20 February 2020 **Ay 21**

Galaxies and Cosmology

Homework Set 4 Due Thursday, March 5, 2020, by 5pm PST

1. **Frequency of Interceptions in an Expanding Universe**
Suppose we live in a universe with $\Omega_{m,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7$, and $H_0 = 70$ km s^{-1} Mpc^{-1}. If the mean effective cross-section $\sigma_{\rm gal}$ of galaxies is redshift-independent, and the co–moving space density of the galaxies Φ_* remains constant, show that the expected number of interceptions (arising from interception of any part of an intervening galaxy) in a line of sight between the observer at $z = 0$ and a source at redshift $z_{\rm s}$ is

$$N = \Phi^* \sigma_{\rm gal} \frac{c}{H_0} \int_0^{z_s} \frac{(1+z')^2}{E(z')} dz' \quad . \tag{1}$$

The line of sight to a typical quasar at $z_{\rm s} = 2$ records ~ 5 independent interceptions of intervening galaxies; the galaxy redshifts can be measured from C^{+++} absorption lines produced by gas in each galaxy, falling at observed wavelengths $1549 \times (1 + z_{\rm gal,i})$ Å, where 1549 Å is the rest wavelength of the C^{+++} transition and $z_{\rm gal,i}$ is the redshift of the ith intervening galaxy. Numerically integrate equation 1 using $z_{\rm s} = 2$ to calculate N in terms of Φ^* and $\sigma_{\rm gal}$. Using a reasonable value for Φ^*, estimate the value of $\sigma_{\rm gal}$ required such that $N \simeq 5$. What is the implied physical size (i.e., radius in kpc) of the gas distribution around a typical galaxy to account for the number of interceptions? Is is reasonable to associate the C^{+++} absorption lines with normal galaxies?

2. **Galaxy Formation on the Back of an Envelope**
Suppose that a dark matter halo of mass 10^{12} M$_\odot$ has virialized at redshift $z = 3$, and that it has

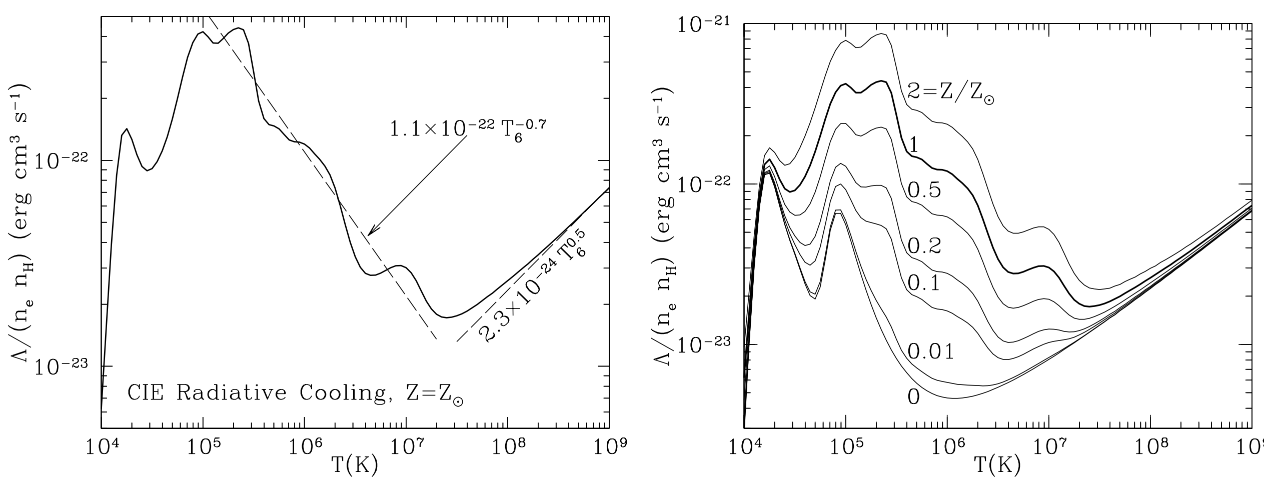

Figure 1: (Left) Cooling function vs. temperature for solar metallicity gas. (Right) Cooling function vs. metallicity.

the same baryon-to-dark matter ratio as the average in the universe (i.e., $M_{\rm b}/M_{\rm DM} \simeq 0.19$). You may assume that the baryons and dark matter in the halo just after virialization have constant density $\bar{\rho}_{\rm coll}$, and that $\Omega_{m,0} \simeq 0.3$, $H_0 = 70$ km s^{-1} Mpc^{-1}.

a) Using the model for spherical collapse in an expanding universe as discussed in class, calculate the "virial radius" $R_{\rm vir}$ of the halo immediately after it has collapsed at $t = t_{\rm coll} = 2t_{\rm ta}$, where $t_{\rm ta}$

is the time of "turnaround" for the perturbation. [hint: the mean density of the collapsed object $\bar{\rho}_{\rm coll}$ is related to the mean density of the universe at $t = t_{\rm coll}$, $\bar{\rho}(z = 3)$, which in turn is related to the matter density at $z = 0$, and $\bar{\rho}_{\rm coll} = 3M_{\rm vir}/4\pi R_{\rm vir}^3$.]

b) Calculate the "virial velocity" $\sigma_{\rm vir}$, for the halo, defined through $\sigma_{\rm vir}^2 = GM_{\rm vir}/R_{\rm vir}$. Use the value obtained to calculate the expected virial temperature $T_{\rm vir}$ for the baryons within $R_{\rm vir}$, and the dynamical time $t_{\rm d}$ for the collapsed halo.

c) The cooling function for gas, $\Lambda(T)/n^2$, can be used to estimate the radiative cooling rate (the rate of energy loss from radiation) of the gas, as a function of T, shown in Figure 1. Use your estimated $T_{\rm vir}$ and the mean number density of baryons n to calculate the cooling rate per unit volume (i.e., $\Lambda(T)$ in ergs s^{-1} cm^{-3}) for the gas.

d) From the cooling rate per unit volume and the thermal energy per unit volume associated with the baryons, estimate the timescale for gas to cool, $t_{\rm cool} \sim E_{\rm therm}/\Lambda(T)$. How does $t_{\rm cool}$ compare with the dynamical time $t_{\rm d}$?

e) Finally, based on the righthand panel of the figure above, at approximately what gas-phase metallicity would the cooling rate slow to the point that $t_{\rm cool} \sim t_{\rm d}$?

3. **Supermassive Black Holes in Garden-Variety Galaxies**

The active nucleus of a particular low–luminosity Type I Seyfert galaxy is believed to be powered by gas accreting onto a black hole of mass 10^6 M$\odot$.

a) Evaluate the orbital period of gas in the accretion disk at a radius of 1 pc.

b) Estimate the radius at which a star like the sun will be torn apart by tidal forces.

c) The innermost radius at which gas can orbit the black hole is 3 Schwarzschild radii. Estimate the shortest orbital period in the accretion disk, and hence the mimimum timescale on which you might expect the X-ray emission to vary.

d) What is the average effective temperature of the accretion disk associated with the black hole? Where would you expect the thermal spectrum to peak (at what frequency or wavelength)?

e) Suppose that the accretion disk is radiating at the Eddington limit. How long could that accretion rate be supported in a typical galaxy? What would be the corresponding lifetime of a luminous quasar with a black hole mass of 10^9 M$\odot$, in years? How do both correspond to the typical age of a galaxy?