

**AY 127**  
**HW #3**  
**Due: April 27, 2011**

**1. Helium recombination.**

(a) Write down the Saha equation for the equilibrium of He I, He II, and He III.

(b) Assuming the hydrogen is fully ionized, estimate the redshift  $z_2$  at which 50% of the helium recombines from He III to He II, and the redshift  $z_1$  at which 50% of the helium recombines from He II to He I.

**2. Growth of perturbations in the far future.**

Using the ordinary differential equation for the rate of growth of perturbations in the matter,

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0,$$

find the growing and decaying modes for the evolution of  $\delta_m$  in the far future (i.e. after the matter density becomes negligible) in: (a) a  $\Lambda$ -dominated universe; and (b) an open universe with  $\Lambda=0$  and  $\Omega_m < 1$ . Show that in both cases, the growth of structure freezes:  $\delta_m$  asymptotes to a constant.

**3. Linear perturbations with neutrinos.**

Suppose that the three neutrino species have masses  $m_{\nu 1}$ ,  $m_{\nu 2}$ , and  $m_{\nu 3}$ . The sum of the three masses is  $\Sigma m_\nu$ .

(a) For a neutrino at the peak of the neutrino blackbody spectrum, find the momentum  $p$  as a function of redshift  $z$ . At what redshift does each species of neutrino become nonrelativistic ( $p < mc$ )? Express your answer in terms of the neutrino mass (in eV).

(b) What is  $\Omega_\nu h^2$  in terms of  $\Sigma m_\nu$  (in eV)?

(c) Now consider the growth of a perturbation in the regime where the neutrinos are nonrelativistic, but whose wavelength is short compared to the distance a neutrino travels in the age of the Universe. In this case, the neutrinos can be considered "hot" – they are smoothly distributed (like the CMB), and the perturbations in the matter are growing. Show that in this case, the background expansion remains  $a \sim t^{2/3}$ , but that the perturbation equation

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G\bar{\rho}_m\delta_m = 0$$

now has a growing mode with  $\delta_m \sim t^\mu$ , where

$$\mu = \frac{1}{6} \left[ -1 + \sqrt{\frac{\Omega_v + 25\Omega_m}{\Omega_v + \Omega_m}} \right].$$

Show that this reduces to the expected result when  $\Omega_v \ll \Omega_m$ .

(d) What is the condition (in terms of ranges of  $z$  and  $m_{\text{vi}}$ ) for the assumptions in part (c) to be valid?