Ay 124 – Homework #1

Posted on Friday, Jan. 21 – Due by 5 pm on Friday, Jan. 28 (directly to the TA)

1. [9 points, 3 each] What are the apparent bolometric magnitudes of:
   a. Sun-like star 50 pc away?
   b. 100 Watt lightbulb on the Moon?
   c. A galaxy containing ~ 3×10^{10} stars of an average luminosity ~ 0.5 $L_\odot$ 20 Mpc away?

   (Assume for simplicity that all of these objects have spectra identical to that of the Sun.)

2. [35 points] Compute star counts (number per magnitude per deg^2) as a function of the V band towards the Galactic pole, assuming the exponential height of 300 pc, in the range V ~ 12 to 22 mag, for:
   a. A population of Solar-type stars (G dwarfs), with the local density of 0.003 pc^{-3}. [15 points]
   b. A population of M dwarfs with masses of 0.2 $M_\odot$. Assume the Salpeter IMF to figure out their local number density, and the lower MS scaling relation L ~ $M^{3.5}$ (ignoring the color terms…) to estimate their luminosities. [17 points]
   c. Comment on what do you think the total star counts would look like, if you actually did it for a full mass spectrum of stars obeying the Salpeter IMF. [3 points]

3. [40 points] Consider a young star cluster in which stars have been made according to the standard Salpeter IMF: $dN/dm \sim m^{-(1+x)}$, where x = 1.35, and ranging from m_{min} = 0.08 $M_\odot$ to m_{max} = 80 $M_\odot$. Assume that the scaling relation between mass and luminosity L ~ M^4 applies for all masses.
   a. What is the average stellar mass? Derive a formula for it, for a general power-law IMF, and compute the value for the parameters given above.
   b. Ditto for the average stellar luminosity.
   c. If the cluster mass is 10^3 $M_\odot$, what is its absolute bolometric magnitude?
   d. What fractions of the total mass are contributed by the stars above and below the solar mass?
   e. Ditto for the luminosity.

4. [16 points] Consider a globular cluster containing N = 10^5 stars with an average mass $\langle m \rangle$ = 0.3 $M_\odot$, and an effective mean radius R = 3 pc.
   a. What are its binding energy, the mean velocity of stars, and the relaxation time?
   b. Now consider a binary in the cluster core, where both components have masses of 0.5 $M_\odot$, in a circular orbit with a diameter of 10 a.u. Compute the binding energy of this binary, and compare it with the mean binding energy *per star* in the cluster as a whole. Is this a soft or a hard binary?