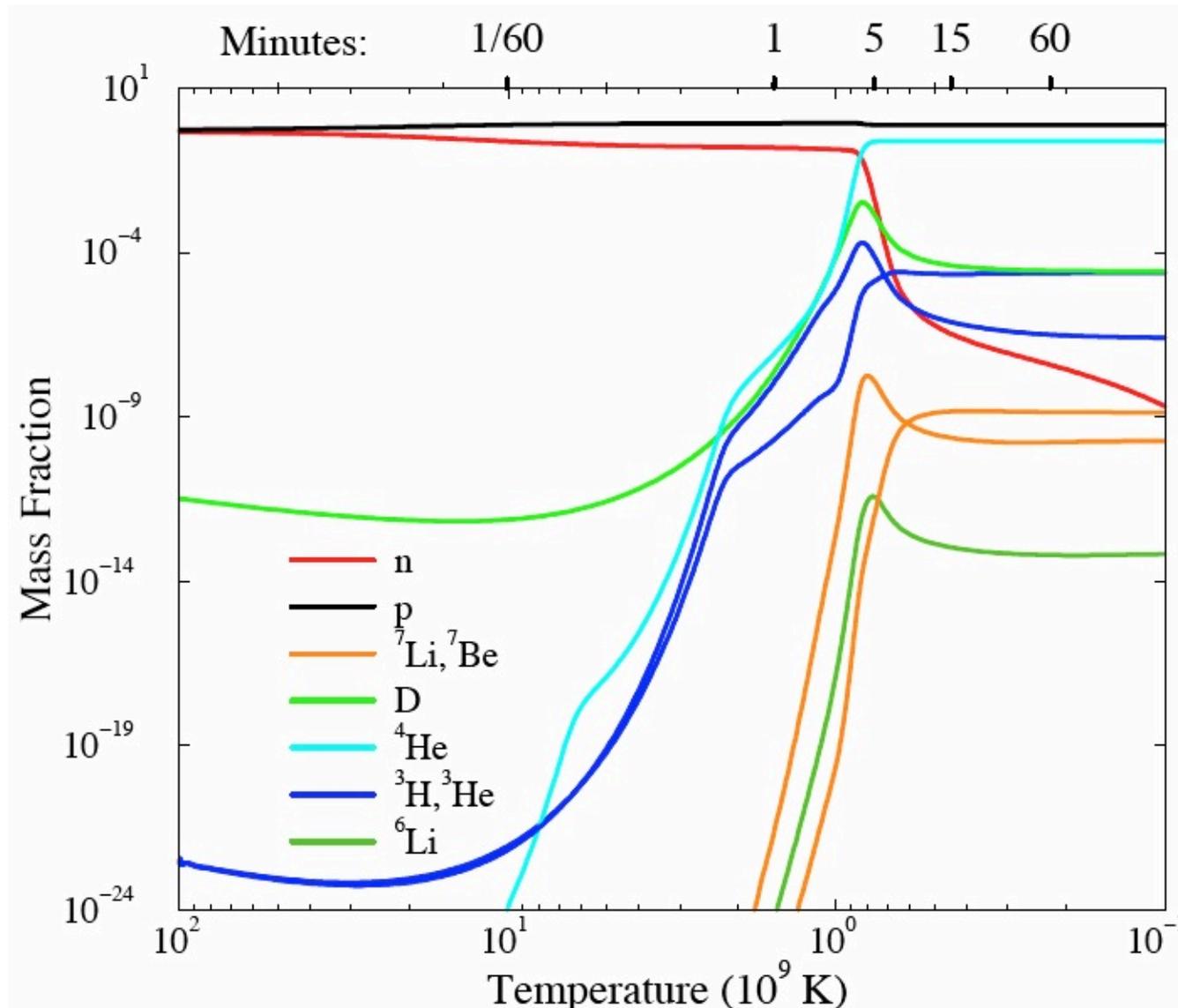


Ay 127, Winter 2017: The Early Universe



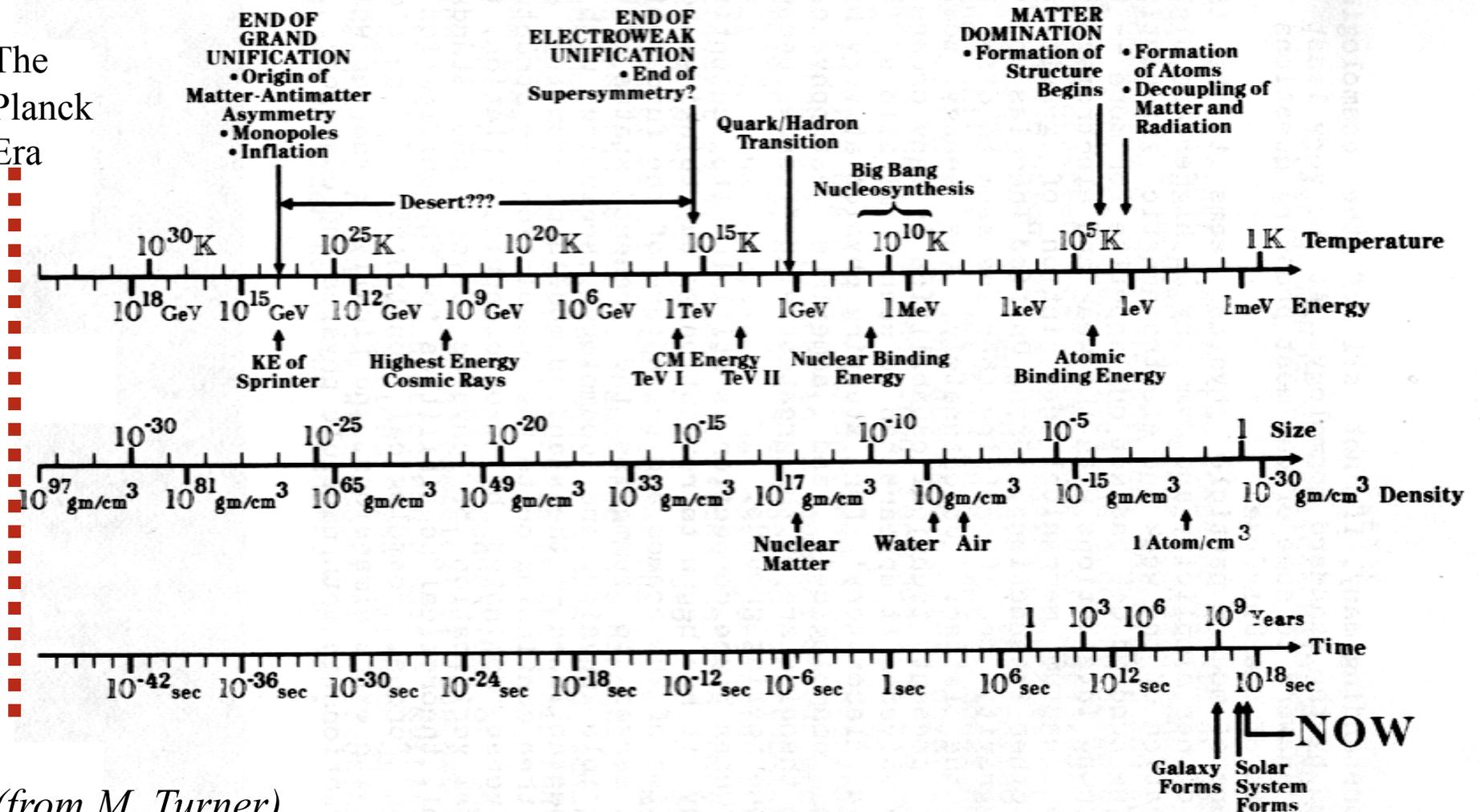
The Key Ideas

- Pushing backward in time towards the Big Bang, the universe was hotter and denser in a fairly predictable manner (aside from surprising “glitches” such as the inflation...)
- At any given time, the temperature translates into a characteristic mass of particles, which dominate that epoch: the Universe as the ultimate accelerator?
- As the energies increase, different physical regimes and different fundamental interactions come into play
- The closer we get to the Big Bang (i.e., further away from the experimentally probed regime), the less certain the physics: the early Universe as the laboratory of physics beyond the standard model?
- Our extrapolations must break down by the epoch of $\sim 10^{-43}$ sec \sim Plank time, where quantum gravity must be important

The Cosmic Thermal History

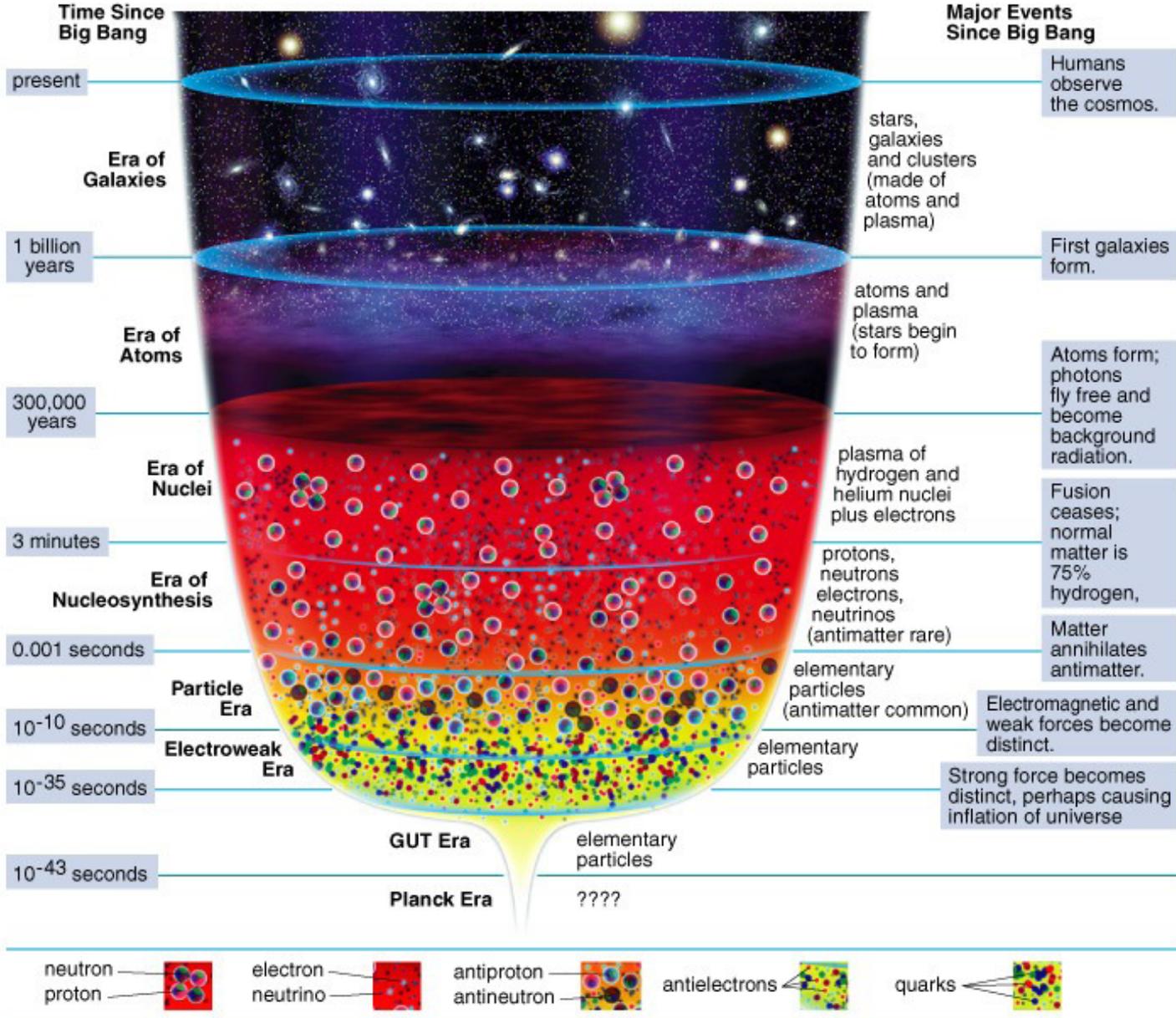
... on a logarithmic time axis - a theorist's delight!

The Planck Era



(from M. Turner)

Another Schematic Outline:



Some Key Moments in the Thermal History of the Universe:

- **Planck era, $t \sim 10^{-43}$ sec:** quantum gravity, ... ??? ...
- **Inflation, $t \sim 10^{-33}$ sec:** vacuum phase transition, exponential expansion
- **Grand Unification, $t \sim 10^{-32}$ sec:** strong and electroweak interactions split
- **Baryogenesis, $t \sim 10^{-6}$ sec:** quark-hadron transition
- **Nucleosynthesis, $t \sim 1$ ms to 3 min:** D, He, Li, Be form
- **Radiation to matter dominance transition, $t \sim 10^5$ yr:** structure begins to form
- **Recombination, $t \sim 380,000$ yr:** hydrogen becomes neutral, CMBR released, dark ages begin
- **Reionization, $t \sim 0.3 - 1$ Gyr:** first galaxies and QSOs reionize the universe, the cosmic renaissance

Thermal History of the Early Universe

Age	Temperature	Processes
$t < 10^{-10}$ s	$T > 10^{15}$ K	GUT
$10^{-10} < t < 10^{-4}$ s	$10^{15} > T > 10^{12}$ K	e^+ , e^- , quarks, γ , ν
$t \sim 10^{-4}$ s	$T \sim 10^{12}$ K	Quarks \rightarrow n, p $\mu^+ \mu^- \rightarrow \nu_\mu, \bar{\nu}_\mu$
$10^{-10} < t < 10^{-4}$ s	$10^{12} > T > 10^{10}$ K	e^+ , e^- , n, p, γ , ν_e
$t \sim 0.01$ s	$T \sim 10^{11}$ K	assymetry in n, p
$t \sim 4$ s	$T \sim 5 \times 10^9$ K	e^+ , $e^- \rightarrow \nu_e, \bar{\nu}_e$ $n \rightarrow p + e^- + \bar{\nu}_e$
$t \sim 100$ s	$T \sim 10^4$ K	nucleosynthesis
$t \sim 10^{11}$ s	$T \sim 16\,500$ K	Matter domination
$t \sim 10^{13}$ s	$T \sim 3000$ K	Decoupling

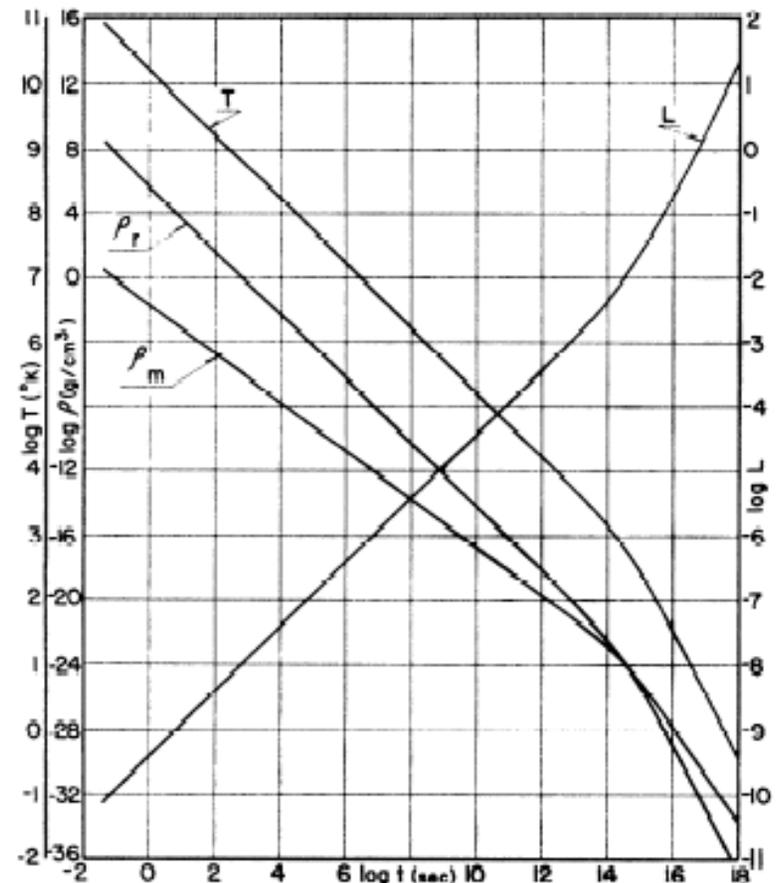
Empirical Evidence

- **The CMBR:** probes the recombination era, $t \sim 10^5$ yr, $z \sim 1100$, based on a well understood atomic and macroscopic physics
- **Nucleosynthesis:** probes the $t \sim 10^{-3} - 10^2$ sec era, $z \sim 10^9$, compare the model predictions with observed abundances of the lightest elements, based on a well understood nuclear physics
- **Matter-antimatter asymmetry:** probes the baryogenesis era, $t \sim 10^{-6}$ sec, $z \sim 10^{12}$, but only in suggesting that some symmetry breaking did occur
- **Predictions of the inflationary scenario:** flatness, uniformity of CMBR, absence of monopoles, the right type of density fluctuation spectrum - it all supports the idea that inflation did happen, but does not say a lot about its detailed physics
- Cosmological observations can indicate or constrain physics well outside the reach of laboratory experiments

CMBR and the Recombination Era

Prediction of CMB is trivial in Hot Big Bang model:

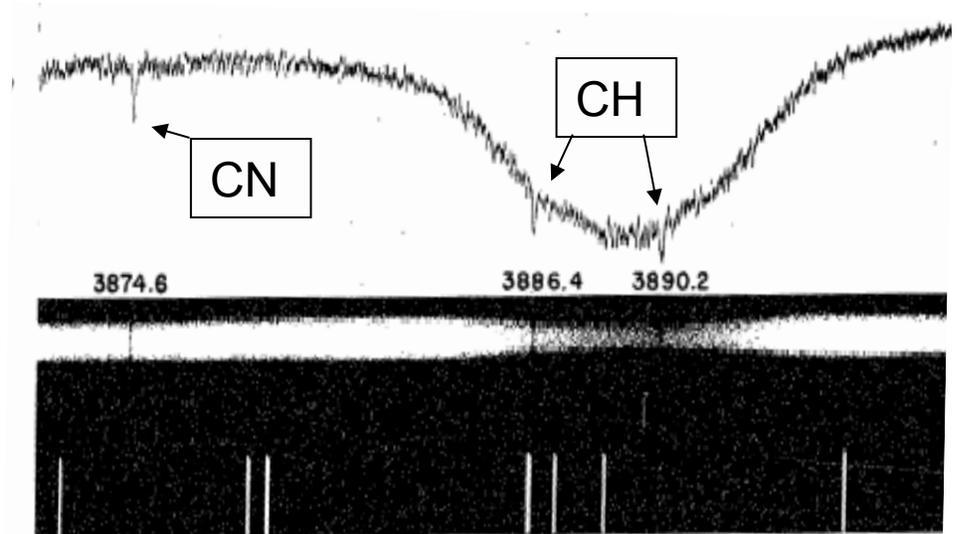
- Hot, ionised initial state should produce thermal radiation
- Photons decouple when universe stops being ionised (**last scattering**)
- Expansion by factor a cools a blackbody spectrum from T to T/a
- Therefore we should now see a cool blackbody background
 - Alpher and Herman, 1949, “A temperature now of the order of 5 K”
 - Dicke et al., 1965, “<40 K”
 - note that the Gamow, Alpher & Herman prediction had been nearly forgotten at this time!



The CMBR Discoveries

First seen in 1941 (yes, 1941!)

- Lines seen in stellar spectra identified as interstellar CH and CN (Andrew McKellar, theory; Walter Adams, spectroscopy)
- Comparison of lines from different rotational states gave “rotational temperature” of 2-3 K
- Unfortunately Gamow *et al.* did not have known about this
- Hoyle made the connection in 1950:
 "[the Big Bang model] would lead to a temperature of the radiation at present maintained throughout the whole of space much greater than McKellar's determination for some regions within the Galaxy."
- So, Penzias & Wilson made the recognized discovery in 1964



Discovery of the Cosmic Microwave Background (CMBR): A Direct Evidence for the Big Bang

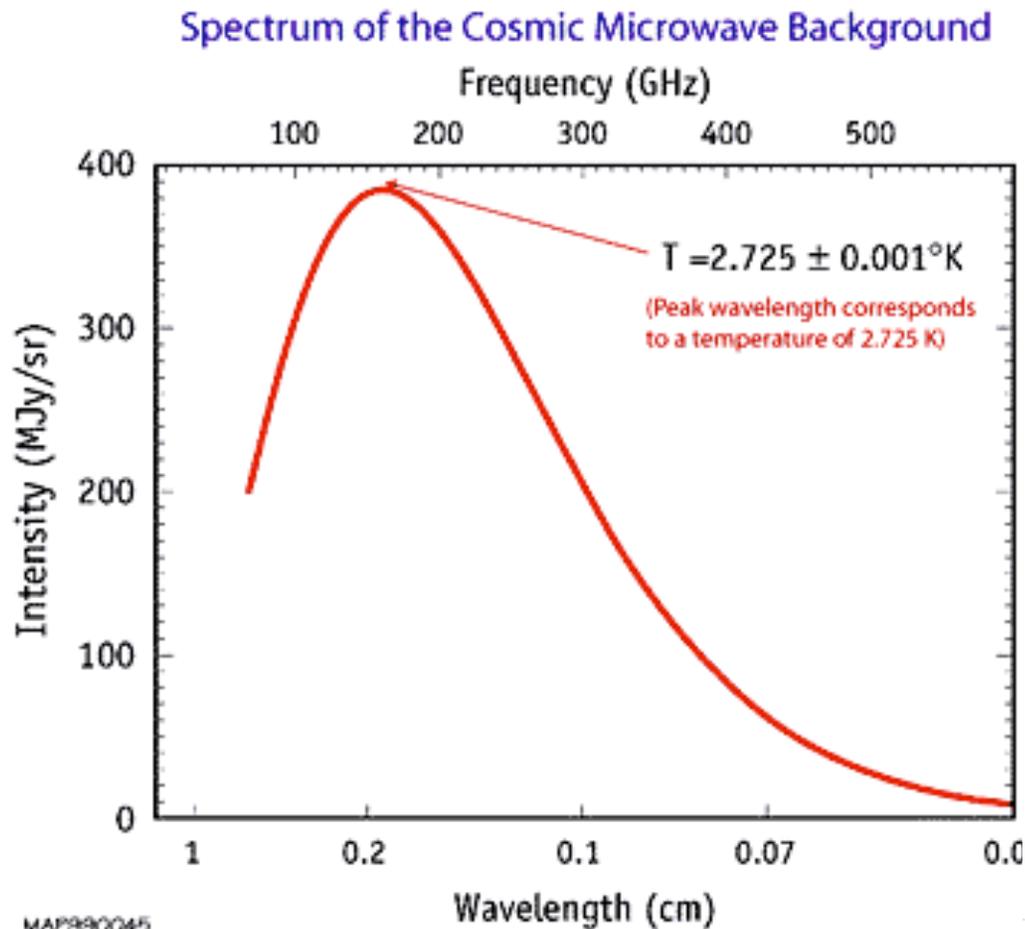


Arno Penzias &
Robert Wilson (1965)

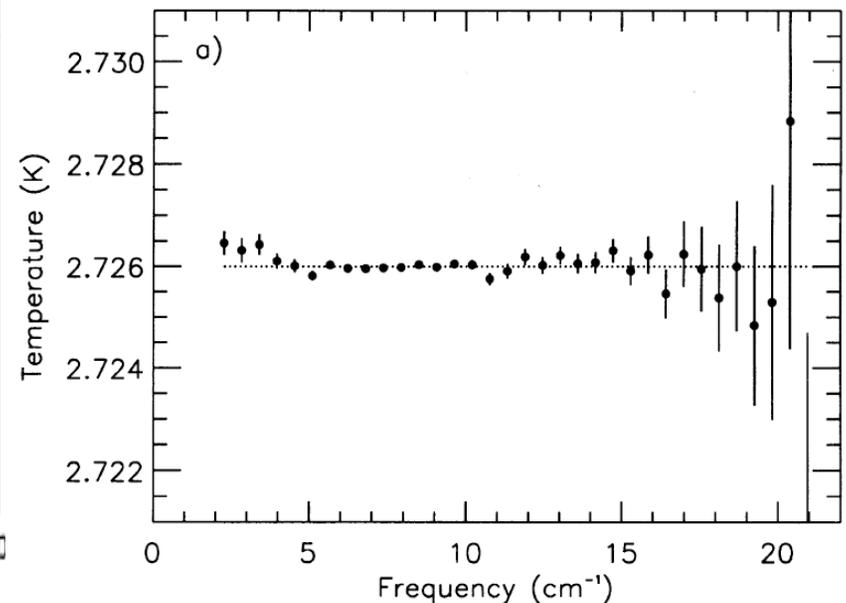
Nobel Prize, 1978



The CMBR Spectrum: A Nearly Perfect Blackbody



Residuals from the BB strongly limit possible energy injection (e.g., from hypothetical decaying DM particles) during and after the recombination - no new physics here...



Temperature of Recombination

Mean photon energy: $\langle E \rangle \sim 3k_B T$

Ionisation energy of H: $E = 13.6 \text{ eV}$

Photoionisation Temperature: $T = \frac{13.6 \text{ eV}}{3k_B} = 50000\text{K}$

But there are many more photons than H ions: $n_\gamma \approx 10^9 n_p$

Thus, the T can be lower and have enough photons with high enough energies to ionize H!

Boltzmann distribution: $n_\gamma (> E) \propto \exp\left(-\frac{E}{k_B T}\right)$

So the actual T of recombination is:

$$T = \frac{13.6 \text{ eV}}{3k_B 9 \ln(10)} = 2500\text{K}$$

And thus, $z_{rec} \sim 1100$

The Extent of Recombination

This phase transition (ionized to neutral gas) has a finite thickness: most of the plasma recombines before the last scattering, which is the CMBR photosphere we see

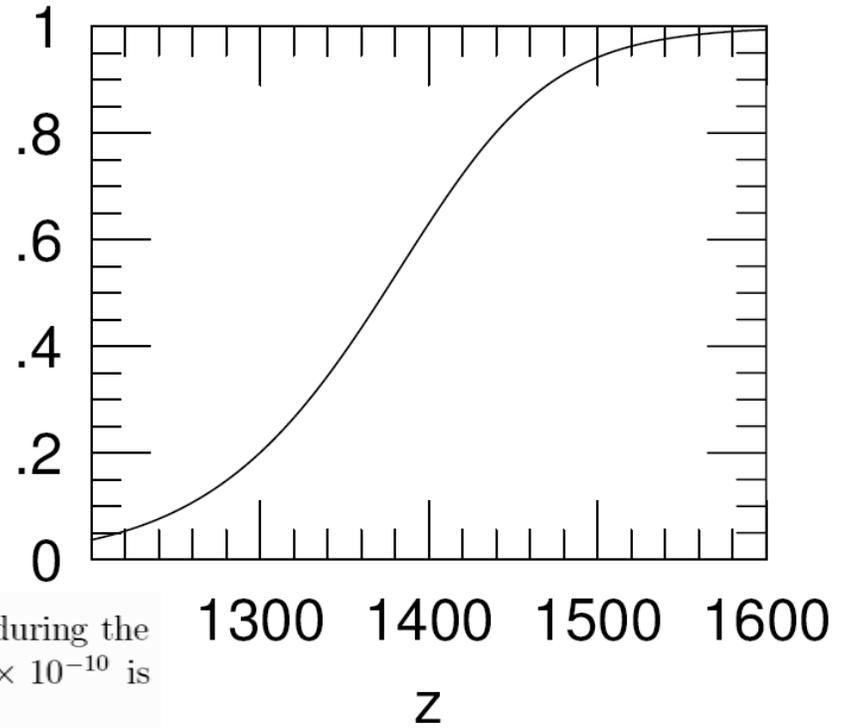


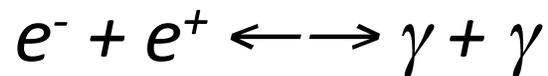
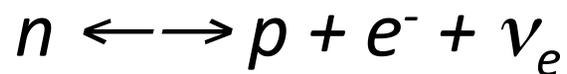
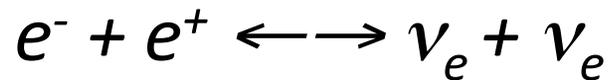
Figure 9.4: The fractional ionization X as a function of redshift during the epoch of recombination. A baryon-to-photon ratio of $\eta = 5.5 \times 10^{-10}$ is assumed.

Table 9.1: Events in the early universe

event	redshift	temperature (K)	time (megayears)
radiation-matter equality	3570	9730	0.047
recombination	1370	3740	0.24
photon decoupling	1100	3000	0.35
last scattering	1100	3000	0.35

Into the Nucleosynthesis Era

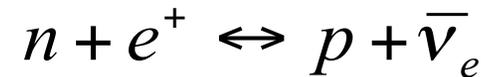
- In the pre-nucleosynthesis universe, the radiation produces pairs of electrons and positrons, as well as protons and antiprotons, neutrons and antineutrons, and they can annihilate; $e^+ e^-$ reactions produce electron neutrinos (ν_e) and antineutrinos:



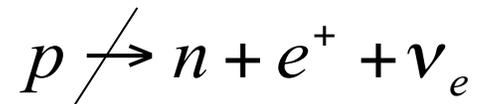
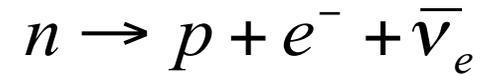
- This occurs until the temperature drops to $T \sim 10^{10}$ K, $t \sim 1$ sec
- In equilibrium there will be slightly more protons than neutrons since the neutron mass is slightly (1.293 MeV) larger
- This leads to an asymmetry between protons and neutrons ...

Asymmetry in Neutron / Proton Ratio

Mass difference between n and p causes an asymmetry via reactions:



It is slightly easier (requires less energy) to produce p than n :



Thus, once e^+ , e^- annihilation occurs only neutrons can decay

We can calculate the equilibrium ratio of n to p via the Boltzmann equation,

$$X_n = \frac{N_n}{N_n + N_p} \sim 0.16 \exp\left(-\frac{t}{1013s}\right)$$

at $T \sim 10^{12}$ K, $n/p = 0.985$

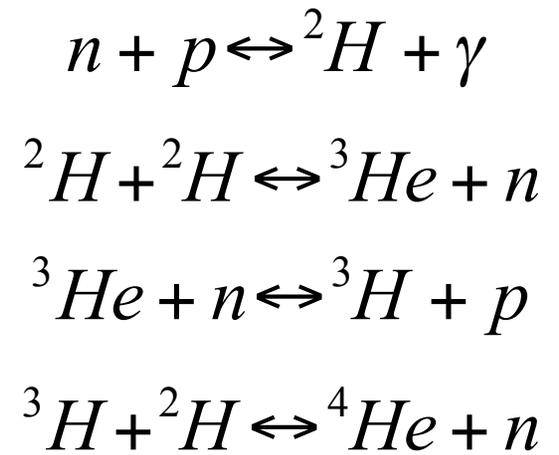
The n/p ratio is “frozen” at the value it had at when $T = 10^{10}$ K, $n/p = 0.223$, i.e., for every 1000 protons, there are 223 neutrons

Big Bang Nucleosynthesis (BBNS)

Free neutrons are unstable to beta decay, with *mean lifetime* = 886 sec, $n \rightarrow p + e^- + \nu_e$. This destroys $\sim 25\%$ of them, before they can combine with the protons

When the temperature drops to $\sim 10^9$ K (t=230s), neutrons and protons combine to form deuterium, and then helium:

Note that these are not the same reactions as in stars (the *pp* chain)!

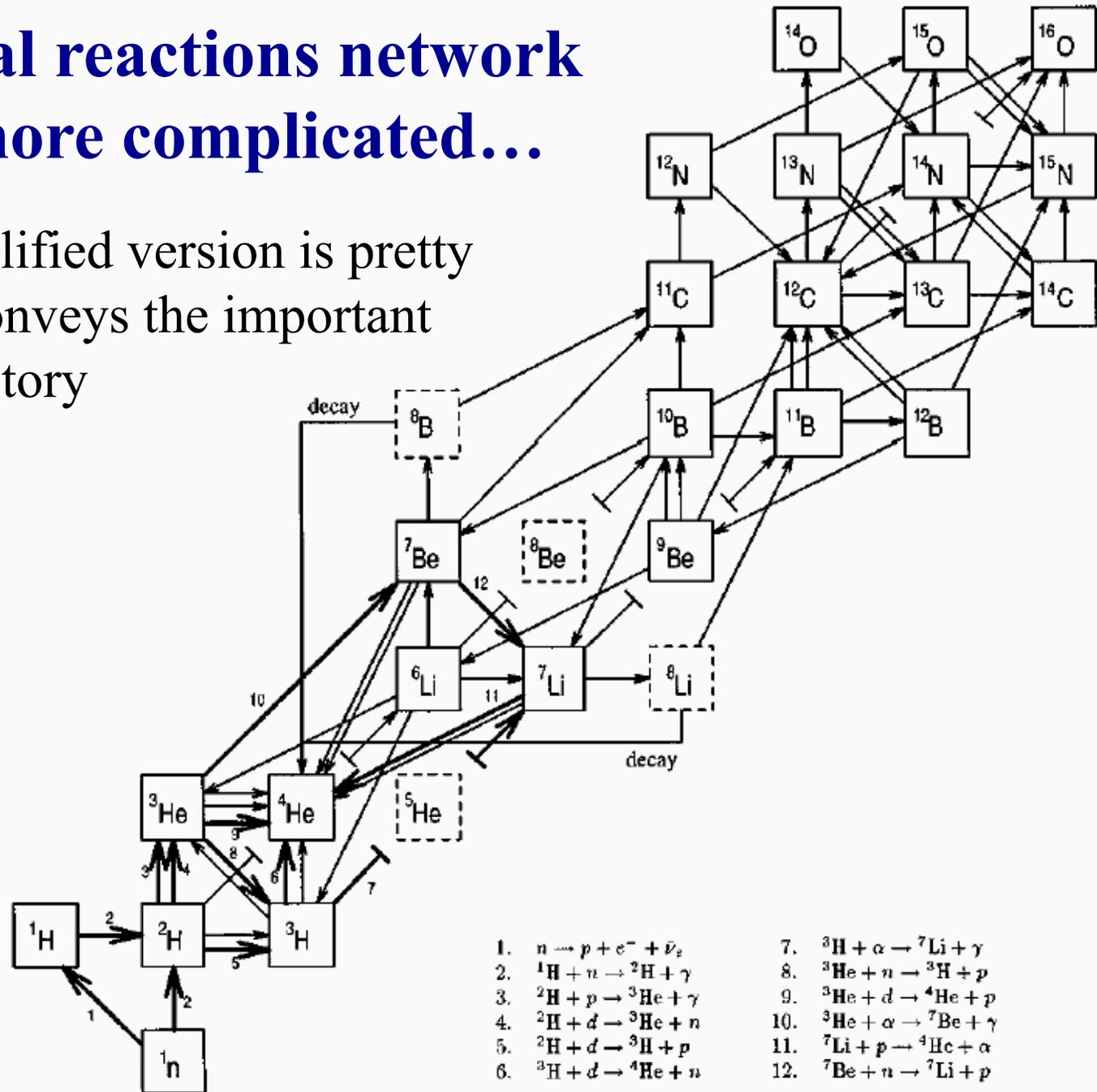


Photons break the newly created nuclei, but as the temperature drops, the photodissociation stops

At $t \sim 10^3$ sec and $T < 3 \times 10^8$ K, the density also becomes too low for fusion, and BBN ends. This is another “freeze-out”, as no new nuclei are created and none are destroyed

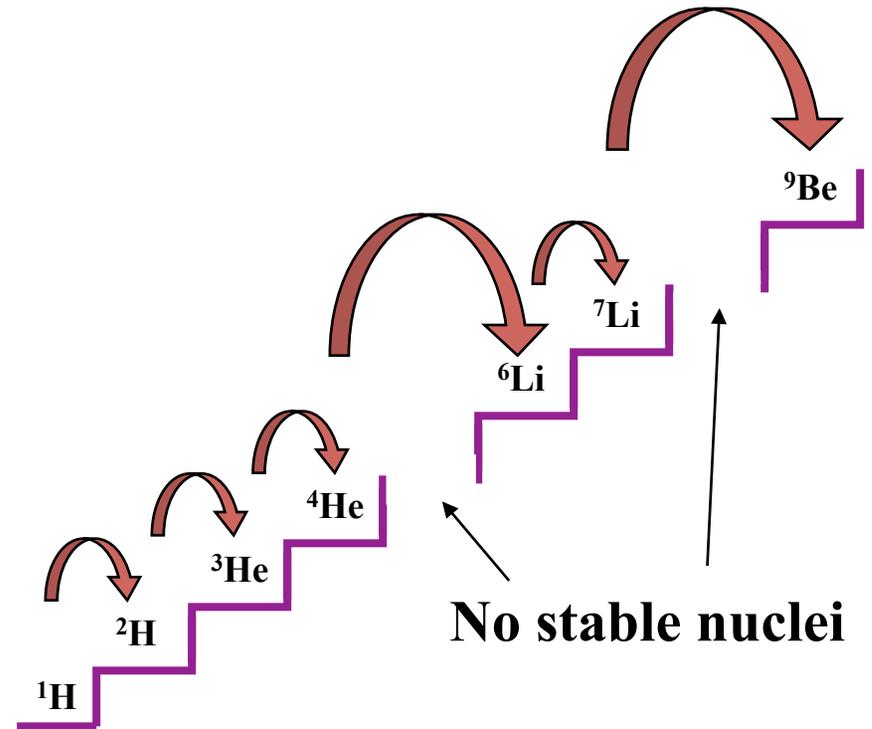
The actual reactions network is a tad more complicated...

But the simplified version is pretty close, and conveys the important parts of the story



Stable Mass Gaps in the Periodic Table

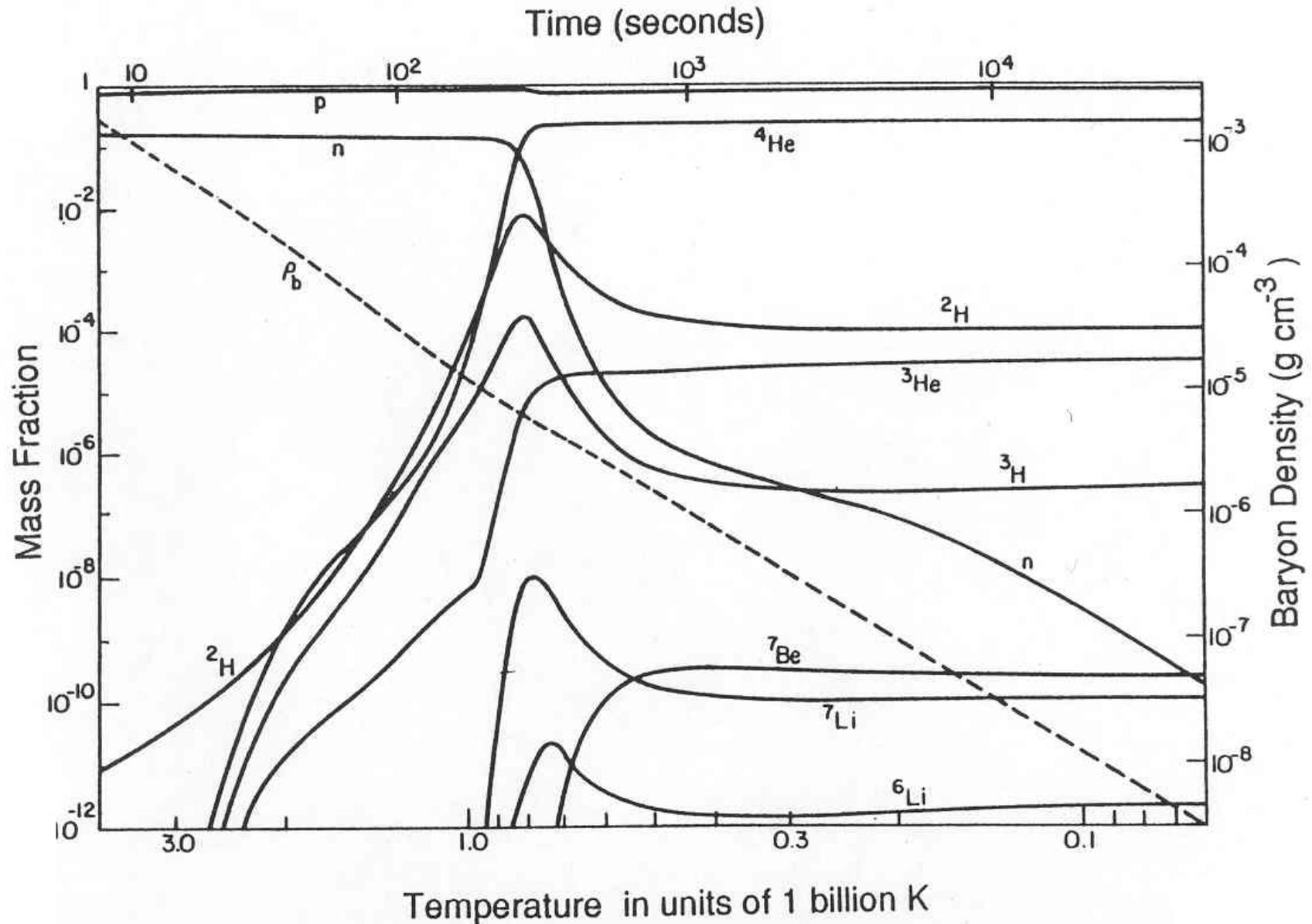
1																	2
H																	He
3	4											5	6	7	8	9	10
Li	Be											B	C	N	O	F	Ne
11	12											13	14	15	16	17	18
Na	Mg											Al	Si	P	S	Cl	Ar
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
55	56	57	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
87	88	89	104	105	106	107	108	109	110								
Fr	Ra	Ac	Unq	Unp	Unh	Uns	Uno	Une	Uun								
58	59	60	61	62	63	64	65	66	67	68	69	70	71				
Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu				
90	91	92	93	94	95	96	97	98	99	100	101	102	103				
Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr				



Since there are no stable mass-5 nuclides, combining He and tritium to get Li requires overcoming the Coulomb repulsion. So almost all of the neutrons end up in He instead!

There is another gap at mass-8, so BBN ends with Li, with only trace amounts of Be produced

The Evolution of Abundances in BBNS



Big Bang Nucleosynthesis End

At this point n/p ratio has dropped to ~ 0.14 . The excess protons account for about 75% of the total mass, and since essentially all neutrons are incorporated into He nuclei, the predicted primordial He abundance is $\sim 25\%$ - about as measured

Thus neutron/proton asymmetry caused by their mass difference and the beta decay of neutrons determines primordial abundance of He and other light elements

Because all the neutrons are tied up in He, its abundance is not sensitive to the matter density. In contrast, the abundances of other elements produced in the early universe, D, ^3He , and ^7Li are dependent on the amount of baryonic matter in the universe

The universe expanded too rapidly to build up heavier elements!

BBNS Predictions

- The BBNS makes detailed predictions of the abundances of light elements: ^2D , ^3He , ^4He , ^7Li , ^8Be
- These are generally given as a function of the baryon to photon ratio $\eta = n_n/n_\gamma$, usually defined in units of 10^{10} , and directly related to the baryon density Ω_b : $\eta_{10} = 10^{10}(n_n/n_\gamma) = 274 \Omega_b h^2$
- As the universe evolves η is preserved, so that what we observe today should reflect the conditions in the early universe
- Comparison with observations (consistent among the different elements) gives:

$$\Omega_{\text{baryons}} h^2 = 0.021 \rightarrow 0.025$$

- This is in a spectacularly good agreement with the value from the CMB fluctuations: $\Omega_{\text{baryons}} h^2 = 0.024 \pm 0.001$

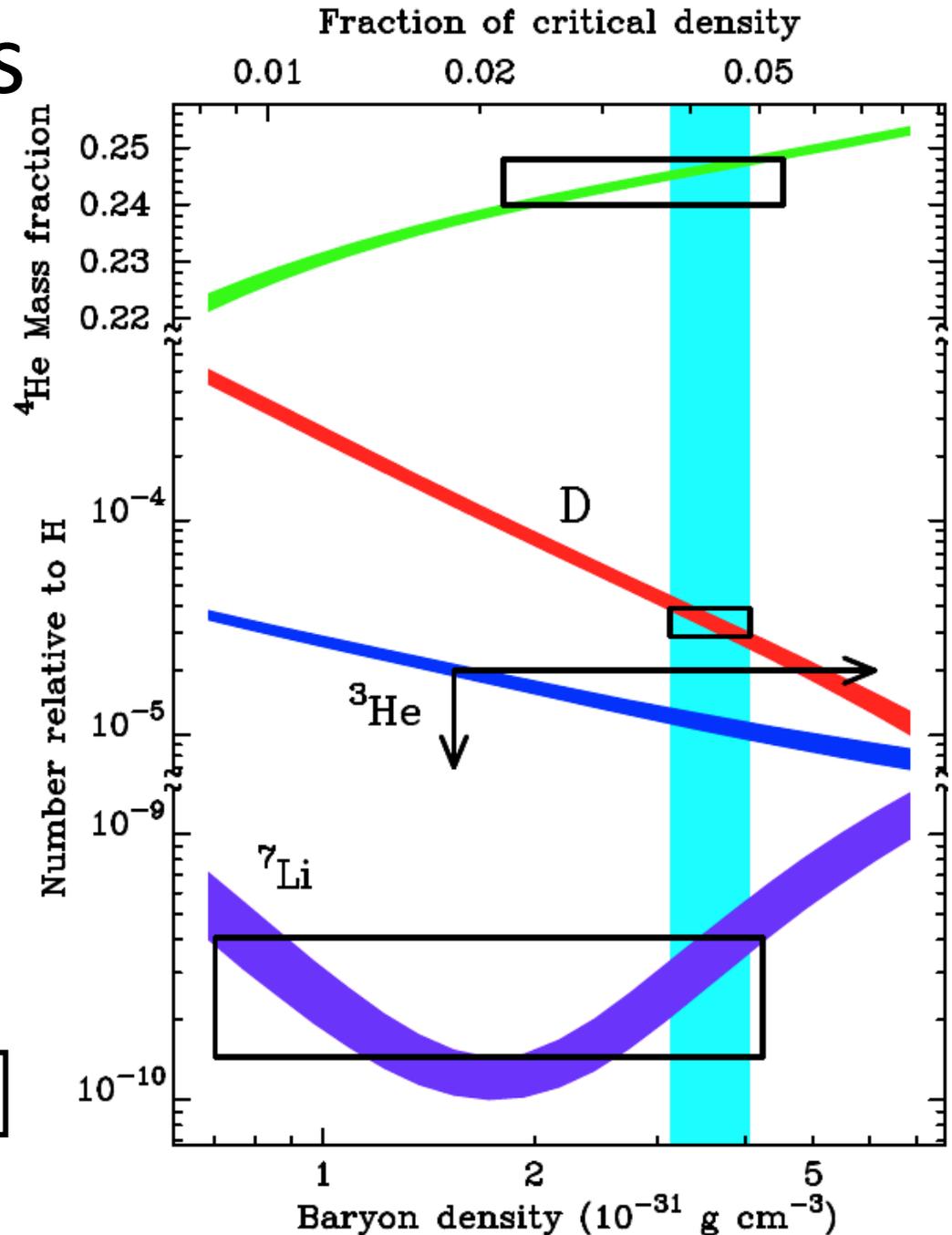
BBNS Predictions

^4He : the higher the density, the more of it is made \rightarrow

^2D , ^3He : easily burned into ^4He , so abundances are lower at higher densities \rightarrow

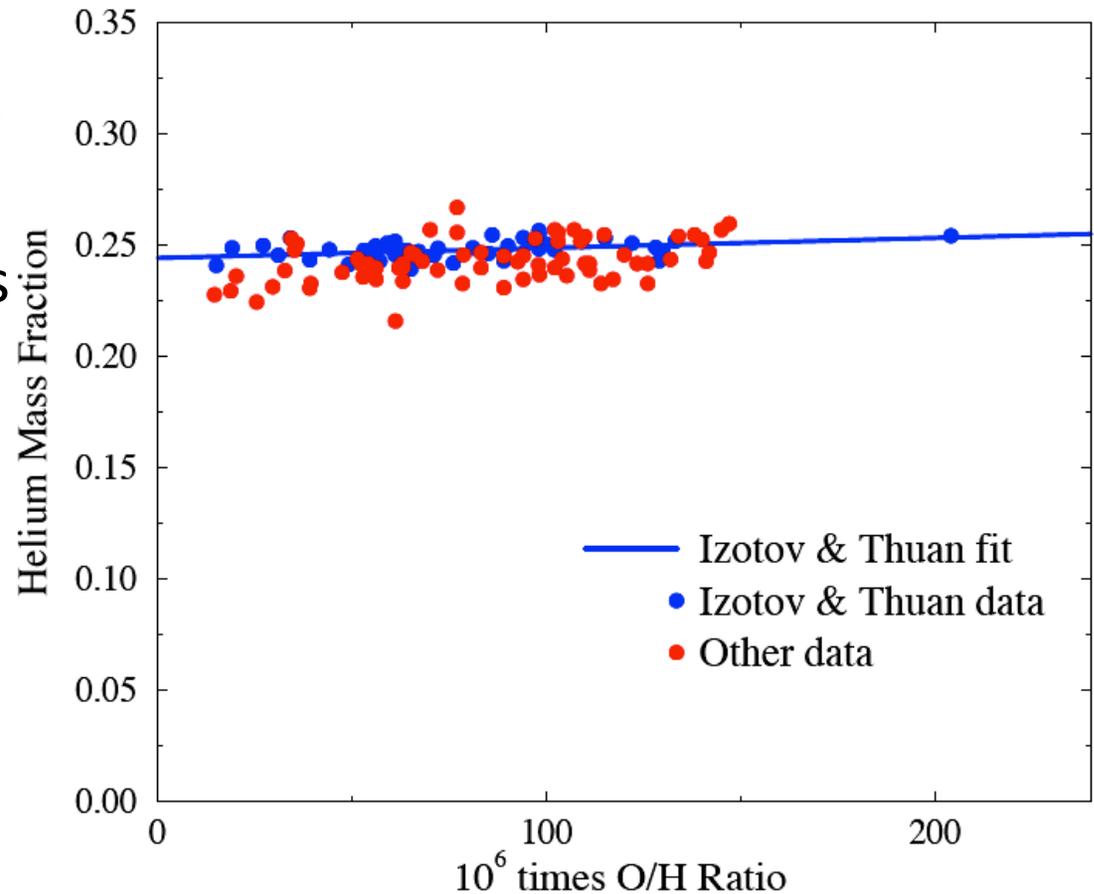
^7Li : ... complicated \rightarrow

Boxes indicate observed values



Helium-4 Measurements

- He is also produced in stars, but this “secondary” abundance is expected to correlate with abundances of other nucleosynthetic products, e.g., oxygen
- Observe ^4He from recombination lines in extragalactic HII regions in low-metallicity starforming galaxies
- The intercept at the zero oxygen abundance should represent the primordial (BBNS) value
- The result is: $Y_{\text{BBNS}} = 0.238 \pm 0.005$



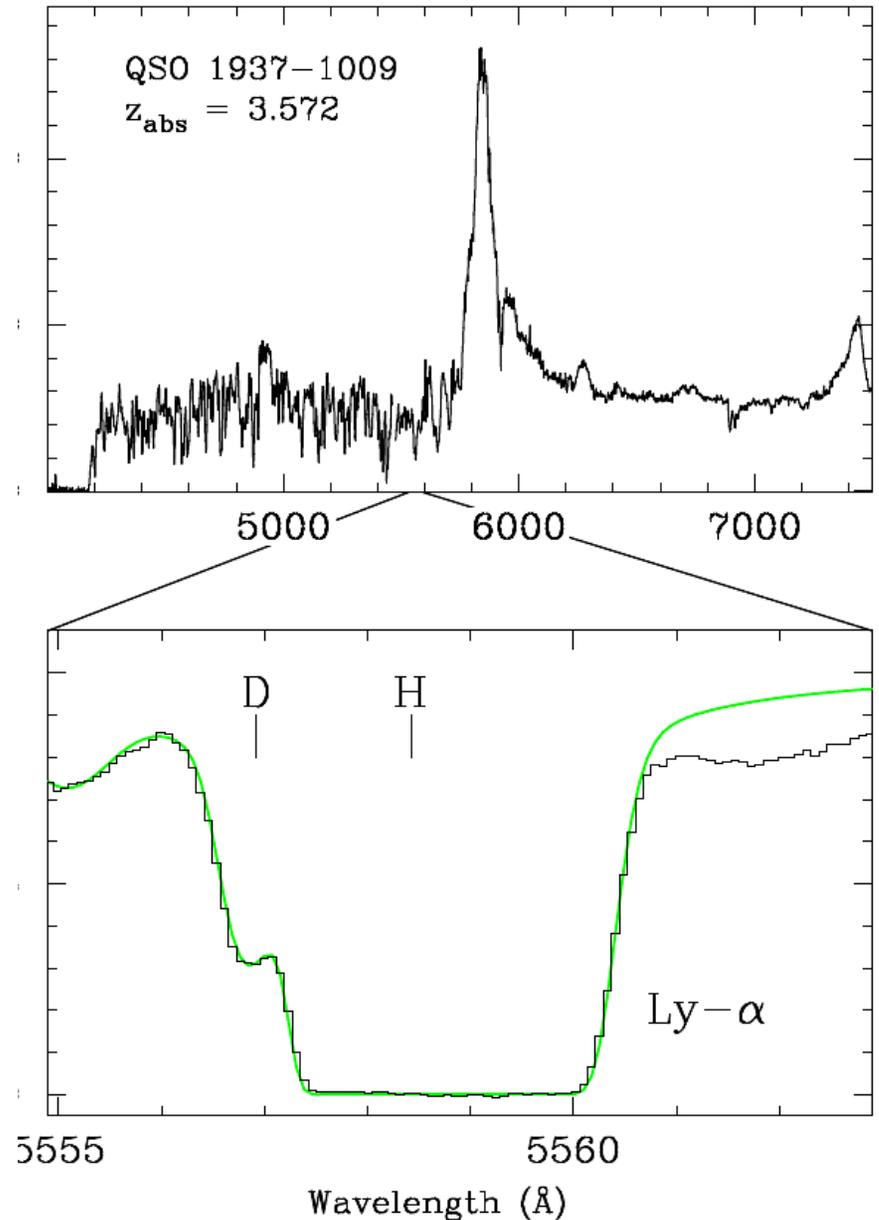
Deuterium Measurements

Deuterium is easily destroyed in stars, and there is no known astrophysical process where it can be created in large amounts after the BBNS

Thus, we need to measure it in a “pristine” environment, e.g., in QSO absorption line systems

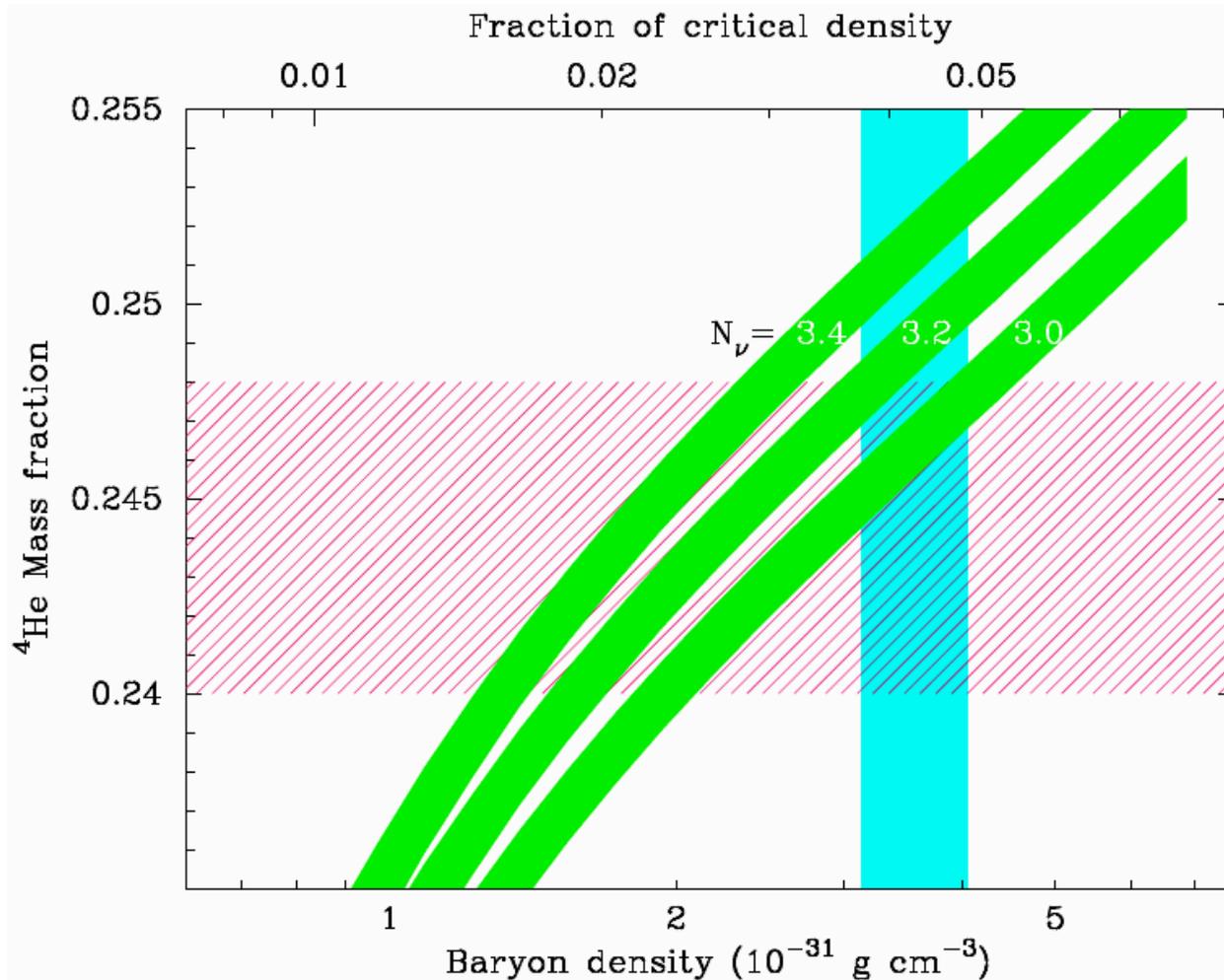
It is a tricky measurement and it requires high resolution spectra from 8-10 m class telescopes

The result is: $\frac{D}{H} = 2.74 \times 10^{-5}$



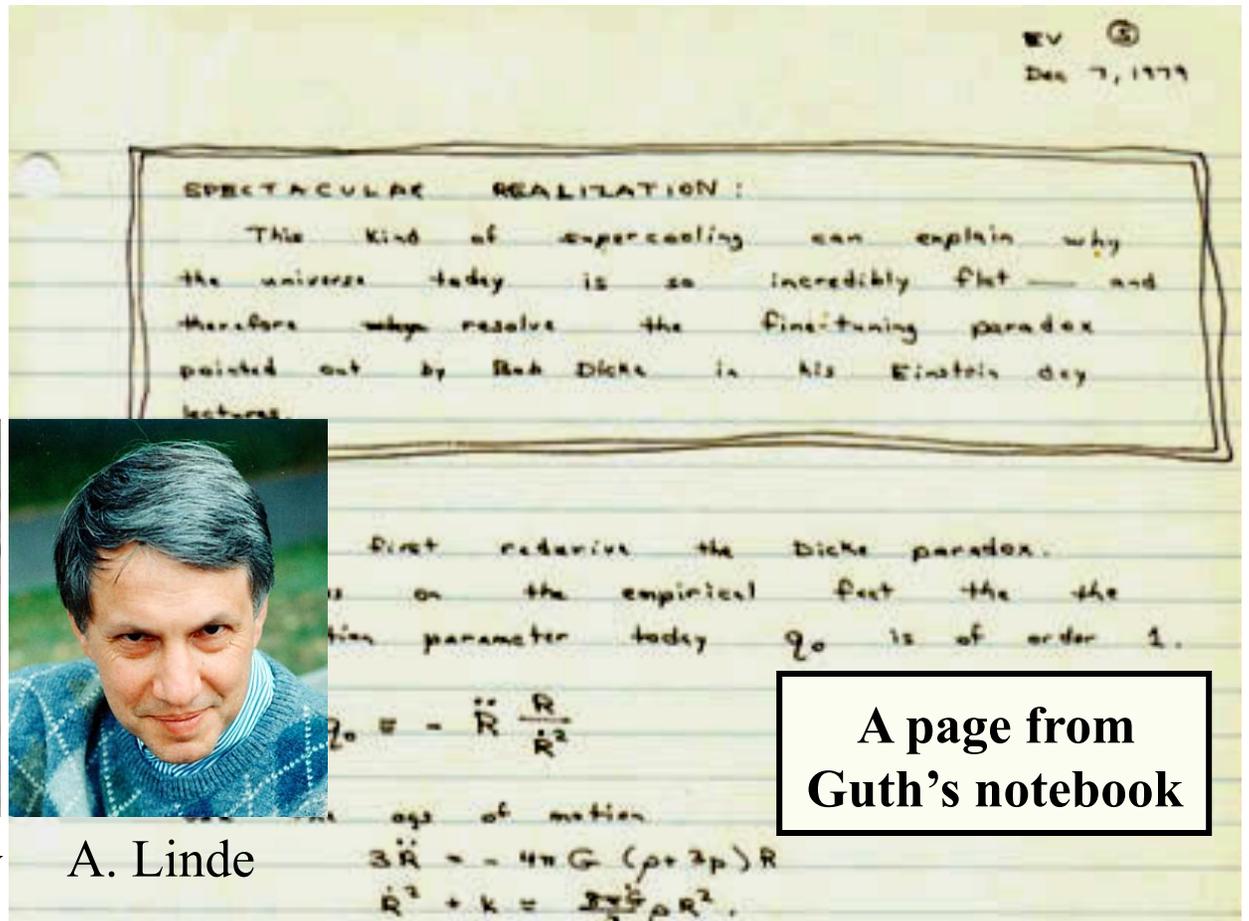
BBNS and Particle Physics

BBNS predictions also depend on the number of lepton (neutrino) families. Indeed, only 3 are allowed:



The Idea of Inflation

- Alan Guth (1980); precursors: D. Kazanas, A. Starobinsky
- Explains a number of fundamental cosmological problems: flatness, horizon, origin of structure, absence of topological defects...
- Developed further by P. Steinhardt, A. Albrecht, A. Linde, and many others



A. Guth



A. Starobinsky



A. Linde

The Inflationary Scenario

It solves **3 key problems** of the Big Bang cosmology:

- 1. The flatness problem:** why is the universe so close to being flat today?
- 2. The horizon problem:** how comes the CMBR is so uniform?
- 3. The monopole problem:** where are the copious amounts of magnetic monopoles predicted to exist in the BB cosmology?

... It also *accounts naturally for the observed power spectrum of the initial density perturbations*

... *It predicts* a similar, scale-invariant spectrum for the cosmic gravitational wave background

... And it implies a much, much(!) bigger universe than the observable one

The Flatness Problem

The expanding universe evolves away from $\Omega_{\text{tot}} = 1$:

$$1 - \Omega(t) = -\frac{kc^2}{H(t)^2 a(t)^2 R_0^2} = \frac{H_0^2 (1 - \Omega_0)}{H(t)^2 a(t)^2}$$

$$\left(\frac{H(t)}{H_0}\right)^2 = \frac{\Omega_{r0}}{a^4} + \frac{\Omega_{m0}}{a^3}$$

This creates an *enormous fine-tuning problem*: the early universe must have been remarkably close to $\Omega_{\text{tot}} = 1$ in order to have $\Omega_{\text{tot}} \sim 1$ today !

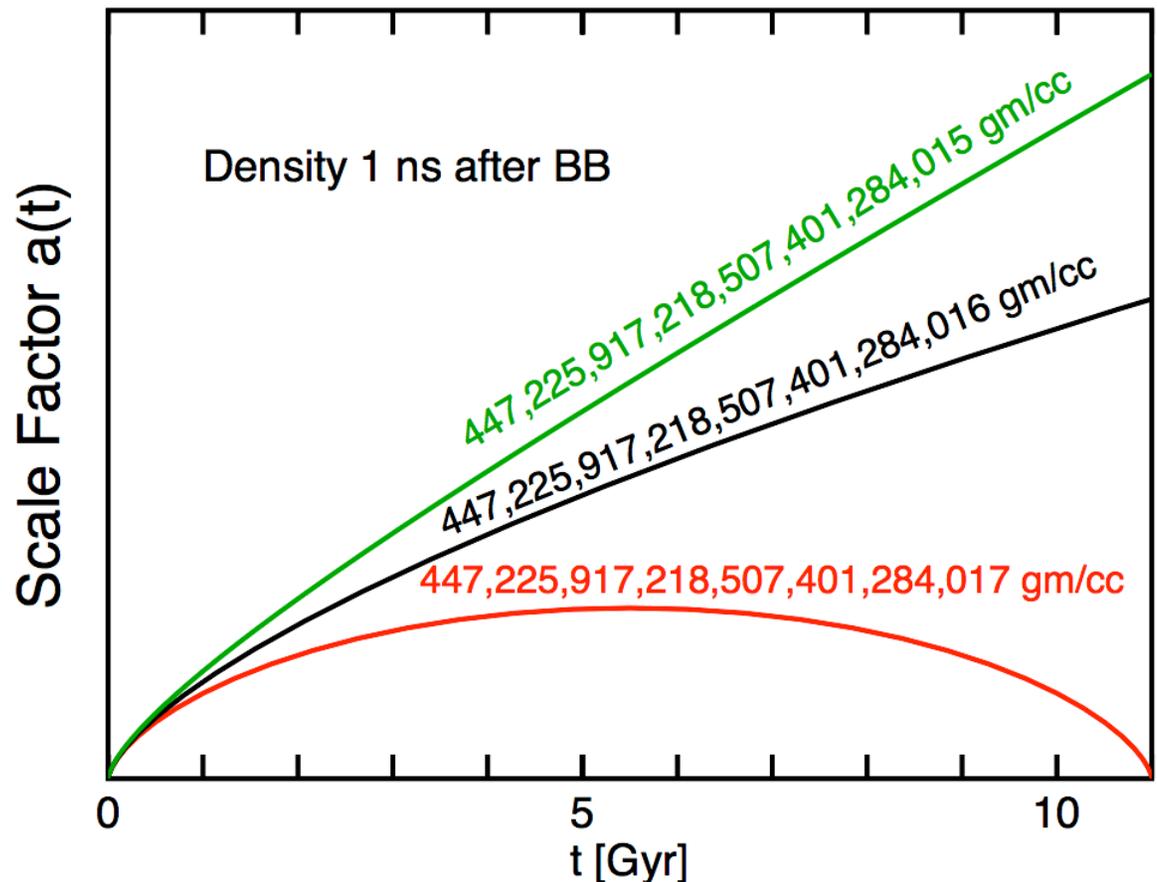


Fig. 11.— Scale factor vs. time for open, critical and closed Universes with density at 1 nanosecond after the Big Bang indicated. (from N. Wright)

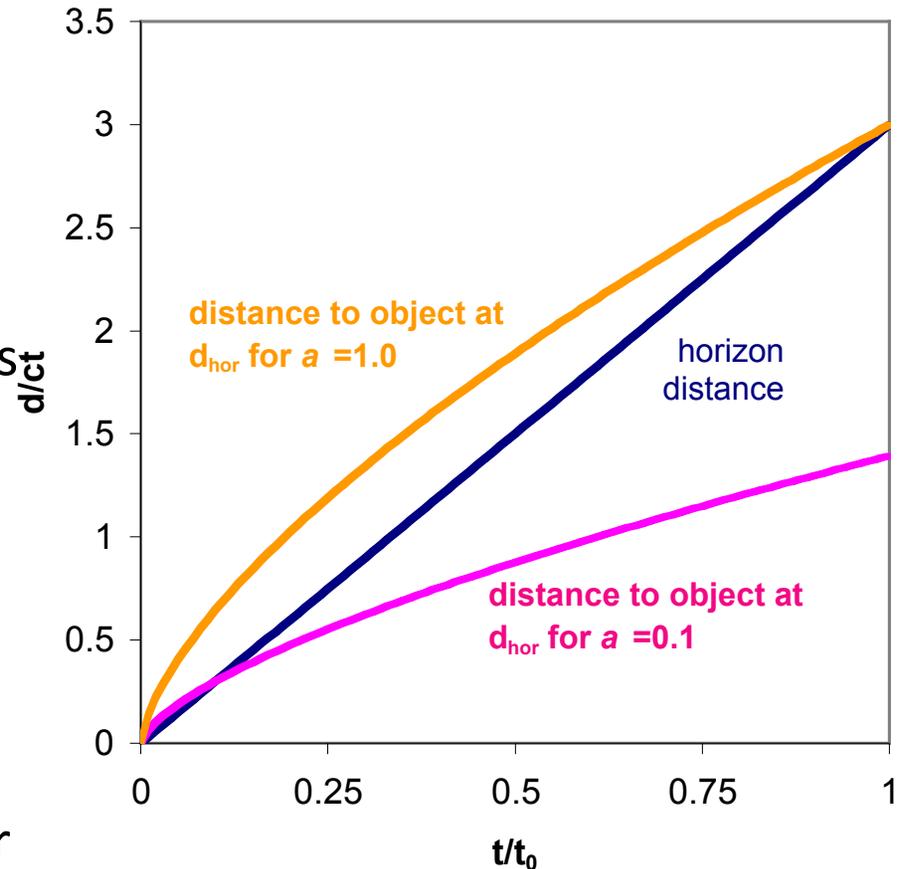
The Horizon Problem

Consider matter-only universe:

- Horizon distance $d_H(t) = 3ct$
- Scale factor $a(t) = (t/t_0)^{2/3}$
- Therefore horizon expands faster than the universe, so new" objects are constantly coming into view

Consider CMBR:

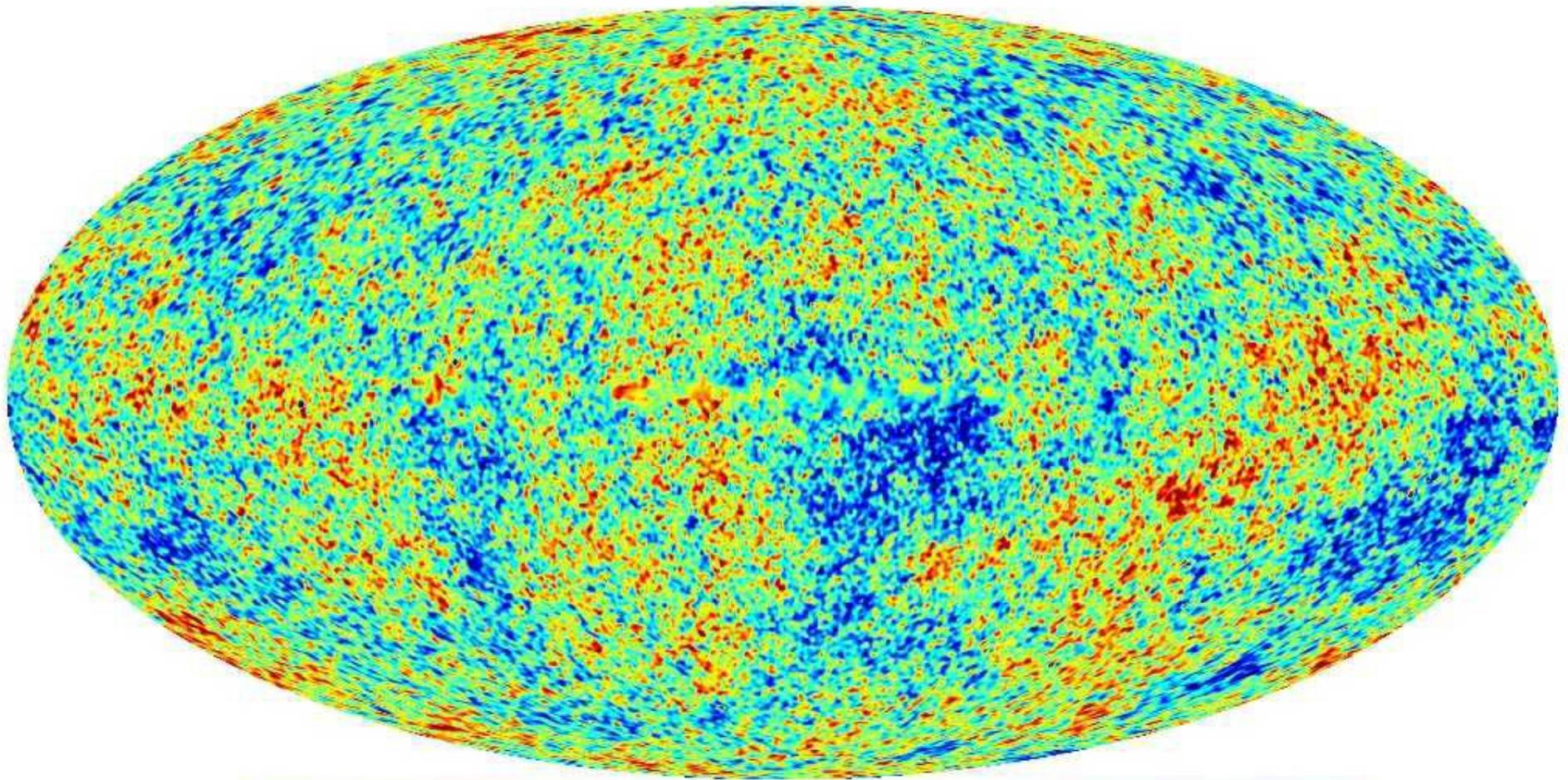
- It decouples at $1+z \sim 1000$
- i.e., $t_{\text{CMB}} = t_0/10^{4.5}$
- Then $d_H(t_{\text{CMB}}) = 3ct_0/10^{4.5}$
- Now this has expanded by a factor of 1000 to $3ct_0/10^{1.5}$
- But horizon distance now is $3ct_0$
- So angle subtended on sky by one CMB horizon distance is only $\sim 2^\circ$



→ Patches of CMB sky $> 2^\circ$ apart should not be causally connected!

CMBR is Uniform to $\Delta T/T \sim 10^{-6}$

Yet the projected size of the particle horizon at the decoupling was $\sim 2^\circ$ - these regions were causally disconnected - so how come?



-200 μ K  200 μ K

The Monopole Problem

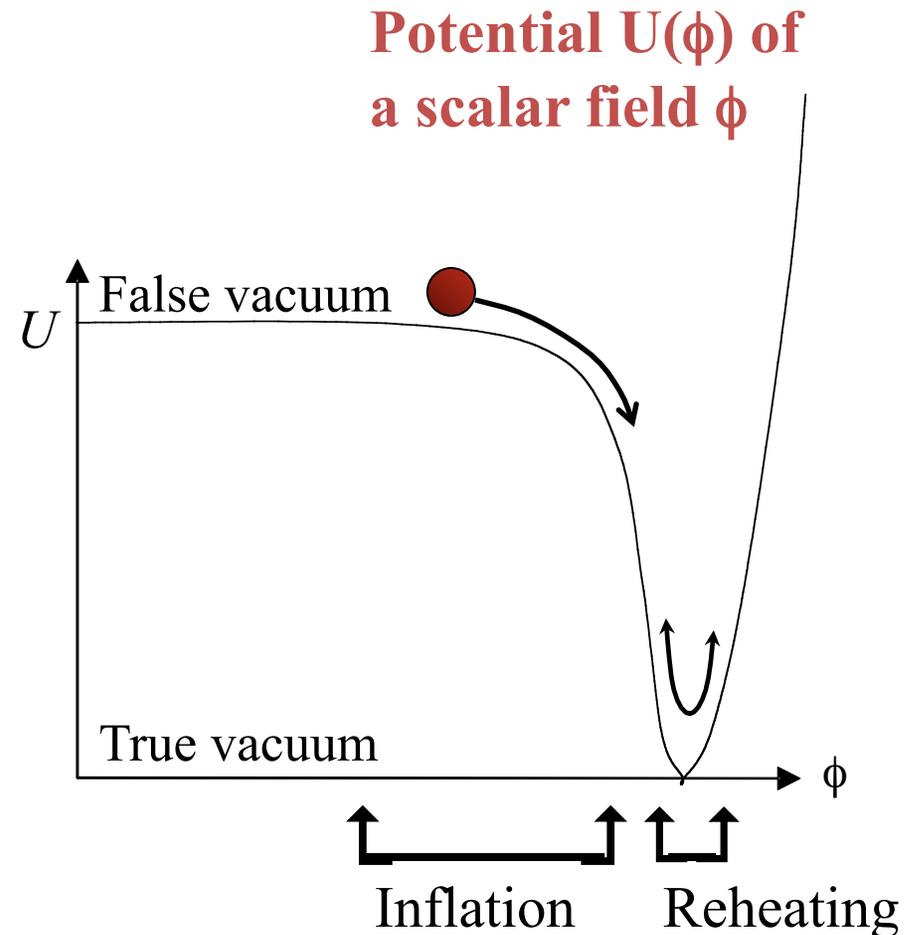
- Magnetic monopoles are believed to be an inevitable consequence of Grand Unification Theories (GUTs)
 - Point-like topological defects arising during the phase transition when the strong and the electroweak forces decouple
- Expect enormous numbers of them
 - Mass $\sim 10^{16} m_p$; dominate all other matter density by a factor of $\sim 10^{12}$ and thus close the universe and drive it to a Big Crunch a long time ago ...
- Not observed! So, where are they?

Inflationary Universe Scenario

- If there is a Theory of Everything (TOE) that unifies all four forces it will break spontaneously at the Planck time ($t \sim 10^{-43}$ sec) into the gravitation and a unified version of the magnetic, electroweak, and strong forces – a Grand Unified Theory (GUT)
- The GUT will hold until $T \sim 10^{28}$ K, or $t \sim 10^{-34}$ sec. At this point the universe entered a period of “**false vacuum**”: the energy level higher than the lowest, “ground” state
- Symmetry breaking in GUT theories is associated with massive Higgs bosons, which are quanta of a scalar field that has an associated potential which describes the energy of the field
- The false vacuum is a metastable state, with its vacuum energy acting as a “negative pressure” causing the universe to expand exponentially as it “rolls down the scalar field”

Inflation With a Scalar Field

- Need potential U with broad nearly flat plateau near $\phi = 0$
- This is the metastable **false vacuum**
- Inflation occurs as ϕ moves slowly away from 0
- It stops at drop to minimum U - the **true vacuum**
- Decay of inflaton field ϕ at this point **reheats** universe, producing photons, quarks, etc. - *all of the matter/energy content of the universe is created in this process*
- This is equivalent to latent heat of a phase transition



The Cosmic Inflation

Recall that the energy density of the physical vacuum is described as the *cosmological constant*. If this is the dominant density term, the Friedmann Eqn. is: $\left(\frac{\dot{a}}{a}\right)^2 = \frac{\Lambda_i}{3}$

The solution is obviously:

$$a(t) \propto e^{H_i t}$$

In the model where the GUT phase transition drives the inflation, the net expansion factor is:

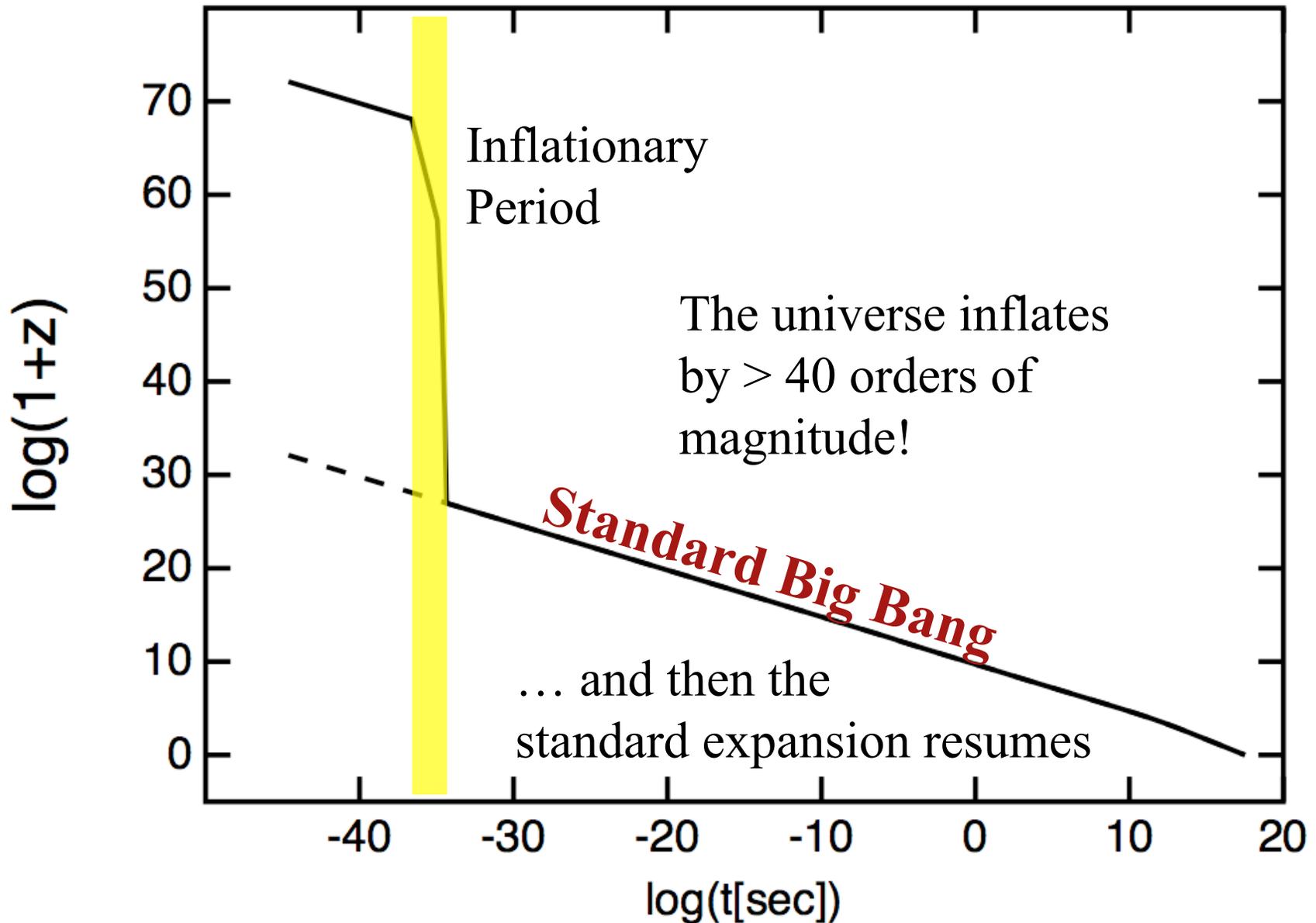
$$\frac{a(t_f)}{a(t_i)} \sim e^{100} \sim 10^{43}$$

The density parameter evolves as: $|1 - \Omega(t)| \propto e^{-2H_i t}$

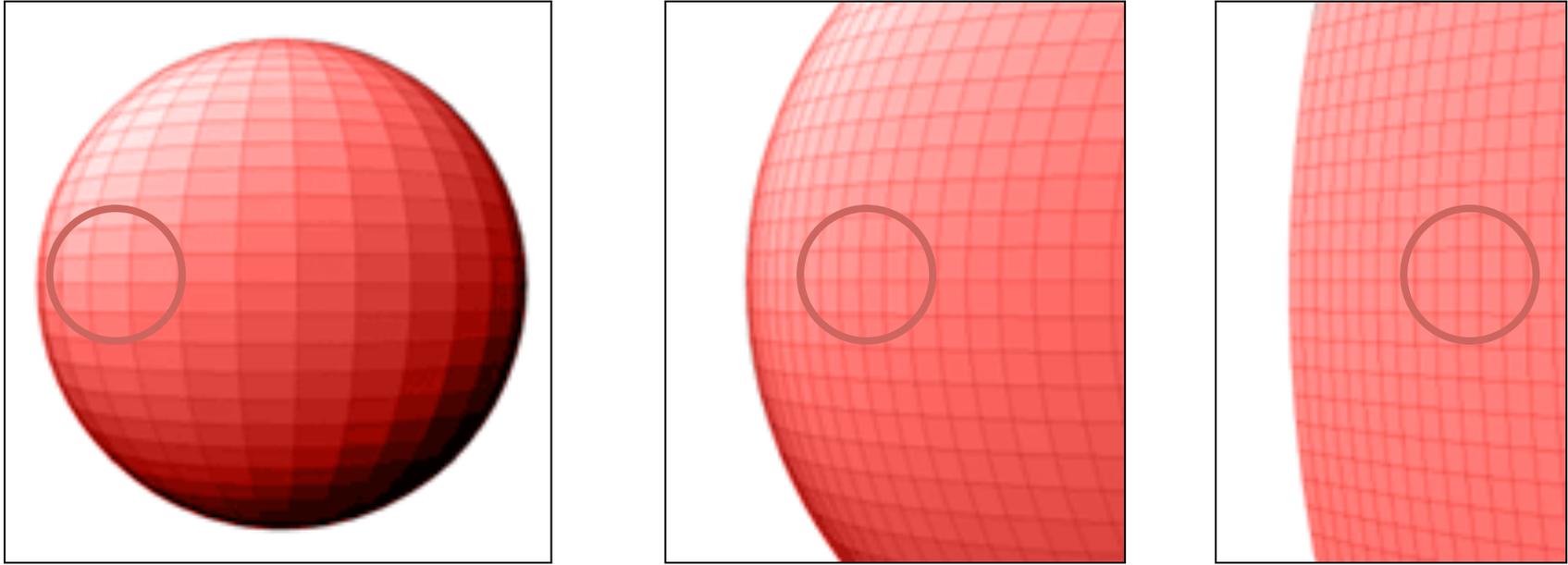
Thus:

$$|1 - \Omega(t_f)| \sim e^{-2N} \sim e^{-200} \sim 10^{-87}$$

The Inflationary Scenario



Inflation Solves the Flatness Problem

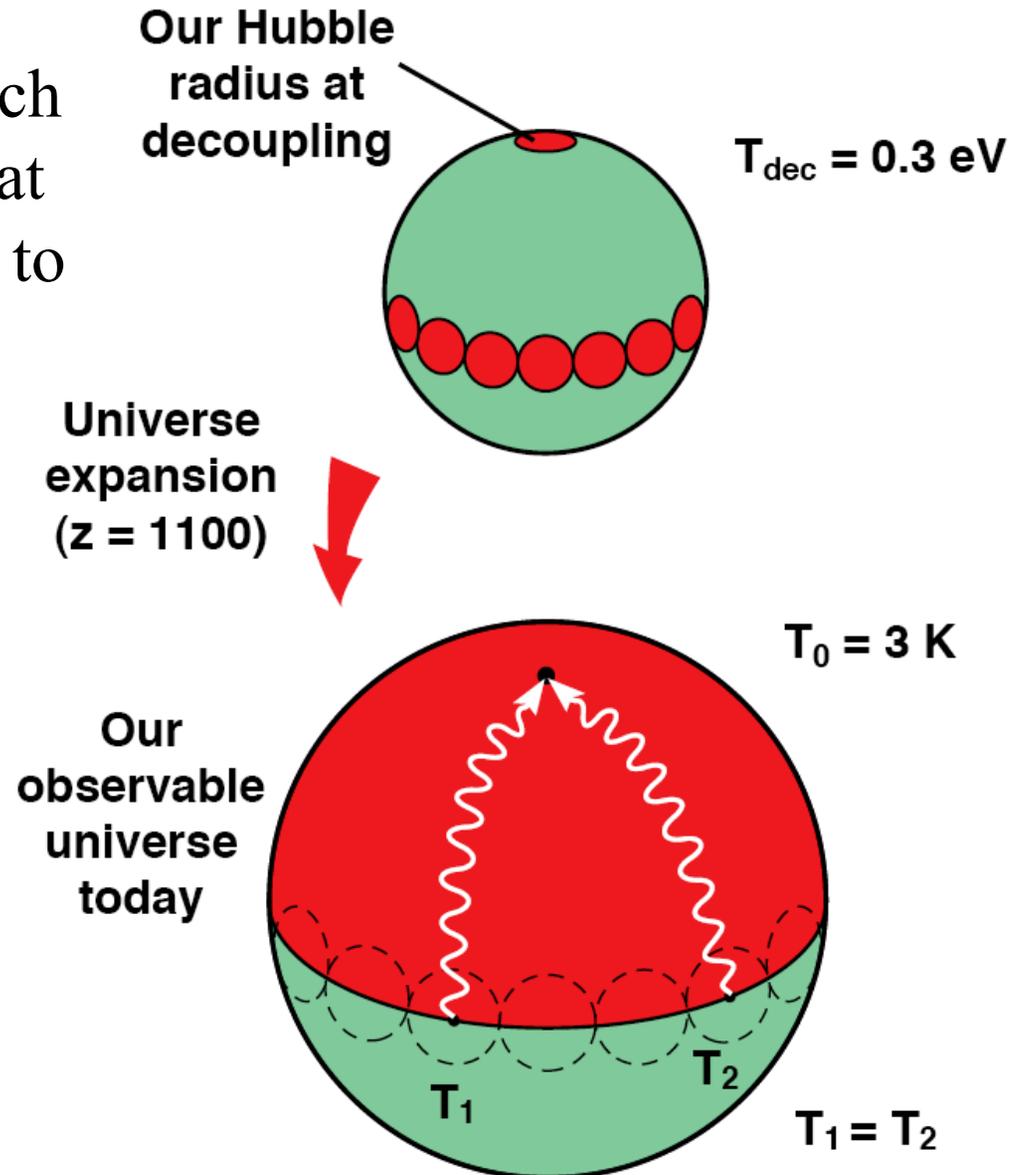


As the universe inflates, the local curvature effects become negligible in comparison to the vastly increased “global” radius of curvature: the universe becomes extremely close to flat locally (which is the observable region now). Thus, at the end of the inflation, $\Omega = 1 \pm \epsilon$

Inflation Solves the Horizon Problem

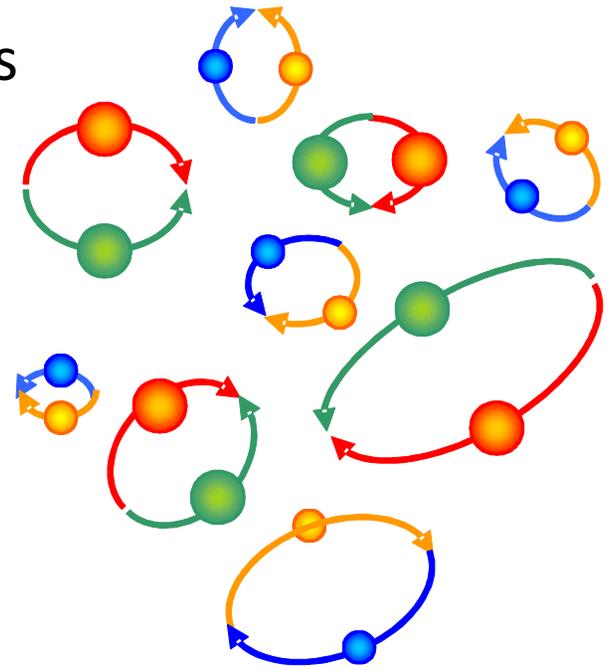
Regions of the universe which were causally disconnected at the end of the inflation used to be connected before the inflation - and thus in a thermal equilibrium

Note that the *inflationary expansion is superluminal*: the space can expand much faster than c



Inflation and Structure Formation

- Uncertainty Principle means that in quantum mechanics vacuum constantly produces temporary particle-antiparticle pairs
 - This creates minute density fluctuations
 - Inflation blows these up to macroscopic size
 - They become the seeds for structure formation
- Expect the mass spectrum of these density fluctuations to be approximately scale invariant
 - This is indeed as observed!
 - Not a “proof” of inflation, but a welcome consistency test



The Inflation as a Phase Transition

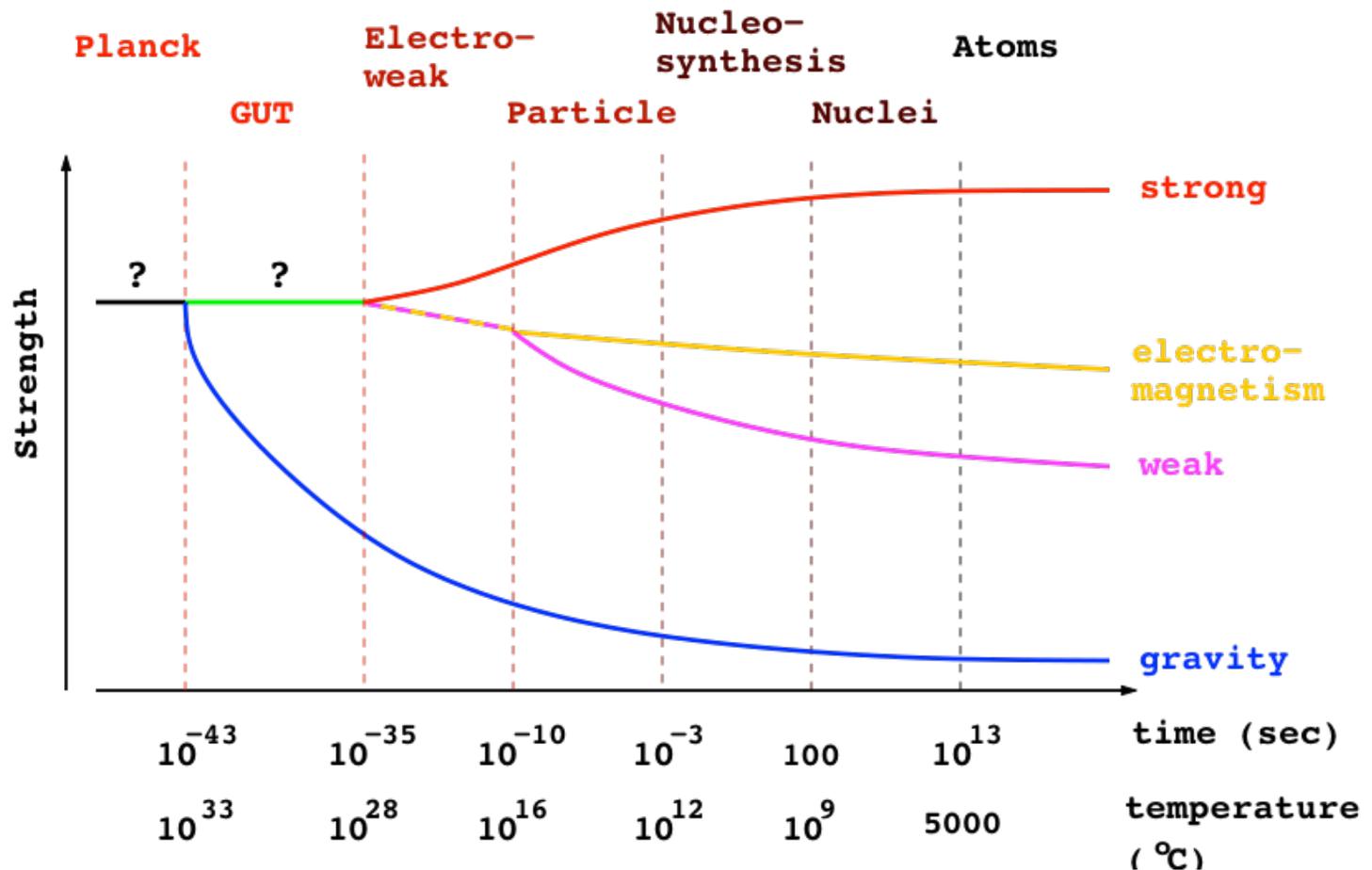
The universe undergoes a phase transition from a state of a false vacuum, to a ground state; this releases enormous amounts of energy (“latent heat”) which drives an exponential expansion



Regions of non-inflating universe are created through the nucleation of bubbles of true vacuum. When two such bubbles collide, the vast energy of the bubble walls is converted into the particles. This process is called reheating

Physical Interactions in the Early Universe

As we get closer to $t \rightarrow 0$ and $T \rightarrow \infty$, we probe physical regimes in which different fundamental interactions dominate. Their strength is a function of energy, and at sufficiently high energies they become unified



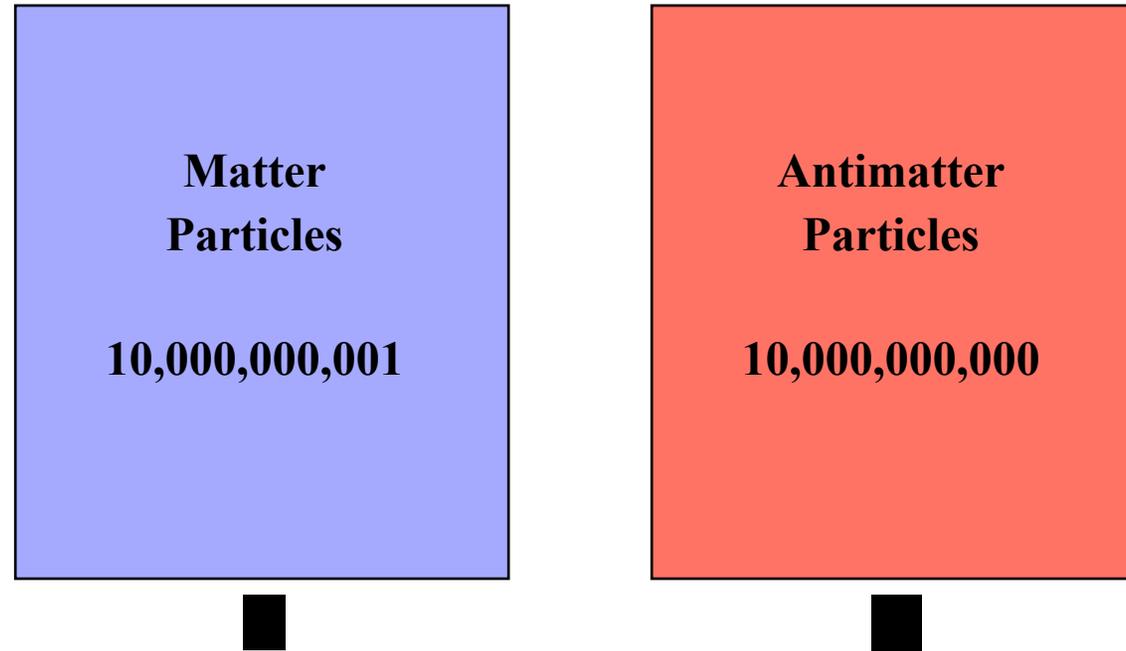
The Electroweak Era: up to 10^{-10} sec

- At $T \sim 10^{28}$ K, three distinct forces in the universe: Gravity, Strong, and **Electroweak**: unified Electromagnetism and Weak nuclear force
- At $T < 10^{15}$ K, Electromagnetism and Weak nuclear force split; this is the **Electroweak phase transition**
- Limit of what we can test in particle accelerators

The GUT Era: up to 10^{-35} sec

- At $T > 10^{29}$ K, electroweak force and strong nuclear force join to form the GUT (grand unified theory) interaction
- Relatively solid theoretical framework (but may be wrong), but not directly testable in experiments
- This **GUT phase transition** may be driving the Inflation (but there are other candidates)

Matter - Antimatter Asymmetry



They basically have all annihilated away
except a tiny difference between them

This process leads to the preponderance of photons over the
leftover baryons today by the same factor ...

Where does it come from?

The Cosmic Baryogenesis

- The conditions required for the creation of more matter than anti-matter were first derived by A. Sakharov in 1967:
 - 1. Baryon number violation**
 - Otherwise have same no. of particles and antiparticles
 - Never been observed
 - Predicted to occur in several GUT theories
 - 2. C and CP violation**
 - Parity of antiparticles is opposite to that of particles
 - CP violation discovered in 1964 (Cronin and Fitch)
 - 3. Departure from non-thermal equilibrium**
 - Otherwise all reactions go both ways
 - Provided by the expansion of the universe
- Now believed to be the mechanism responsible for the matter - antimatter asymmetry

Planck Units

Proposed in 1899 by M. Planck, as the “natural” system of units based on the physical constants:

Name	Dimension	Expression	Approx. SI equivalent measure
Planck time	Time (T)	$t_P = \frac{l_P}{c} = \sqrt{\frac{\hbar G}{c^5}}$	5.39121×10^{-44} s
Planck length	Length (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	1.61624×10^{-35} m
Planck mass	Mass (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	2.17645×10^{-8} kg
Planck charge	Electric charge (Q)	$q_P = \sqrt{\hbar c 4\pi\epsilon_0}$	$1.8755459 \times 10^{-18}$ C
Planck temperature	Temperature (Θ)	$T_P = \frac{m_P c^2}{k} = \sqrt{\frac{\hbar c^5}{G k^2}}$	1.41679×10^{32} K

They may be indicative of the physical parameters and conditions at the era when gravity is unified with other forces ... assuming that **G**, **c**, and **h** do not change ... and that there are no other equally fundamental constants

“Derived” Planck Units

Name	Dimension	Expression	Approx. SI equivalent measure
Planck energy	Energy (ML ² T ⁻²)	$E_P = m_P c^2 = \sqrt{\frac{\hbar c^5}{G}}$	1.9561 × 10 ⁹ J
Planck force	Force (MLT ⁻²)	$F_P = \frac{E_P}{l_P} = \frac{c^4}{G}$	1.21027 × 10 ⁴⁴ N
Planck power	Power (ML ² T ⁻³)	$P_P = \frac{E_P}{t_P} = \frac{c^5}{G}$	3.62831 × 10 ⁵² W
Planck density	Density (ML ⁻³)	$\rho_P = \frac{m_P}{l_P^3} = \frac{c^5}{\hbar G^2}$	5.15500 × 10 ⁹⁶ kg/m ³
Planck angular frequency	Frequency (T ⁻¹)	$\omega_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}}$	1.85487 × 10 ⁴³ s ⁻¹
Planck pressure	Pressure (ML ⁻¹ T ⁻²)	$p_P = \frac{F_P}{l_P^2} = \frac{c^7}{\hbar G^2}$	4.63309 × 10 ¹¹³ Pa
Planck current	Electric current (QT ⁻¹)	$I_P = \frac{q_P}{t_P} = \sqrt{\frac{c^6 4\pi\epsilon_0}{G}}$	3.4789 × 10 ²⁵ A
Planck voltage	Voltage (ML ² T ⁻² Q ⁻¹)	$V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{G 4\pi\epsilon_0}}$	1.04295 × 10 ²⁷ V
Planck impedance	Resistance (ML ² T ⁻¹ Q ⁻²)	$Z_P = \frac{V_P}{I_P} = \frac{1}{4\pi\epsilon_0 c} = \frac{Z_0}{4\pi}$	2.99792458 × 10 ¹ Ω