesimals and dust.
- Interactions between small bodies and the giant planets should have caused the giant planets' orbits to migrate and evolve. Analysis of small-body orbits may yield a picture of how and why the planets attained their current marginally stable configuration and why this configuration looks so different from other known planet systems.
- All of the above also apply to the formation and evolution of extrasolar planet systems.

The following summarizes the work I've been doing and outlines a few projects on related topics that I plan to investigate in the future.

2 Generalized Lagrange points and resonances for highly eccentric orbits

The Lagrange points are well-known fixed points in the dynamics of a test particle moving in the same plane as a massive planet in a circular orbit around a much heavier star. Associated with the two stable Lagrange points are families of orbits known as 'tadpoles' and 'horseshoes'. The Trojan asteroids, ∼1700 objects in two groups which lead and trail Jupiter in its orbit, are the best-known examples of objects in tadpole orbits. While these standard Lagrange points, tadpoles, and horseshoes correspond to low-eccentricity

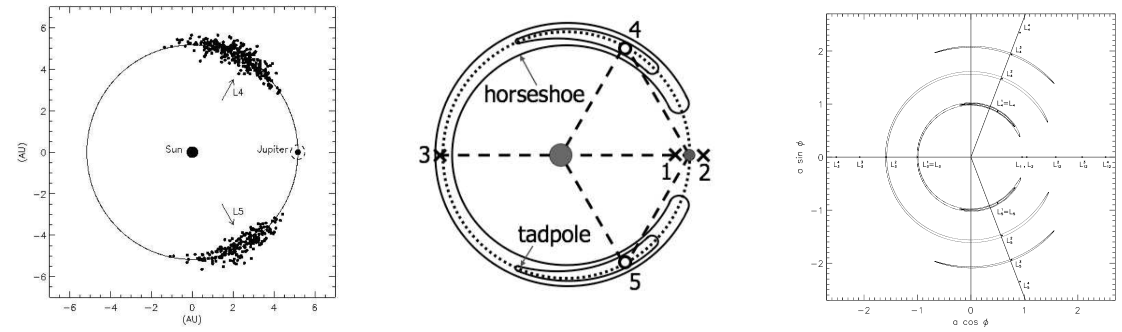

Figure 1: Illustrations of the Lagrange points. The center panel is a schematic of the five standard Lagrange points with examples of tadpole and horseshoe orbits. The left panel is a plot of the Trojan asteroids—objects in tadpole orbits—relative to Jupiter and the sun (taken from Scott Sheppard's web site). The right panel shows the first few Lagrange point analogues with some tadpole and horseshoe analogues.

orbits, we found an infinite number of Lagrange point analogues and tadpole and horseshoe analogues that are associated with test particles in highly eccentric long-period orbits. We developed a framework for the evolution of high-eccentricity long-period test particle orbits and used it to explore mean-motion resonances between a particle and the massive bodies.

Our framework might be applied to the origins of long-period comets and the structure of Oort cloud. This population is believed to have formed from small bodies originally on roughly circular orbits within Neptune's orbit; these were kicked by the giant planets into very eccentric orbits ∼10,000 AU in size which were then circularized by galactic tides and passing stars. One surprising result of our work is that for sufficiently small planets (planet/star mass ratio $\mu < \sim 5 \times 10^{-6}$), a test particle initially close to the planet cannot escape from the system.

Briefly, other results of this work include 1) a condition $a \sim \mu^{-2/5}$ for the onset of chaos at large semimajor axis a; 2) a redefinition of resonance orders for the high-eccentricity regime in which a $p : p + q$ resonance is called 'pth order' instead of the usual 'qth order' to reflect the importance of interactions at periapse; and 3) a simple explanation for the presence of asymmetric librations in exterior $1 : N$ resonances and the absence of these librations in other exterior resonances.

3 Size spectrum of Kuiper belt objects

The observed size distribution of Kuiper belt objects (KBOs)—small icy and rocky solar system bodies orbiting beyond Neptune—is well described by a power law at large KBO sizes. However, recent work by Bernstein et al. (2003, astro-ph 0308467) indicates that the size distribution breaks and becomes shallower for KBOs smaller than about 70 km in size. We showed that we expect such a break at KBO radius ∼40 km since destructive collisions are frequent for smaller KBOs. Specifically, we assumed that KBOs are gravity-dominated bodies with negligible material strength. This gave a power-law slope $q \simeq 3$ where the number $N_{>r}$ of KBOs larger than a size r is given by $N_{>r} \propto r^{1-q}$. The break location followed from this slope through a self-consistent calculation: while bodies smaller than the size at the break are effectively in collisional equilibrium, bodies larger than the break size have never undergone catastrophic collisions. The existence of this break, the break's location, and the power-law slope we expect below the break are consistent with the findings of Bernstein et al. (2003). The agreement with observations indicates that KBOs as small as ∼40 km are effectively strengthless.

4 Plane of the Kuiper belt

With Mike Brown I did some work on measuring the effective plane of the Kuiper belt. Although Kuiper belt surveys tend to assume the highest concentration of KBOs is at the ecliptic plane, there is no reason to expect that the earth and the Kuiper belt would share the exact same plane. Indeed, the measured plane of the Kuiper belt is inconsistent with the invariable plane and the planes of Jupiter and Neptune at the greater than 3σ level. Linear secular perturbation theory implies that the plane of the Kuiper belt should oscillate about the position of the invariable plane. Specifically, while Neptune has much more gravitational influence over the Kuiper belt plane than any other single planet, Neptune's orbit plane oscillates about the position of the invariable plane due to secular interactions with the other planets. Neptune's oscillations act as a periodic driving force on the Kuiper belt plane. The position predicted by secular theory for the plane of the Kuiper belt is within 1σ of our measured position.

5 Future projects

A few projects which grew out of the above and which I plan to investigate in future are outlined below.

Escaping planets The relatively high eccentricities often seen in the orbits of extrasolar planets were completely unexpected when the first ones were found. None of the standard planet-formation scenarios had a robust way to create such eccentric orbits: planets accreted from particles in a disk should have formed with very small eccentricities. Though these eccentricities' origin remains unresolved today, current work in this area typically involves either interactions between a planet and the disk it formed from or interactions between multiple planets in which the lightest planet is ejected from the system.

Our framework for studying particles in eccentric orbits provides a natural starting point for an investigation into two-planet interactions and the process of ejection. For example, we can apply this framework to investigate the chances that the lighter planet—idealized in our framework as a test particle—is ejected or collides with the star or the second planet. We found during our investigation of Lagrange point analogues that when only the lowest-order terms in the interactions are included, an eccentric particle not in resonance with the planet should therefore random walk in semimajor axis, resulting in diffusion in orbit size. However, particles take so long to random-walk to escape that higher-order terms in the kick interactions become important; they lead to a systematic drift in orbit size. The problem is then one of a random walk (lowest-order interaction) on a gentle slope (higher-order interaction) with a cliff on either end (ejection/collision). This is closely related to the 'gambler's ruin' problem in probability, which I plan to investigate and apply to the particle/planet interactions.

Very eccentric planet in a disk Some highly eccentric exoplanets may have coexisted and interacted with the debris disks from which they formed. However, the interactions between a planet with eccentricity $e \sim 1$ and a disk are not well understood, so their long-term effects are largely unknown. An investigation into this topic would also be relevant for disks surrounding other eccentric binaries such as binary stars or binary black holes.

Interactions between a disk and a planet with small e have been studied extensively. The standard approach involves expanding the interaction in a Fourier series and including only those terms which rotate either with the average angular velocity of the planet or slightly faster or slower; these terms are the largest because they are proportional to small powers of e. Unfortunately, when e is large this approach is ineffective: an expansion in powers of the eccentricity converges slowly if at all, so very many terms must be included. I plan to study the $e \simeq 1$ case. At this high-eccentricity extreme a different set of terms in the expansion may emerge as most important; the new classification of resonances introduced in the work on Lagrange point analogues may be relevant. I plan to compare this analysis with numerical simulations of disks being done by Milos Milosavljevic.

Dust in the solar system In principle, the collisional cascade which currently determines the size spectrum of Kuiper belt objects could continue to arbitrarily small KBO sizes. However, a naive extrapolation of the collisional cascade to micron-sized dust particles in the Kuiper belt gives an unrealistically large number of particles. IRAS and possibly ISO should have seen infrared background emission from a dust population this large. Also, this amount of dust implies an optical depth to dust in the Kuiper belt alone of about 2 orders of magnitude larger than the observed optical depth of the zodiacal light.

It seems that either some dust has disappeared or the dust is unexpectedly dim. Work on this topic might shed light on some longstanding problems in solar system formation. Young stars often exhibit large (several hundred AU), optically thick gas/dust/debris disks while planetary systems seem to contain very little free dust; where does all the dust go, and on what timescale does it vanish? After planets finish accreting from a circumstellar disk, what happens to the remaining unaccreted small bodies? I've begun to look into possible mechanisms for eliminating dust, including solar photons, or radiation pressure, Poynting-Robertson drag, and the Yarkovsky effect; solar wind effects; and cosmic ray impacts. I've also been investigating the effects of a cutoff in the number of bodies (due to elimination of dust) on the collisional cascade operating on larger bodies.

I plan to graduate in the summer of 2006 after having worked on these and other projects.