Problem Set #3 **AY 21**

Due 2/18/2010 by 5:00 p.m. in Swarnima's mailbox in 249 Cahill.

(1) Consider the parameters $\Omega = \rho/\rho_c = 8\pi G\rho/3H^2$, $q = -a\ddot{a}/\dot{a}^2$ as functions of cosmic time in a standard Friedmann universe. First show that

$$\dot{a}^2 = \frac{H_0^2\Omega_0}{a} - H_0^2(\Omega_0 - 1)$$

where present–day values are denoted with the $_0$ subscript. Use this relation to show that $q = \Omega/2$ at *all* times, and derive asymptotic values of q and Ω for very early, and very late, times.

(2) In 1917 Einstein and de Sitter published a cosmological model based on a modification of Einstein's General Relativity. The direct Newtonian analogy of the "cosmological constant" is a force per unit mass which grows linearly with distance, so

$$\ddot{a} = -\frac{GM}{a^2} + \frac{\Lambda}{3}a$$

is the new fundamental dynamical equation. Derive an expression for $H^2 = (\dot{a}/a)^2$ in a $\Lambda > 0$ universe. Can such a universe be static? Can it be static *and* stable? Assuming that it is never static, what will the asymptotic expansion laws at late and early times be? Given "reasonable" values of Ω_0 and H_0, what order of magnitude Λ would produce significant cosmological effects? How might such a value be detected experimentally?

(3) Show that a source of fixed linear size will subtend the smallest angle when it is at $z = 1.25$ in a universe with $q_0 = 0.5$ ($\Omega_m = 1$) (in other words, the angular size of an object starts *increasing* again beyond $z = 1.25$!). Explain how/why this could be. Qualitatively, in which direction would you expect the peak to move (in terms of redshift z) in an $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ universe.

(4) Suppose that we live in a matter–dominated, Einstein–de Sitter Universe with $H_0 = 70$ km $s^{-1}Mpc^{-3}$. At what redshift did the Universe become matter-dominated? Was this before or after the epoch of recombination?

(5) Calculate how much mass is within the horizon at the epoch of recombination (express your answer in units of $M_\odot$). What angle does such a region subtend on the sky when observed today? What is the significance of this fact, given that the microwave background is observed to be smooth, in all directions, to about 1 part in 10^5?