Due Monday 1/25/2010, in class.

(1) Consider an infinite, static Universe containing 0.01 galaxies Mpc^{-3}, each of which has been shining with a constant luminosity of $10^{10}\ L_\odot$ since its birth 10 billion years ago (previously being completely dark). What is the energy flux at the earth's surface due to these galaxies? Compare it with that of the sun (Earth–Sun distance $= 1.5 \times 10^{13}$ cm). Is it dark at night? Will it always be so? If not, when will night be like day?

(2) Assuming that galaxies in a particular cluster have luminosities distributed according to the Schechter luminosity function,

$$\Phi(L)d\left(\frac{L}{L^*}\right) = \Phi^*\left(\frac{L}{L^*}\right)^\alpha \mathrm{e}^{-L/L^*} d\left(\frac{L}{L^*}\right)$$

, with $\alpha = -1$.

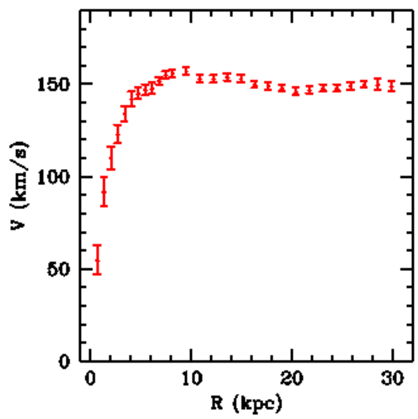
a) Calculate the ratio of the total cluster luminosity to the luminosity of just those galaxies brighter than some luminosity $L_{\min}$.

b) Suppose that the above luminosity function applies to field galaxies, and that all galaxies up to a given flux limit are observed in a small area of sky (in practice, this is how it is done). What will be the luminosity distribution of the *observed* sample? What fraction of the observed galaxies will be within a factor of 2 of the characteristic L^* luminosity?

(3) One of the tools used by astronomers to assess whether a population of objects is distributed uniformly with distance is called the V/V_{max} test. For each object in a sample, one forms V_i/V_{max} by considering the ratio of the spherical volumes enclosed by the actual observed distance of object i, divided by the volume enclosed by the *maximum* distance at which the same object could be placed and still appear in the sample. For uniformly distributed objects, $\langle V/V_{max}\rangle$=0.5.

Suppose the density of galaxies falls off like the inverse 1.8 power of distance about an observer as the correlation function ξ suggests it will, on average. What mean value of V/V_{max} will observers determine for galaxies in a magnitude–limited sample? How many galaxies will be required in a sample to demonstrate that the galaxy distribution is not uniform using the mean V/V_{max} test?

(4) One of the ways that the presence of "dark matter" is inferred in spiral galaxies is through the galaxy rotation curves, where the rotational velocity as a function of distance from the center of the galaxy is plotted, as in the example below:



a) The very surprising result is that the rotational velocity remains essentially constant well beyond where most of the starlight is found; if one were to assume that spiral disks were thin disks undergoing Keplerian rotation, and that their physical sizes were identical to their apparent optical sizes, what would you expect the rotation curve to look like? Draw a plot of v_r versus R, extending beyond the optical size R_{opt}. What dependence of mass with radius is implied by a "flat" rotation curve?

b) Suppose that the observed rotation speed of a 10^{10} L$\odot$ galaxy is 220 km s^{-1}and its optical "edge" is at R_{opt} = 10 kpc. What is the mass–to–light ratio inside this radius, in solar units? Suppose that the mass distribution continued out to $R = 100$ kpc with the same radial dependence; what is M/L for the galaxy now?

c) Suppose now that all galaxies have this same value of M/L, that they obey a Schechter luminosity function with $\Phi^* = 2 \times 10^{-2}$ Mpc^{-3}, and $\alpha = -1$ (assume that $L^* = 10^{10} L_\odot$). What is the total mass density (in g cm^{-3}) due to galaxies? The "critical" density for a closed Universe, as we will discuss later in the course, is $\rho_c = 2 \times 10^{-29}$ g cm^{-3}. Comment on the difference between ρ_{gal} and ρ_c.