

Ay 127 – Spring 2011 – Homework #1

Distributed on Apr. 6, due by 5 pm on Apr. 13 (Return to the TA directly)

The honor system applies as follows: You can discuss the problems among yourselves, how to go about them, but not derive the solutions jointly – everyone should work out their own solutions.

1. Spatially curved universes. Consider a spatially curved universe with no cosmological constant, with Hubble constant H_0 today and matter density parameter Ω_0 .

- (a) Use the dependence of matter density on scale factor to write the Friedmann equation in the form:

$$\dot{a}^2 = H_0^2 \left[\Omega_0 \left(\frac{1}{a} - 1 \right) + 1 \right]$$

(Longair Eq. 7.21).

- (b) For the closed universe case ($\Omega_0 > 1$), show that with the substitution

$$a = \frac{\Omega_0}{2(\Omega_0 - 1)} (1 - \cos \theta)$$

one can solve the above equation for $t(\theta)$ and thus obtain a parametric equation for the expansion history of the universe. (Longair Eq. 7.25).

- (c) Determine the comoving coordinate distance that a photon can travel between any two epochs θ_1 and θ_2 . Show that there is exactly enough time between the Big Bang and the Big Crunch for a photon to circumnavigate the universe exactly once.

2. Microwave background. The cosmic microwave background (CMB) today has a $T = 2.728$ K.

- (a) Assuming $H_0 = 70$ km/s/Mpc, what is the fraction of the critical density in the microwave background, Ω_{CMB} ?
- (b) Using the scaling of the radiation density as a function of redshift, show that early in the history of the universe $\rho_{CMB} > \rho_{matter}$. At what redshift did $\rho_{CMB} = \rho_{matter}$? Assume that today $\Omega_m = 0.3$.

3. Computing cosmological models. For this problem you can either use Ned Wright's web calculator (linked on the class webpage), <http://www.astro.ucla.edu/~wright/CosmoCalc.html> or, for an *extra credit* (we'll increase your score for this problem by 50%), write your own program to integrate the appropriate equations from Friedmann-Lemaitre models. Either way, use your favorite graphing package to plot the results. Assume $H_0 = 70$ km/s/Mpc.

- (a) Compute the total comoving volume (in Gpc^3) and the present age (in Gyr) for a universe with $\Omega_\Lambda = 0$ ($= \Omega_{vac}$), as a function of Ω_m , in the range from 0 to 2, with a step of 0.1.
- (b) Ditto, but for a universe with $\Omega_m = 0$, as a function of Ω_Λ , in the range -1 to 1 , with a step 0.1.
- (c) Ditto, but for a spatially flat model, as a function of Ω_Λ , in the range 0 to 1, with a step of 0.1.
- (d) Compute the $a(t)$ curves, where t is the time since the big bang at a given redshift, for the universes with $[\Omega_m, \Omega_\Lambda] = [0,0]$, $[1,0]$, $[0,1]$ and $[0.3,0.7]$, each with about 10 – 15 time steps spaced roughly uniformly from here to the big bang.