Ay 127 - Spring 2013

Solution Set 3

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1. **Starbursts and the cosmic far-infrared background.** [20 points]

   (a) Physical energy density scales as $(1+z)^4$, so we can write

   $$u_{\rm CIB}(1+z)^4 = \frac{N\langle L\rangle \Delta t}{V_{z=3}},$$

   which means that the total number of starbursts needed to produced the observed energy density today is

   $$N = \frac{V_{z=3}u_{\rm CIB}(1+z)^4}{\langle L\rangle \Delta t} = \frac{4\pi R^3_{{\rm hor},z=3}\mu_{\rm CIB}(1+z)^4}{3\langle L\rangle \Delta t}.$$

   The physical distance traveled by a photon from the Big Bang by a given time in the Universe's history, also known as the particle horizon, is

   $$R_{\rm hor} = \frac{1}{(1+z)}\int_z^\infty \frac{c\,{\rm d}z'}{H(z')}.$$

   For a $\Omega_m = 1$, $\Lambda = 0$ Universe, $R_{\rm hor} = 2c/H(z)$, which means that the radius of the casually-connected Universe at $z = 3$ is approximately 3.30×10^{27} cm. Therefore, plugging in to the above equation, we find

   $$N \approx 7 \times 10^{10}\,\text{galaxies.}$$

   (b) The comoving number density of starburst "relics" is equivalent to their physical number density today:

   $$n = \frac{N}{V_{z=0}} \approx 9 \times 10^{-76}\,{\rm cm}^{-3} = 0.03\,{\rm Mpc}^{-3},$$

   where we have taken the radius of the Universe to be $2c/H_0$.
   Compared to the estimated comoving number density of normal galaxies ($\sim 10^{-3}$ Mpc^{-3}), this seems rather high, but note that we have assumed that the CIB is produced *entirely* by this population of starburst galaxies. In reality, there is a significant contribution from AGN to all cosmic backgrounds.

   (c) If 7 MeV/nucleon is released during the production of helium, the estimated amount of helium produced by each starbursts is

   $$M_{\rm He} \approx \frac{\langle L\rangle \Delta t}{7\,{\rm Mev/nucleon}}\left(\frac{1\,{\rm MeV}}{1.6022\times 10^{-6}\,{\rm erg}}\right)m_{\rm H} = 5.6\times 10^{41}\,{\rm g} = 2.8 \times 10^8\,{\rm M}_\odot.$$

   If 5 times as much helium is produced as metals, then the estimated metal production in each starburst is approximately 5.6×10^7 M$_\odot$, which means the overall metallicity of the system is roughly $Z \approx 5 \times 10^{-4}$.

2. **Black holes and quasar lensing.** [20 points]

   (a) The comoving number of black holes can be determined in a similar way to the number density of hydrogen atoms:

   $$n_{\rm BH} = \left(\frac{2.94 \times 10^{73} {\rm cm}^3}{\rm Mpc^3}\right) \frac{3H_0^2 \Omega_{\rm DM,0}}{8\pi G M_{\rm BH}} = 3.7 \times 10^4 \, {\rm Mpc}^{-3},$$

   where we have assumed $M_{\rm BH} = 10^6 \; {\rm M}_\odot$. Note that there is no factor of $(1+z)^3$, because this is a comoving density instead of a physical density (but comoving = physical at $z = 0$). Since the volume of the Milky Way's dark matter halo is $\approx \frac{4}{3}\pi(0.05)^3 = 5.2 \times 10^{-4}\;{\rm Mpc}^3$, there could (mathematically) be up to 20 massive black holes passing through the galaxy at any given time; in reality, this implies that the probability of finding a black hole in the Milky Way is 100%.

   (b) The crudest estimate of the number of black holes on the sky is simply the total number of black holes in the Universe. We can estimate this by multiplying our comoving number density from part (a) by the comoving volume of the Universe (which we also used in problem 1).

   $$N_{\text{on-sky}} = n_{\rm BH} V_{z=0} = (3.7 \times 10^4 \, {\rm Mpc}^{-3})(2.6 \times 10^{12} \, {\rm Mpc}^3) \sim 10^{17} \text{ black holes}$$

   (c) As you have probably read, the probability of a foreground object lensing a background object occurs when the foreground object is halfway to the source plane. However, we need to think carefully about what this means for 10^6 ${\rm M}_\odot$ black holes lensing background quasars. First, consider the magnification of a source by a point mass,

   $$\mu = \left[1 - \left(\frac{\theta_E}{\theta}\right)^4\right]^{-1},$$

   where θ_E is the Einstein radius for the lensing system and θ is the angular separation between the lens and the apparent position of the background source. Generally, there are two solutions for this angle,

   $$\theta = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right).$$

   If we combine these, we can write the magnification as a function of the Einstein radius and the angle between the source and lens (arbitrarily taking only one of the images):

   $$\mu = \left[1 - \left(\frac{2\theta_E}{\beta + \sqrt{\beta^2 + 4\theta_E^2}}\right)^4\right]^{-1}.$$

   For fixed β, the magnification has a lower limit of 1 (when $\beta = 0$) and increases with increasing θ_E. This means that, despite the peak *efficiency* of lensing occurring when the lens is halfway to the source, it is easier to observe a lensing event when θ_E is large (i.e., when the lens is close).

As an example, consider the Einstein radius for a quasar at $z = 2.5$ and a foreground black hole at $z = 1$,

$$\theta_E = \sqrt{\frac{4GM_{\rm BH}}{c^2}\frac{D_{LS}}{D_L D_S}} \approx 2 \times 10^{-4}\,\text{arcsec},$$

and contrast this with the case of a much closer black hole at $z = 0.001$, which has $\theta_E \approx 2 \times 10^{-2}$ arcsec.

To make things simpler, we will place all of the foreground black holes at the same distance, $z = 0.001$, and then estimate the probability of intercepting a foreground black hole's Einstein radius as we look at a background quasar (c.f., Hogg's "Distance Measures in Cosmology"):

$$dP = n(z)\sigma(z)\frac{c(1+z)^2}{H(z)}dz$$

We can take $\sigma \sim (\theta_E D_L)^2$ and, since we have essentially created a δ-function at $z = 0.001$, this reduces to

$$P \approx n_{\rm BH}(\theta_E D_L)^2\frac{c}{H_0} \sim 10^{-6}.$$

If there are $\sim 10^6$ quasars on the sky, then approximately one of them is being lensed at any given time. Note that your answer may be different, depending on the exact configuration you chose (e.g., if you had the lenses closer or farther away).