Problem Set #3  

AY 21

Due 2/18/2010 by 5:00 p.m. in Swarnima’s mailbox in 249 Cahill.

(1) Consider the parameters \( \Omega = \rho / \rho_c = 8\pi G \rho / 3H^2 \), \( q = -a\ddot{a} / \dot{a}^2 \) as functions of cosmic time in a standard Friedmann universe. First show that

\[ \dot{a}^2 = \frac{H_0^2 \Omega_0}{a} - H_0^2 (\Omega_0 - 1) \]

where present–day values are denoted with the \( 0 \) subscript. Use this relation to show that \( q = \Omega / 2 \) at all times, and derive asymptotic values of \( q \) and \( \Omega \) for very early, and very late, times.

(2) In 1917 Einstein and de Sitter published a cosmological model based on a modification of Einstein’s General Relativity. The direct Newtonian analogy of the “cosmological constant” is a force per unit mass which grows linearly with distance, so

\[ \ddot{a} = -\frac{GM}{a^2} + \frac{\Lambda}{3} a \]

is the new fundamental dynamical equation. Derive an expression for \( H^2 = (\dot{a}/a)^2 \) in a \( \Lambda > 0 \) universe. Can such a universe be static? Can it be static and stable? Assuming that it is never static, what will the asymptotic expansion laws at late and early times be? Given “reasonable” values of \( \Omega_0 \) and \( H_0 \), what order of magnitude \( \Lambda \) would produce significant cosmological effects? How might such a value be detected experimentally?

(3) Show that a source of fixed linear size will subtend the smallest angle when it is at \( z = 1.25 \) in a universe with \( q_0 = 0.5 \) (\( \Omega_m = 1 \)) (in other words, the angular size of an object starts increasing again beyond \( z = 1.25 \)). Explain how/why this could be. Qualitatively, in which direction would you expect the peak to move (in terms of redshift \( z \)) in an \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \) universe.

(4) Suppose that we live in a matter–dominated, Einstein–de Sitter Universe with \( H_0 = 70 \) km \( s^{-1} \) Mpc\(^{-3} \). At what redshift did the Universe become matter-dominated? Was this before or after the epoch of recombination?

(5) Calculate how much mass is within the horizon at the epoch of recombination (express your answer in units of \( M_\odot \)). What angle does such a region subtend on the sky when observed today? What is the significance of this fact, given that the microwave background is observed to be smooth, in all directions, to about 1 part in \( 10^5 \)?