AY 105 Lab Experiment #3: Fundamentals of spectroscopy

This week you will set up a spectrograph on the optical table and use it to visually resolve the D-lines of neutral sodium (NaI $D_1 = 5895.93$ Å, ${}^2P_{1/2} -{}^2 S_{1/2}$; and $D_2 = 5889.96$ Å, ${}^2P_{3/2} -{}^2 S_{1/2}$). You will also iinvestigate how spectral resolution depends upon such spectrograph design parameters as grating (or prism) dispersion, input slit width, camera EFL, camera-collimator angle, etc.

1 Diffraction grating spectrograph

Configuration. You need to set up on your optical table a Na arc lamp light source (powered by a high voltage supply), opal glass diffuser, entrance slit, and Newtonian reflecting collimator (45° flat mirror and paraboloid mirror). This setup should produce a collimated beam parallel to the long axis of the optical table, with the front portion of the table left empty to accommodate the camera axis of the spectrograph. Opposite the collimator needs to be a diffraction grating assembly. Sketch the configuration in your notebook, and record the collimating mirror focal length, grating grooves/mm and blaze angle θ_B. Mark on your sketch the grating blaze direction; is this the correct orientation to use if the camera is to be located on the front of the table? (See the instructor or T.A. if the grating blaze direction isn't what you expect). What blaze wavelength, λ_B, do you derive for your grating from the grating equation,

$$\frac{m\lambda}{a} = (\sin\alpha + \sin\beta)\cos\gamma, \tag{1}$$

evaluated in first-order Littrow configuration ($m = 1$; $\alpha = \beta = \theta_B$; and $\gamma = 0$)? Is this reasonable? In which order would you expect the NaD lines to be brightest with this grating? When the group working at the other table has derived λ_B for their grating, compare your results with theirs and comment on any differences in your lab notebook.

Turn on the spectral line lamp power supply and let the Na arc lamp warm up. Meanwhile, you will need to position your camera lens on the front portion of the optical table and align its optical axis with the center of the grating. Keep the collimator axis to camera axis angle at the grating small, but do not obstruct any part of the collimator beam to the grating with the camera lens. Turn the micrometer to a reading of zero to reveal the two bolt holes in the bottom (stationary) portion of the translation stage. Since the tapped holes in the table comprise a rectangular grid, you will only be able to use one bolt hole in the translation stage. You may use the slotted post-holder "bases" as stops to prevent the translation stage from rotating around the single bolt attaching it to the optical table.

Determine the angle (which in lecture we called 2θ) between the collimator and camera axes as viewed from the grating. Next, turn the knob which rotates the grating so that the grating is normal to the collimator axis (if you are using the "cylindrical" grating mount; $\alpha = 0$) or to the camera axis (if you are using the "rectangular" grating mount, pre-loaded by an external spring; $\beta = 0$). Record the counter reading at this position. With this counter reading as the zero-point, you can use the counter to measure the grating angle α (or β). As a working hypothesis, assume the counter on the cylindrical mount reads α in units of arcminutes; check this by rotating the grating until the grating is normal to the camera axis ($\beta_c = 0$), and compare α at this setting to the trigonometrically derived 2θ from above. Is the agreement consistent with the hypothesis that the counter reads in arcminutes? Note that the two least significant digits on the cylindrical grating rotator range from 00 - 59 (and so the two most significant digits on this unit are then in degrees if indeed the two least significant digits correspond to arcminutes). In the case of the rectangular grating mount, adopt as a working hypothesis a conversion factor of -280 counts per degree for angle β.

What relationship between α and β does Equation 1 predict for the zeroth order ($m = 0$) from the diffraction grating? Given that the geometry of the spectrograph fixes $\alpha - \beta_c = 2\theta$, what values of α and β_c will produce a zeroth order image in your spectrograph? Calculate the grating counter setting corresponding to this value of α (cylindrical) or β_c (rectangular) and if possible rotate the grating accordingly. Position the micrometer eyepiece on-axis behind the camera lens and find a location to bolt it in place (make sure the optical axis of the eyepiece is parallel to the optical table, and coaxial with the camera lens axis). With the lens barrel focus set at ∞ translate the camera lens along its axis using the micrometer until a focus is achieved (the front section of the eyepiece should penetrate the rear opening of the lens at the ∞ focus setting). Fine tune first the eyepiece alignment and possibly also the grating rotation to center the zeroth order image horizontally in the eyepiece. Record the grating rotation setting in your notebook, and use any offset from your prediction to revise your value of θ.

Substitute $\alpha - 2\theta = \beta_c$, (the value of β at the center of the camera field of view) for β in Equation 1 and solve for α in terms of the other variables. The trigonometric identity

$$\sin A + \sin B = 2 \sin \frac{A + B}{2} \cos \frac{A - B}{2} \tag{2}$$

may help you simplify your solution analytically. Evaluate your solution to find α for the NaD lines in first, second, and third orders, and tabulate these angles (both in degrees and in grating counter units) in your lab notebook for future reference. Rotate the grating to the first order α setting and confirm that the $m = 1$ Na spectrum appears in the eyepiece.

Method. Return to Equation 1 and solve it for $\sin \beta$, the sine of the angle between the grating normal and the outgoing diffracted rays as a function of λ, m, $a \cos \gamma$, and the angle of incoming rays α. Take the partial derivative of $\sin \beta$ with respect to λ, and show that substitution from the grating equation and elimination of α leads to the following expression for the dispersive power (i.e., the angular change in the direction of outgoing rays with a change in wavelength $\Delta\lambda/\lambda$) of a diffraction grating:

$$\lambda\frac{\partial\beta}{\partial\lambda} = 2\tan\beta_c\cos^2\theta + \sin 2\theta. \tag{3}$$

The trigonometric identities

$$\begin{aligned} \sin(A+B) &= \sin A\cos B + \cos A\sin B \qquad (4)\\ \sin A\cos B &= \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B) \qquad (5)\end{aligned}$$

may be helpful in deriving this expression (you can work on the derivation at home later if it doesn't work out for you right away, so don't spend more than five minutes of lab time working on it). Evaluate Equation 3 to find the dispersive power of the grating in your spectrograph. Apply this result to determine the angular separation $\Delta\beta$ of the NaI D_1 and D_2 spectral lines in orders $m = 0, 1, 2, 3$ in your spectrograph.

Next write down the expression relating the linear width of the slit Δw to its angular size $\Delta\alpha$ in the collimator. Calculate what slit width in microns corresponds to $\Delta\alpha = \Delta\beta$ for the NaD lines in first order. Set the slit width micrometer first to twice this amount, then equal to this amount, half this amount, then finally one quarter this amount and record in your lab notebook the appearance (intensity versus relative position) of the yellow spectral lines seen in the eyepiece at each slit width setting.

Once each partner has made these observations, set the slit width back to an amount corresponding to the angular separation $\Delta\beta/2$ of the D lines (they should then appear separated by a gap of one individual line-width). Use the eyepiece micrometer to measure the linear separation Δx between the D1 and D2 line centers. Calculate the observed value of $\Delta\beta$ from the measured Δx and the effective focal length of the camera lens, and compare to the $\Delta\beta$ predicted from Equation 3.

At this point you should exchange camera lenses with the other group (skip to the next paragraph if you need to wait for the other group to be ready for the trade, and come back to this paragraph when they've reached this point). Simply uncouple the lens from the bayonet mounting ring on the aluminum plate. Using their camera lens, record the appearance of the spectral lines for the current slit width setting (which gave D lines separated by one line-width with your original camera lens). Record the EFL' of the new camera lens, and use the micrometer eyepiece to measure $\Delta x'$ then recompute the observed value of $\Delta\beta'$. Trade camera lenses back again so you have your original lens.

If you rotate the grating so that α decreases, will the wavelengths seen by the camera eyepiece increase or decrease according to Equation 1? Test your assertion by slowly rotating the grating while your partner watches through the eyepiece until another spectral line comes into view; is it bluer or redder in appearance than the yellow D lines? Adjust the grating α setting so that the line appears centered in the eyepiece, and record the grating tilt counter reading in your lab notebook. If there is still time before the other group is ready to exchange camera lenses for the previous paragraph, record the grating tilt counter readings for other spectral lines you can find in the first order (you are moving into second order when blue lines appear where redder ones were expected; stop and go back if you haven't yet measured all the first order lines). If you still have time, you can work more on the derivation of Equation 3 or continue measuring grating tilts for lines in the second order.

Set the grating rotation to the α angle at which you predicted the NaD lines would appear in second order. Adjust α if necessary to bring the D lines into view. Record the appearance of the lines, paying particular attention to differences between this pair and the first-order D lines. Are the two D lines in second order still separated by one line width? Measure the separation Δx between the two line centers with the micrometer eyepiece. Repeat this procedure for $m = 3$.

Analysis. Complete the derivation of Equation 3 if time did not permit you to do so during lab, and likewise if time did not permit convert your Δx data in the focal plane of the camera lens to corresponding $\Delta\beta$ values and compare these to predictions using Equation 3. Convert the grating rotation counter readings you took for other lines in the first order Na spectrum into α value, and then from Equation 1 determine the wavelengths of the lines. Do the same for any Δx or grating tilt data you have for second order.

Discussion. Comment on the appearance of the NaD lines in first order when the slit width was varied from twice to one quarter of the D line spacing. How narrow a slit was needed to resolve (i.e., separate) the pair of lines? Did the resolution continue to improve as the slit width became narrower? What limiting factor was reached, or would be reached eventually? Short of such a limit, is a narrower slit always better? (think of the astronomical application where only a small number of photons arrive from a distant and faint galaxy—is a narrower slit better in this instance? If not, what is optimum?).

What difference did borrowing the other group's camera lens make in your spectrograph? How did the ratio of D line width to D line spacing change? What does this imply about the resolution, and the factor which was limiting the resolution of your spectrograph? If you were designing a spectrograph to work with a CCD detector array in the focal plane of the camera lens (where the micrometer eyepiece reticle and crosshair were), what difference would camera EFL make and how would you determine the most appropriate camera EFL to use? (In the CCD data you still wish to distinguish two lines at the limiting spectral resolution).

What happened to the D line width to line spacing ratio when the order number was increased from $m = 1$ to $m = 2$ and 3? What was happening meanwhile to the grating dispersive power defined by Equation 3? Reconsider the CCD spectrograph design question from the previous paragraph: if you ultimately wanted no more resolution than was necessary to resolve the NaD lines, would there be any advantage at all to be gained from increasing the dispersion of the grating? (Don't forget about the faint galaxy target from two paragraphs above. Is there an optimum grating dispersion to use?)

2 Prism spectrograph

Configuration. Your optics drawer contains a 30° − 60° − 90° prism, 60 mm in size, made of Schott glass SF2 (one group will work with a newer, smaller rism, but it is the angles which define its properties!). You will find the diffraction grating assembly on the optical table replaced by a shelf on which you can set one of the triangular prism faces. For the first part of this lab, you should position the prism on the shelf so that the 60 × 60 mm square prism face (along the longer leg of the triangular face, not the hypotenuse) is normal to the optical axis of the collimator, and the apex of the prism is closest to the back wall. Turn on the Na lamp power supply, and center the prism in the collimator beam (using a large slit width will produce a yellow beam which you can see in a darkened room). Note that light from the collimator then enters the prism normally, and no dispersion occurs at the prism entrance face. Calculate the angle at which light is incident upon the exit face of the prism, and for $\lambda = 5893$ Å (prism index $n_D = 1.64752$) what the angle of refraction will be when the light emerges from glass into air.

Align the camera lens and eyepiece with the refracted NaD light emerging from the prism, and as before try as best you can to attach the camera translation stage and eyepiece post holder base to the optical table. Look into the eyepiece and focus the translation stage micrometer until the edges of the oversize slit appear sharp.

Method. Once focused, one partner should look through the eyepiece while the other partner narrows the slit until either the NaD lines are resolved or the slit is fully closed (the partner closing the slit jaws should pay close attention to the decreasing slit opening, and stop turning the micrometer adjustment when the slit becomes fully closed; *DO NOT FORCE THE SLIT JAWS TOGETHER by going beyond the fully closed point*). If the D-lines are resolved, record the slit opening when this first occurs. Then open the slit wide again, and switch roles so the other partner first adjusts focus and then watches to see if the D lines become separated as the slit is narrowed. Record your results, measuring Δx if the lines are resolved. Exchange camera lenses with the other group and have both partners try again. While using the 180 mm focal length camera lens, look for any other lines in the eyepiece field of view besides the yellow NaD lines and measure their linear separations from NaD with the eyepiece micrometer.

At this stage, join forces with the other group using the optical table with the 13.25-inch focal length collimator and the 300 mm EFL camera lens. Locate the second prism symmetrically with the first about the exit beam from the first prism; that is, hypotenuse first with apex away from the front of the table, so that light enters this second prism at the same angle it left the first one, and emerges finally at right angles to the 60 × 60 mm face of the second prism, as in the left diagram of Figure 1.

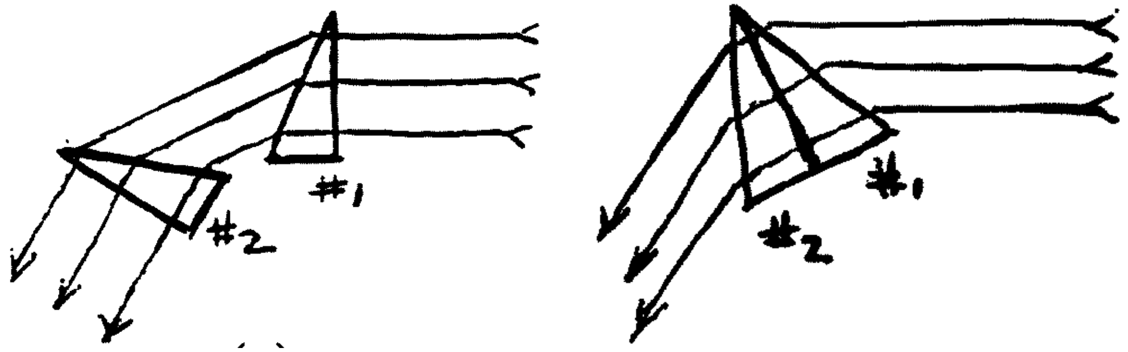

Figure 1: Experimental setup for prisms.

Reposition the camera lens and eyepiece to be aligned with the exit beam, and repeat the slit narrowing process while each person looks through the eyepiece in turn. Record your results. Elect one person to measure the Δx in the 300 mm lens focal plane between the two NaD lines. Finally, rearrange the prisms as shown in the right half of Figure 1 and repeat the steps outlined previously in this paragraph.

Analysis. In the lab you will also find a page from the Schott Glass catalog which gives "Constants of the Dispersion Formula" describing $n(\lambda)$, the refractive index of the prism material as a function of wavelength λ in μm:

$$n^2(\lambda) = A_0 + A_1\lambda^2 + A_2\lambda^{-2} + A_3\lambda^{-4} + A_4\lambda^{-6} + A_5\lambda^{-8} \tag{6}$$

For the single prism configuration you began with, compute the dimensionless dispersive power $\lambda\ \partial\beta/\partial\lambda$ for the prism from Snell's Law and the known angle at which all wavelengths of light are incident on the exit face of the prism. Evaluate the prism dispersive power at $\lambda = 5983$ Å and derive the value of $\Delta\beta$ expected between the pair of D lines. If an analytical expression for $\partial n/\partial\lambda$ proves too difficult to obtain from equation (3), you may use instead the numerical estimate $(n_e - n_{632.8})/(\lambda_e - 632.8\text{ nm})$ with values taken from the data sheet.

Discussion. How large a slit width would subtend this same angle in the focal plane of the collimating mirror? Comment on the comparison of this slit width with your single prism results visually in the eyepiece. Assuming an effective focal length of 25.4 mm for the 10× eyepiece, what angular magnification was achieved by your camera, lens + eyepiece lens "telescope?" Multiply the theoretically derived $\Delta\beta$ by this angular magnification and comment on the answer in light of your observations visually.

Interpret the separations of the other spectral lines measured using the 180 mm focal length camera lens. How does the dispersive power $\lambda\ \partial\beta/\partial\lambda$ for the prism compare to that of your grating from Section 1 of the lab, both in terms of magnitude and dependence on wavelength? In which two-prism configuration from Figure 1 was the resulting dispersion highest, and why?

In conclusion, summarize what you think are the advantages and disadvantages of gratings versus prisms as spectrograph dispersing elements.