Ay 127 Midterm Solutions

2) a) $\Omega_{tot} = \Omega_m = 1 \quad \rightarrow \quad H^2 = H_0^2 \frac{\Omega_{m,0}}{a^3}$, $\dot a = H_0 \underbrace{\sqrt{\Omega_{m,0}}}_{1} a^{-1/2}$

$$\Omega_k = 0 \rightarrow D_A(a) = a \cdot r(a) = a \cdot c \int_{t(a)}^{t_0} \frac{dt}{a(t)} = a \cdot c \int_a^1 \frac{da'}{a' \dot a'}$$

$$= a \frac{c}{H_0} \int_a^1 \frac{da'}{\sqrt{a'}} = \frac{2ca}{H_0}\left(1 - \sqrt{a}\right)$$

$$D_A(z) = \frac{2c}{H_0(1+z)}\left(1 - \frac{1}{\sqrt{1+z}}\right)$$

$$\dot a = H_0 a^{-1/2} \Rightarrow \sqrt{a}\, da = H_0\, dt \Rightarrow t(a) = \frac{2}{3}\frac{1}{H_0} a^{3/2}, \quad t_0 = \frac{2}{3}\frac{1}{H_0}$$

b) comoving particle horizon $\eta(a) = c\int_0^a \frac{da'}{a' \dot a'} = \frac{c}{H_0}\int_0^a a^{-1/2}\, da = \frac{2c}{H_0}\sqrt{a}$

@ $z = 1100$: $\eta = 258$ Mpc

proper size of particle horizon $x(a) = a\,\eta(a) = 0.23$ Mpc

c) $\theta = \frac{x(a)}{D_A(a)} = 0.03 \,\hat{=}\, 1.72°$

d) $\Omega_{tot} = \Omega_m = 0 \quad \rightarrow H^2 = \frac{\dot a^2}{a^2} = H_0^2 (0) \frac{\overbrace{(1 - \Omega_{tot})}^{1}}{a^2} \Rightarrow \dot a = H_0$

$$r(a) = c\int_a^1 \frac{da'}{a' \dot a'} = \frac{c}{H_0}\int_a^1 \frac{da'}{a'} = \frac{c}{H_0} \ln\left(\frac{1}{a}\right)$$

$\Omega_k = 1 \rightarrow \mathcal{R}(a) = \frac{c}{H_0 a}$ and $D_A(a) = a\, \mathcal{R} \sinh\left(\frac{r(a)}{\mathcal{R}}\right) = \frac{c}{H_0} \sinh\left(a \ln\left(\frac{1}{a}\right)\right)$

$\dot a = H_0 \rightarrow t(a) = \frac{a}{H_0} \qquad t_0 = H_0^{-1}$

e) $\eta(a) = c\int_0^a \frac{da'}{a'^2 H(a)} = \frac{c}{H_0}\int_0^a \frac{da'}{a'} = \frac{c}{H_0} \ln(a')\Big|_0^a = \infty$

$x(a) = a\,\eta(a) = \infty \quad \rightarrow$ no particle horizon, observable universe fills entire actual univer

f) $\theta = \frac{x(a)}{D_A(a)} = \infty$

g) The observed angular scale for the first peak of CMB fluctuations corresponds to the sound particle horizon $x_s(a) \simeq \frac{x(a)}{\sqrt{3}}$ in this case as $c_s \simeq \frac{c}{\sqrt{3}}$ until a_{rec}. This is a strong hint that we live in a flat universe ($\theta_s(\Omega_m = \Omega_{tot} = 1) \simeq 1°$, $\theta_{obs} \simeq 0.9°$), but we need further information (= observations) to deduce the existence of Λ.