Ay 127 – Spring 2011 – Final Exam

Distributed on Wednesday, June 1, 2011

Due by 9 am on Monday, June 6, 2011 (directly to the TA)

The Rules: Closed books, no internet, etc. Open course lecture notes, *your own* notes, and *your own* homework sets/solutions from this course. You can use a calculator and tables of physical and astronomical constants or units. No collaboration. You cannot discuss the exam with anyone, until everyone in the class has turned in their test.

You have a maximum of 3 hours from the moment you start until the moment you finish. Please mark your exam with the start and stop times. You have to hand it to the TA or one of the instructors directly.

Please write legibly – it is in your own best interest. Good luck!

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1. Short Problems [10 points each; they are independent]

1A. Define or explain briefly (in a few sentences at most):

a. Stellar population synthesis models, and what are their main ingredients?
b. "Downsizing" in the context of galaxy evolution.
c. AGN feedback, and why is it important?
d. The reionization era and why is it important, and at what redshifts did it happen?
e. Blazars and how they differ from other types of AGN; why are they so interesting? (Give at least 3 good reasons.)
f. Kerr black hole vs. Schwarzschild black hole; which one can produce a more luminous AGN, for the same accretion rate?
g. Types of absorbers in quasar spectra, and how do they relate to galaxies?
h. The cosmic web, and how do we study it?

1B. Consider a quasar powered by an Eddington-limited accretion to a black hole, with an efficiency of 10%.

a. Derive the function describing the evolution of the black hole's mass and the quasar's luminosity, and the numerical value of the characteristic time scale entering these formulas.
b. Assume that the BH starts with a seed mass of $M_{init} = 100\ M_\odot$ at $z \sim 30$. How long will it take to reach $M_{\rm bh} \sim 10^9\ M_\odot$? Does that present a difficulty, given the observations of luminous QSOs at $z \sim 6$? Note: you'll need to estimate the ages of the universe at these redshifts, which accounts for half the credit in this sub-problem (1.B.b).

1C. Compute the Compton cooling timescale of fully ionized gas at $(1+z) = 10$. Would Compton cooling have had a significant effect on the thermodynamics of gas during reionization?

1D. To an order of magnitude, what fraction of the energy density of the Universe is in gravitational binding energy? Is this positive or negative? Does it have a significant effect on the cosmic expansion rate? (Note: in general relativity one must treat the notion of "gravitational energy" very carefully, but it turns out that the Newtonian binding energy does gravitate in the usual way.)

2. Warm Dark Matter [30 points]

Consider a dark matter candidate of mass m that was in thermal equilibrium and decoupled at a temperature T_d. The dark matter particle is assumed to have been relativistic at its decoupling epoch, i.e., $mc^2 << kT_d$. After this decoupling, the dark matter particles do not interact (except gravitationally. (The candidates of this type are usually spin 1/2 or 3/2 fermions that are their own antiparticles, but since this is an order-of-magnitude problem you don't need to know this.) This type of dark matter is a type of Warm Dark Matter (WDM). Assume a cosmology with $\Omega_m = 0.3$ and $H_0 = 70$ km/s/Mpc.

Answer the following questions to an order of magnitude only, i.e. you may neglect factors of 2, 7/8, 4/11, etc. but not the effective number of degrees of freedom $g_*(T_d)$ (as this may be large). If you can't solve an earlier part of the problem you may express answers to later parts of the problem in terms of the answers to the earlier parts.

Hint: There are a lot of similarities between the behavior of WDM and neutrinos.

a. What is the ratio $n_{\mathrm{dm}} / n_\gamma$ of the number density of dark matter particles to CMB photons at the present epoch? Assume $g_*(T_d) \approx 100$.

b. Determine the number density n_{dm} of DM particles today.

c. Given the known density of dark matter, and assuming our candidate is all of the dark matter, what is the mass m?

d. At what redshift z_{nr} did the DM particles become non-relativistic? This should be during the radiation-dominated epoch. Explain why it would be a problem for the theory if you obtained a redshift in the matter-dominated epoch.

e. Compute the comoving distance r traveled by the DM particles before they become non-relativistic. Explain what this implies for structure formation in WDM models.

The following formulas may be helpful:

- Entropy density of a thermalized plasma with no chemical potential:

$$s = \frac{2\pi^2}{45\hbar^3 c^3} g_*(T) k^4 T^3.$$

- Number density of photons in a blackbody:

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} \left(\frac{kT_\gamma}{\hbar c}\right)^3,$$

  where $\zeta(3) = 1.2$ is the Riemann ζ-function.

3. All galaxies in the Universe and their black holes [30 points]

a. From deep galaxy counts, it is estimated that there are ~ 40 billion galaxies within the observable universe (not including probable multitudes of dwarf galaxies). Assuming that the mean age of these galaxies is ~10 Gyr, and that each one goes through an AGN episode once, with a mean duration of ~ 10^8 yr, estimate the projected surface density of quasars on the sky (in deg^{-2}).

b. If the volume number density of quasars at $z \sim 0$ is ~100 Gpc^{-3}, and the normalization of the galaxy luminosity function is $\varphi_* \sim 0.01\ \mathrm{Mpc}^{-3}$, what is the probability of a galaxy containing a quasar at $z \sim 0$? Their commoving density was ~ 1000 times higher at $z \sim 2$, the peak of the quasar era. Compare these numbers with the average probability evaluated in (a).

c. Using the same assumptions about the galaxy luminosity function, and assuming the look-back time to the onset of galaxy formation of ~ 10 Gyr, estimate the number of $L \sim L_*$ (i.e., non-dwarf) galaxies in the entire observable universe. State your assumptions. Compare this with the total observed number of galaxies over the entire sky. Discuss the differences, if there are any.

d. Estimate the total energy released in the production of a $M_{\rm bh} \sim 10^7\ M_\odot$ black hole, assuming the average efficiency of 10%. Estimate the binding energy of a typical galaxy, and compare the two. Comment of the effects of the AGN feedback on their host galaxies (both radiative and mechanical). What if $M_{\rm bh} \sim 10^9\ M_\odot$ (e.g., as in a luminous quasar)?

e. Just to make this problem more fun, answer this: What would be the form of the galaxy 2-point correlation function if all galaxies were distributed uniformly in space? All galaxies were on sheets/walls? All galaxies were in filaments?

4. Diffuse X-ray background [30 points]

The observed brightness of the cosmic X-ray background (CXRB) is $\nu I_\nu \approx 3\times10^{-10}$ W m^{-2} sr^{-1}.

a. Compute the corresponding volume energy density. Compare with the energy density of the CMB, assuming $T_{\rm CMB} = 2.735$°K.

b. It is estimated that on average there is a SMBH with $M_{bh} \sim 10^7\ M_\odot$ per average ($L \sim L_*$) galaxy. Assuming that the efficiency of accretion in converting the rest mass into energy was ~ 10% (i.e., 90% ends in the SMBH, 10% is radiated away), and that the mean redshift of emission was $\langle z\rangle \sim 2$, compute the energy density today, generated by the making of these SMBHs. Compare it with the numbers for the CXRB and CMBR computed in (a). (Note: you will need to estimate the comoving density of L_* galaxies today.)

c. Consider a quasar with $L_{\rm bol} \approx 5\times10^{12}\ L_\odot$, at $z = 2$. For simplicity, assume an Einstein – de Sitter universe with $H_0 = 50$ km/s/Mpc. Compute the luminosity distance to the quasar. (Note: if you don't know how to evaluate or derive this distance quickly, make a reasoned estimate for half the credit in this sub-question, and move on.)

d. Compute its absolute and apparent bolometric magnitudes, assuming $M_{bol_\odot} = 4.8$ mag.

e. Assuming that 30% of the entire luminosity of this quasar is emitted in X-rays, compute the observed X-ray flux (in cgs or SI units).

f. How many such quasars would it take to generate *all o*f the observed CXRB? Compute their projected surface density on the sky (number per deg^2). How does this compare with what you know about the observed surface density of QSOs on the sky?