

**AY 127**  
**HW #2**  
**Due: April 20, 2011**

**1. Exercises with  $g^*$**

Add up the effective number of polarizations  $g^*$  for all the particles in the Standard Model, and show that you get the value reported in class.

**2. Transition from Radiation to Matter Domination.**

In this problem, you will work out the expansion history of the Universe during the transition from radiation to matter domination.

- a) Use the Friedmann equation to obtain the Hubble parameter  $H(a)$  in terms of  $\Omega_m$ ,  $H_0$ , and  $a_{\text{eq}}$ . Neglect terms associated with spatial curvature or the cosmological constant (and explain why you can do this).
- b) Use the substitution  $y = a/a_{\text{eq}}$ , and find the differential equation relating  $y$  to  $t$ .
- c) Solve this equation to obtain:

$$t = t_1 \left[ \frac{2}{3}(1+y)^{3/2} - 2(1+y)^{1/2} + \frac{4}{3} \right]$$

For  $\Omega_m = 0.3$  and  $H_0 = 70$  km/s/Mpc, what is  $t_1$  in years and what is  $a_{\text{eq}}$ ?

- d) What is the power law exponent relating  $t$  and  $y$  (or  $a$ ) long before equality or long after?

[Note: if you are feeling really ambitious, you can solve this in closed form for  $a(t)$ , but this is rarely done and is not part of the assignment.]

**3. Equilibrium Timescales in the Early Universe.**

In class we worked out the equilibrium timescale for the plasma (electrons + ions) to come to equilibrium at the temperature of the CMB. In this problem you will work out two additional timescales: (i) the Compton equilibrium timescale, and (ii) the double Compton timescale.

This is an order of magnitude problem only – there is no need to do complicated integrals, nor keep factors of 2,  $\pi$ , etc. You may assume that  $kT \ll m_e c^2$ .

You may assume for this problem that the baryon:photon ratio (or to within a factor of 1.16 the electron:photon ratio) is  $\eta = n_e/n_\gamma = 6 \times 10^{-10}$ . The intermediate results are most conveniently expressed in terms of the temperature  $T$ , fundamental constants, and  $\eta$ . The

resulting timescales should be expressed both analytically, and numerically in terms of  $T_6$  (the temperature in units of  $10^6$  K).

(i) The Compton equilibrium timescale: This is the timescale for photons to redistribute their energy, and form a modified blackbody spectrum.

- Compute the scattering rate  $\Gamma_{es}$ , i.e. the number of times that a particular photon scatters off an electron per second.
- Compute the fractional change in energy  $\delta E/E$  that a photon experiences in such a collision due to the motion of the electron.
- Assuming a random walk, compute the number of scatterings a photon needs in order to redistribute its energy.
- Compute the timescale required for this redistribution of energy to take place.

(ii) The Double Compton equilibrium timescale: This is the timescale for photons to be created and destroyed, and hence the timescale for the modified blackbody to be converted into a true blackbody spectrum. The dominant process is Double Compton scattering,

$$e^- + \gamma(\nu) \rightarrow e^- + \gamma(\nu') + \gamma(\nu''),$$

where  $\nu$  is the frequency of the initial photon,  $\nu'$  ( $\approx \nu$ ) is the final frequency of the scattered photon, and a secondary photon of low frequency  $\nu''$  ( $\ll \nu$ ) is also emitted.

- What is the typical frequency of photons  $\nu$  present in the Universe?
- When such a photon scatters off an electron, compute the typical velocity change  $\Delta v$  of the electron (implied by momentum conservation).
- In terms of the emission of a photon of frequency  $\nu''$ , apply the uncertainty principle and suppose that the change of the electron's velocity can be assumed to have occurred in time  $\Delta t \sim 1/\nu''$ . What is the acceleration  $a_e$ ?
- Using the formula for power from an accelerating charge, estimate the amount of energy radiated by the electron in photons of frequency  $\sim \nu''$  during the collision.
- What is the expected number of secondary photons of frequency  $\sim \nu''$  emitted?
- You should find that an equal number of photons are emitted in each range of frequency. Argue that this means that the total probability  $P$  to produce a secondary photon in a scattering contains a factor of  $\ln(\nu''_{\max}/\nu''_{\min}) = \ln \Lambda$ . (For order-of-magnitude calculations,  $\ln \Lambda$  is typically  $\sim 10$ .) Estimate  $P$ . [Extra credit: give a plausible argument for what physically determines  $\nu''_{\max}$  and  $\nu''_{\min}$ .]
- Based on the scattering rate  $\Gamma_{es}$  computed in (i) and  $P$ , estimate the timescale for a photon to spawn a secondary photon in a Double Compton event. This is the timescale at which the number of photons should come to thermal equilibrium.