4 February 2020 **Ay 21**

Galaxies and Cosmology

Homework Set 3

Due Tuesday, February 18, 2020, by 5pm PST

1. **Co-Moving Volume and Galaxy Counts**
   The co–moving volume out to a redshift z is the volume calculated at the present time enclosed by a 3–sphere centered on Earth and passing through sources from which spectral lines would have a redshift z. Calculate the co–moving volume in Mpc^3 (assuming $H_0 = 70$) for an Einstein–de Sitter Universe (i.e., $\Omega = 1$) out to $z = 1,3,\infty$. Use a Schechter function as in problem set #1 to estimate the number of galaxies brighter than $0.1\mathrm{L}^*$ with redshifts $z < 3$. The actual number is observed to be $\sim 3 \times 10^{10}$. Compare your estimate with this number and discuss the implications.

2. **No Gambling Allowed in This Universe**
   In 1917 Einstein and de Sitter published a cosmological model based on a modification of Einstein's General Relativity. The direct Newtonian analogy of the "cosmological constant" is a force per unit mass which grows linearly with distance, so

   $$\ddot{a} = \frac{-GM}{a^2} + \frac{\Lambda}{3}a \tag{1}$$

   is the new fundamental dynamical equation. Derive an expression for $H^2 = (\dot{a}/a)^2$ in a $\Lambda > 0$ universe. Can such a universe be static? Can it be static *and* stable? Assuming that it is never static, what will the asymptotic expansion laws at late and early times be? Given "reasonable" values of Ω_0 and H_0, what order of magnitude Λ would produce significant cosmological effects? How might such a value be detected experimentally?

3. **Cosmological Misnomers**
   Consider the density parameter $\Omega_\mathrm{m} = \rho/\rho_c = 8\pi G\rho/3H^2$ in a standard Friedmann universe with zero cosmological constant.

   a) First show that

   $$\dot{a}^2 = \frac{H_0^2\Omega_0}{a} - H_0^2(\Omega_0 - 1) \tag{2}$$

   where present–day values are denoted with the $_0$ subscript.

   b) Until the late 1990s (i.e., before Dark Energy became part of the standard model for the universe), one of the most sought-after numbers in cosmology was the "deceleration parameter", defined as $q = -a\ddot{a}/\dot{a}^2$, which if measured at the present epoch has $q_0 = -\ddot{a}(t_0)a(t_0)/\dot{a}^2(t_0) = \Omega_0/2$. Use the relation in part (a) to show that $q = \Omega/2$ at *all* times, and derive asymptotic values of q and Ω for very early, and very late, times.

   c) Show that, for the more general case of a universe with non-zero values of both $\Omega_{\mathrm{m},0}$ and $\Omega_{0,\Lambda}$, $q_0 = \Omega_{\mathrm{m},0}/2 - \Omega_{\Lambda.0}$. Speculate on why the use of q_0 as a cosmological parameter has gone out of favor in recent years.

4. **Extreme Close-Up**
   Show that a source of fixed linear size will subtend the smallest angle when it is at $z = 1.25$ in a universe with $q_0 = 0.5$ ($\Omega_0 = 1$) (in other words, the angular size starts *increasing* again beyond $z = 1.25$!). Does a similar effect happen for a flat universe with $\Omega_\mathrm{m} = 0.3$, $\Omega_{\Lambda,0} = 0.7$? If so, at what redshift would the "turnaround" occur?