23 January 2020 **Ay 21**

Galaxies and Cosmology

Homework Set 2

Due Tuesday, February 4, by 5pm PST

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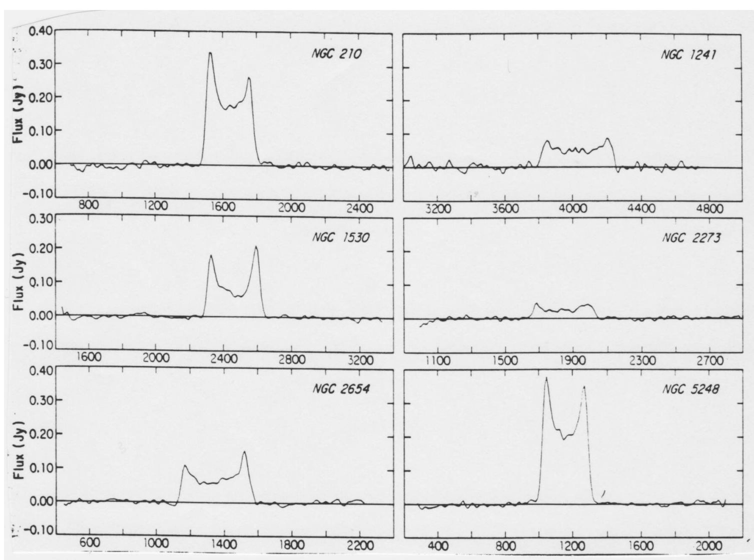
1. **Pushing Cepheids to the Limit**
   The controversy over the distances to the so–called "spiral nebulae" was settled by Edwin Hubble in 1924 when he successfully resolved individual stars in M31 (The Andromeda Galaxy). If one were to observe a Cepheid variable star in M31 that turns out to have a light curve period of 60 days and the average visual magnitude of the star is $\langle m_V \rangle = 18$, use the period–luminosity relation

   $$\langle M_V \rangle = -1.35 - 2.78 \log_{10} P,$$

   to determine the distance to M31. Here P is the period of the Cepheid in days and $\langle M_V \rangle$ is its average visual magnitude. Using the Hubble Space Telescope, it would be reasonable to obtain measurements of Cepheids as faint as $m_V \simeq 26$. What is the approximate distance out to which Cepheids of similar luminosity to that above could be used for distance measures?

2. **Tully-Fisher Relation for Spiral Galaxies**
   In the figure below are observed velocity profiles of a few spiral galaxies measured in the 21cm line of neutral H. The abscissae have been converted to the radial velocity as seen by us, and the intensity is proportional to the number of neutral H atoms as a function of line-of-sight velocity.



   The Tully–Fisher relation for spiral galaxies can be expressed as

   $$M_B = -4.42 - 6.15 log_{10} \Delta v,$$

   where Δv is the velocity width of the galaxy profile in km s^{-1}, and M_B is the absolute blue magnitude of the galaxy.

   a) Assume that the galaxies NGC 1241 and NGC 5248 have apparent blue magnitudes of $m_B = 14$ and $m_B = 12$, respectively. Calculate the distances to these two galaxies. From these and their radial velocities, estimate the value of the Hubble constant. For simplicity, also assume that the galaxies are seen edge–on (inclination angle $i = 90\,$deg, so that the velocity width corresponds to the maximum rotational velocity.

b) In most cases, the galaxies observed are not seen edge–on. How do the results obtained in part (a) depend on the inclination angle i?

c) Qualitatively, try to explain the *shape* of the observed velocity profiles (e.g., why do they have these "horns" at the extrema?) [Hint: think about the *quantity* of gas that will have a particular radial velocity.]

3. **Galaxy Cluster Back of the Envelope Estimates**
Consider a cluster of galaxies with a radial velocity dispersion $\sigma_{\rm v} \simeq 1500$ km s^{-1}, and the mean radius $\langle R \rangle = 1.5$ Mpc. It contains ~ 500 galaxies, with a mean luminosity per galaxy of $\langle L \rangle \simeq 10^{10}\ L_\odot$.

a) Estimate the total mass of the cluster, in solar units.

b) Determine the cluster M/L, also in solar units (i.e., in units of $M_\odot/L_\odot$.)

c) What is the characteristic temperature of the intracluster gas (i.e., the "virial temperature")?

d) High temperature gas such as that in the ICM radiates primarily via thermal Bremsstrahlung (a.k.a. "free-free" emission, due to electron/proton scattering events, which produces emission with characteristic photon energies comparable to the thermal energy of the "hot" electrons. Estimate the characteristic photon frequency.

e) The approximate rate of cooling per unit volume due to this process is given by

$$\Lambda \approx 1.4 \times 10^{-23} \left(\frac{T}{10^8\ \mathrm{K}}\right)^{0.5} \left(\frac{n_{\rm e}}{\mathrm{cm}^{-3}}\right)^2 \mathrm{ergs\ s^{-1} cm^{-3}} \tag{1}$$

where $n_{\rm e}$ is the electron volume density. Assuming that the hot gas accounts for $\sim 20\%$ of the total cluster mass, what is the expected luminosity? How does it compare to the total luminosity of the stars in the cluster galaxies?

4. **Real World Applications of Not-Quite-Standard Candles**
Suppose you have a "standard candle" which (somehow) you have established has an intrinsic "scatter" of 0.35 magnitudes (1 sigma), and that it can be used successfully to measure relative distances (radial velocities) out to $cz = 10,000$ km s^{-1}. How many measurements of such objects, over the whole sky, would be required to verify that the volume probed is "at rest" with respect to the cosmic microwave background (the net error in velocity should be smaller than 150 km s^{-1}). How many measurements of the same objects would be necessary, in principle, to measure H_0 to $\sim 10\%$ accuracy if the volume is not subject to large scale flows or significant peculiar velocities? State all of your assumptions–there is no exact "right" answer, but your reasoning is important. Comment on why, in the real world, things aren't so easy. [Hint: remember that the direction of peculiar velocities will not always be along our line of sight...]