Ay 127 – Spring 2009 – Homework #3

Distributed on May 8, due by 5 pm on May 15 (Return to the TA directly)

The honor system applies as follows: You can discuss the problems among yourselves, how to go about them, but not derive the solutions jointly – everyone should work out their own solutions.

1. Transfer functions.

a. Suppose that Universe contained only baryons and radiation, and no dark matter or dark energy, i.e., $\Omega_b = 1$, $H_0 = 50$ km/s/Mpc, and $T_{cmb} = 2.73$ K. Qualitatively draw the transfer function $T(k)$, assuming adiabatic initial perturbations. Give at order of magnitude level the comoving wavenumbers and corresponding values of T at important features.

b. Explain the qualitative differences between the baryons-only and the CDM model.

2. Massive halo abundance. For this problem, you may use the BBKS approximation to the transfer function, Eq. (14.37) of Longair. Assume an initial perturbation spectrum that is scale-invariant with $\Delta\zeta^2(k) = 2.5 \times 10^{-9}$, and cosmological parameters $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 70$ km/s/Mpc. You may neglect the baryons.

a. Compute the power spectrum $P(k)$ today.

b. Compute the radii $R(M)$ of balls containing 10^{12}, 10^{13}, 10^{14}, and $10^{15}\ M_\odot$.

c. Compute the standard deviations of the density field $\sigma(M)$ top-hat smoothed over a radius $R(M)$, for $M = 10^{12}$, 10^{13}, 10^{14}, and $10^{15}\ M_\odot$ at $z = 0$. [Note: you will have to do numerical integrals for this part.]

d. Using the Press-Schecter argument, estimate the fraction of the matter that lives in haloes of $M > 10^{12}\ M_\odot$, and similarly for 10^{13}, 10^{14}, and $10^{15}\ M_\odot$.

3. Galaxy cluster. Consider a cluster of galaxies with a radial velocity dispersion $\sigma = 1500$ km s^{-1}, and the mean radius $\langle R \rangle = 1.5$ Mpc. It contains approx. 500 galaxies, with a mean luminosity $\langle L \rangle = 10^{10}\ L_\odot$.

a. What is the estimated mass of the cluster?

b. What is the mass-to-light ratio, in Solar units?

c. What is the temperature of the intra-cluster gas (assume a pure hydrogen)?

d. What is the typical energy and wavelength of emitted photons (explain)?

4. Peculiar velocities. Suppose you have a standard candle which has an intrinsic scatter of 0.35 mag (1 σ), and that it can be used successfully to measure relative distances (radial velocities) out to $cz = 10{,}000$ km/s. How many measurements of such objects, over the whole sky, would be required to verify that the volume probed is at rest with respect to the CMB, with the net error < 100 km/s? How many measurements of the same objects would be necessary, in principle, to measure H_0 to a ~10% accuracy if the volume is not subject to large scale flows or significant peculiar velocities? State all of your assumptions – there is no exact right answer, but your reasoning is important. Comment on why, in the real world, things aren't so easy. [Hint: remember that the direction of peculiar velocities will not always be along our line of sight...]