Ay 21– Problem Set #4

Due Thursday, March 4, 2010 by 5:00 pm, in Swarnima Manohar's mailbox in 249 Cahill.

(1) We have seen that the Einstein–de Sitter Universe is generally thought to be adequate to describe the Universe at large redshift (recall what happens to Ω for small values of the expansion factor R even in a low $\Omega_{m,0}$ universe). Now, consider a spherical, uniform density matter perturbation in an Einstein–de Sitter Universe, which can be regarded as a miniature "closed" Universe. Show that the sphere reaches its maximum radius (i.e., it starts to collapse) when the density in the sphere is $9\pi^2/16$ times the mean density of the surrounding medium. (Hint: use the cycloidal solution of the general dynamical equation that we have looked at in several different contexts in class. Then solve for the time corresponding to the maximum expansion of the perturbation, and see what the density is in the surrounding medium (the $\Omega = 1$ part) at that time). This is actually very close to the kind of calculation used to work out when a galaxy of a given mass scale will collapse and "form".

(2) A distant, rich cluster of galaxies ($z = 0.1$) has a 1-D velocity dispersion of 1000 km s^{-1}, and a characteristic angular size of 5 arc minutes. It is filled with hot gas at fairly uniform density. Estimate its temperature. We observe X-ray flux of 10^{-10} ergs s^{-1} cm^{-2} from the hot gas. Estimate the total mass of gas; is it enough to bind the cluster? Will it cool in a Hubble time? (Retain the scaling with H_0 when you answer this question).

(3) The active nucleus of a particular low–luminosity Type I Seyfert galaxy is believed to be powered by gas accreting onto a black hole of mass 10^6 $M_\odot$.

a) Evaluate the orbital period of gas in the accretion disk at a radius of 1 pc.

b) Estimate the radius at which a star like the sun will be torn apart by tidal forces.

c) The innermost radius at which gas can orbit the hole is 3 Schwarzschild radii. Estimate the shortest orbital period in the accretion disk, and hence the mimimum timescale on which you might expect the X-ray emission to vary.

d) What is the average effective temperature of the accretion disk associated with the black hole? Where would you expect the thermal spectrum to peak (at what frequency or wavelength)?

e) Suppose that the accretion disk is radiating at the Eddington limit. How long could that accretion rate be supported in a typical galaxy? What would be the corresponding lifetime of a luminous quasar with a black hole mass of 10^9 years? How do both correspond to the typical age of a galaxy?