1. **Short questions.** [24 pts]

(a) [6 pts] Suppose the correlation function of galaxies is approximated by $\xi(r) = (r/r_0)^{-\gamma}$ where r_0 is the correlation length. Two types of galaxies ("sample 1" and "sample 2") have biases b_1 and b_2. What is the ratio of their correlation lengths, $r_{0,1}/r_{0,2}$ in terms of b_1, b_2, and γ?

(b) [6 pts] Explain why CMB observations do not show giant arcs behind galaxy clusters. (Order of magnitude arguments will suffice.)

(c) [12 pts] Suppose the dark matter consists of non-interacting (or weakly interacting) spin-$\frac{1}{2}$ particles. Using arguments based on degeneracy pressure, and the properties of the Milky Way's dark matter halo, what can be said about the particle's mass? (Again, you may work to an order of magnitude.)

2. **Clusters and CMB polarization.** [30 pts]

Consider a galaxy cluster of virial mass $M_{\rm vir} = 5 \times 10^{14} M_\odot$ located at $z = 0.1$. Assume a flat ΛCDM universe with $\Omega_m = 0.3$, $\Omega_b = 0.05$, and $H_0 = 70$ km/s/Mpc. You may work in the approximation $z \ll 1$ to avoid doing messy numerical integrals.

(a) [10 pts] What is the virial radius $r_{\rm vir}$ of the cluster in kpc? What is the apparent size of this virial radius in arcmin?

(b) [10 pts] Averaged over a disk whose is the virial radius, what is the optical depth due to Thomson scattering within the cluster? (You may assume the baryons are mainly in the form of ionized intracluster gas.)

(c) [10 pts] The cluster is expected to exhibit microwave polarization due to scattering of the quadrupole moment of the CMB anisotropy. To an order of magnitude level, what is the polarized signal in μK?

3. **Cosmic X-ray background.** [46 pts]

The observed brightness of the cosmic X-ray background (CXRB) is $\nu I_\nu \approx 3 \times 10^{-10}$ W/m^2/sr.

(a) [4 pts] Compute the corresponding volume energy density. Compare with the energy density of the CMB.

(b) [13 pts] It is estimated that on average there is a SMBH with $M_{\rm BH} \sim 10^7 M_\odot$ per average ($L \sim L_\star$) galaxy. Assuming that the efficiency of accretion in converting the rest mass into energy was $\sim 10\%$ (i.e. 90% ends up in the SMBH, 10% is radiated away), and that the mean redshift of emission was $\langle z \rangle \sim 2$, compute the energy density today, generated by the making of these SMBHs. Compare it with the numbers for the CXRB and CMB compuated in (a). (Note: you will need to estimate the comoving density of $L_\star$ galaxies today.)

(c) [8 pts] Consider a quasar with $L_{\rm bol} \approx 5 \times 10^{12} L_\odot$ at $z = 2$. For simplicity, assume an Einstein-de Sitter universe with $H_0 = 50$ km/s/Mpc. Compute the luminosity distance to the quasar. (Note: if you dont know how to evaluate or derive this distance quickly, make a reasoned estimate for only 3 points credit, and move on.)

(d) [5 pts] Compute its absolute and apparent bolometric magnitudes, assuming $M_{\odot\,\mathrm{bol}} = 4.8$ mag.

(e) [6 pts] Assuming that 30% of the entire luminosity of this quasar is emitted in X-rays, compute the observed X-ray flux (in cgs or SI units).

(f) [10 pts] How many such quasars would it take to generate all of the observed CXRB? Compute their projected surface density on the sky (number per deg^2). How does this compare with what you know about the observed surface density of QSOs on the sky?