Dark Matter Candidates

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Abstract. An overview is given of various dark matter candidates. Among the many suggestions given in the literature, axions, inert Higgs doublet, sterile neutrinos, supersymmetric particles and Kaluza-Klein particles are discussed. The situation has recently become very interesting with new results on antimatter in the cosmic rays having dark matter as one of the leading possible explanations. Problems of this explanation and possible solutions are discussed, and the importance of new measurements is emphasized. If the explanation is indeed dark matter, a whole new field of physics, with unusual although not impossible mass and interaction properties may soon open itself to discovery.
1. Introduction: The Dark Matter Problem

The dark matter problem has been a part of astrophysics for at least 75 years – since Zwicky’s observation of a large velocity dispersion of the members of the Coma galaxy cluster [1]. Similarly, the problem of galactic rotation curves - the stars rotate “too fast” to be bound by Newtonian gravity if all matter is visible - can be traced back to Babccock’s measurements of the Andromeda galaxy 1939 [2]. It took, however, several decades before it was recognized as a real problem, and in its modern form it goes back to the late 1970’s and early 1980’s when the so-called cold dark matter paradigm appeared [3] (in this context, cold means matter moving with non-relativistic velocities when structure formed in the universe). Today, a wealth of impressive data from studies of the microwave background radiation, supernova distance measurements, and large-scale galaxy surveys have together solidified the Standard Model of cosmology, where structure formed through gravitational amplification of small density perturbations with the help of cold dark matter. Without the existence of dark matter the density contrast seen in the universe today could not have formed, given the small amplitude of density fluctuations inferred from anisotropies of the cosmic microwave background.

Present-day cosmology of course also has another, mysterious component: a cosmological constant Λ or a similar agent (such as time-varying quintessence exerting negative pressure such that the expansion of the universe is today accelerating). For the purpose of this article, however, this dark energy plays little role other than to fix the background metric and thus influencing late-time structure formation. In fact, most large-scale n-body simulations are now carried out in this cosmological Standard Model, the ΛCDM model. Modern models of cosmology contain a brief period of enormously accelerated expansion, inflation, which gives a nearly scale-invariant spectrum of primordial fluctuations, which together with the fact that the universe observationally appears to be very flat (i.e., the total energy density is equal to the critical density) are cornerstones of the Standard Model of cosmology.

There exist several extensive reviews of particle dark matter [4, 5, 6, 7] as well as a recent book [8], in particular covering the prime candidate which has become something of a template for dark matter, namely the lightest supersymmetric particle. In this review, I will focus mainly on recent developments. I will also discuss some of the less often mentioned possibilities, like axion dark matter and sterile neutrinos, and also some new interesting - though speculative, types of dark matter models that may perhaps explain the surprising new measurements of a large flux of positrons in the cosmic rays [9, 10]. The enhanced cross sections needed in these models, in particular the so-called Sommerfeld enhancement, will also be discussed.

1.1. Models for Dark Matter

Almost all current models of dark matter use the standard concept of quantum field theory to describe the properties of elementary particle candidates (for exceptions, see for instance [11, 12]). This means that they can be characterized by the mass and spin
Table 1. Properties of various Dark Matter Candidates

<table>
<thead>
<tr>
<th>Type</th>
<th>Particle Spin</th>
<th>Approximate Mass Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axion</td>
<td>0</td>
<td>µeV-meV</td>
</tr>
<tr>
<td>Inert Higgs Doublet</td>
<td>0</td>
<td>50 GeV</td>
</tr>
<tr>
<td>Sterile Neutrino</td>
<td>1/2</td>
<td>keV</td>
</tr>
<tr>
<td>Neutralino</td>
<td>1/2</td>
<td>10 GeV - 10 TeV</td>
</tr>
<tr>
<td>Kaluza-Klein UED</td>
<td>1</td>
<td>TeV</td>
</tr>
</tbody>
</table>

of the dark matter particle. The mass of proposed candidates spans a very large range, as illustrated in Table 1.

The density of cold dark matter (CDM) is now given to an accuracy of a few percent. With $h$ being the Hubble constant today in units of $100\text{ km s}^{-1}\text{ Mpc}^{-1}$, the density derived from the 5-year WMAP data \[13\] is

$$\Omega_{\text{CDM}}h^2 = 0.1131 \pm 0.0034, \quad (1)$$

with the estimate $h = 0.705 \pm 0.0134$.

Using the simplest type of models of thermally produced dark matter (reasonably far away from thresholds and branch cuts) this corresponds to an average of the annihilation rate at the time of chemical decoupling of \[4\]

$$\langle \sigma_A v \rangle = 2.8 \cdot 10^{-26} \text{ cm}^3\text{s}^{-1}. \quad (2)$$

The fact that this corresponds to what one gets with a weak interaction cross section for particles of mass around the electroweak scale around a few hundred GeV is sometimes coined the “WIMP miracle” (WIMP standing for Weakly Interacting Massive Particle), but it may of course be a coincidence. However, most of the detailed models proposed for the dark matter are in fact containing WIMPs as dark matter particles.

The rate in Eq. (2) is a convenient quantity to keep in mind, but it has to be remarked that this is the value needed at the time of freeze-out, when the temperature was typically of the order of $(0.05 - 0.1)M_X$ (with $M_X$ the mass of the dark matter particle) and the velocity $v/c \sim 0.2 - 0.3$. There are now publicly available computer codes \[14, 15\] that solve the Boltzmann equation numerically, taking various effects into account, such as co-annihilations which may change the effective average annihilation cross section appreciably if there are other states than the one giving the dark matter particle which are nearly degenerate in mass. There are also computer packages available (e.g., \[16\]) that can perform joint Bayesian likelihood analysis of the probability distribution of combinations of parameters, in particular for supersymmetric dark matter models.

As we will see later, the simple Eq. (2) may be modified by orders of magnitude in the halo today, for example by the Sommerfeld enhancement – if there are zero velocity bound states in the annihilating system (cold dark matter particles should today move with typical Galactic velocities of $v/c \sim 10^{-3}$). One should also be aware of the large astrophysical uncertainties present when estimating the observable annihilation rate.
today, as it may be influenced by the presence of substructure in the dark matter distribution, such as discovered in large simulations of structure formation [17, 18].

As an example, for indirect detection of gamma rays in the Galactic halo the annihilation rate towards a direction making the angle $\Psi$ with respect to the galactic centre is conveniently given by the factorized expression [19]

$$\Phi_{\gamma}(\psi) = 0.94 \cdot 10^{-9} \left( \frac{N_{\gamma} \langle \sigma v \rangle}{10^{-29} \text{cm}^3\text{s}^{-1}} \right) \left( \frac{100 \text{GeV}}{M_\chi} \right)^2 J(\psi) \text{ m}^{-2} \text{s}^{-1} \text{sr}^{-1} \quad (3)$$

where the dimensionless function

$$J(\psi) = \frac{1}{8.5 \text{kpc}} \cdot \left( \frac{1}{0.3 \text{GeV/cm}^3} \right)^2 \int_{\text{line of sight}} \rho^2(l) \, dl(\psi) , \quad (4)$$

with $\rho(l)$ being the dark matter density along the line of sight $l(\Psi)$. (Note the numerical factor in Eq. (3) differs by a factor $1/2$ from that given in the original reference [19]; this takes into account the fact that the annihilating particles are identical, as is the case for supersymmetric neutralinos. See the footnote in connection to Eq. (21) of the publication [20] for a detailed explanation.)

The particle physics factor $N_{\gamma} \langle \sigma v \rangle$, which is the angle and velocity averaged annihilation rate times the number of photons created per annihilation, can usually be rather accurately computed for a given dark matter candidate. In particular, for cross sections containing an $s$-wave piece, usually $\sigma v$ does not depend on velocity to an excellent approximation, for the typical small Galactic velocities $v/c \sim 10^{-3}$ in the halo. However, the value of $\sigma v$ may in some cases, in particular for the Sommerfeld enhanced models, depend on velocity or rather, after the angular average, on the velocity dispersion $v_{\text{disp}}$, in the simplest case like $1/v_{\text{disp}}$. Also, $\rho^2(l)$ may vary rapidly along the line of sight if there exists substructure in the halo dark matter distribution. Therefore, in general the integration does not factorize as in Eq. (3) and has to be performed over the full phase space of the dark matter distribution, which for a given halo – like that of the Milky Way – unfortunately is poorly known.

The procedure in this situation has been to introduce a “boost factor”, which unfortunately does not seem to have a unique definition. In particular, the boost of the detection rates for a given model will depend on the particle of detection, and the energy. One possible definition of the boost factor would be

$$B_{\text{tot}} = B_{\rho} \times B_{\sigma v} = \left( \frac{\rho^2(r) \Delta V}{\rho_0^2(r) \Delta V} \right) \times \left( \frac{\langle \sigma v \rangle_{v=b_{\text{disp}}} v_{\text{disp}}}{\langle \sigma v \rangle_{v=b_{\text{F}}} v_{\text{F}}} \right) \Delta V , \quad (5)$$

with $v_{\text{disp}}$ the velocity dispersion in the object under study and $v_{\text{F}}$ the typical velocity at freeze-out. It is important to note that this boost factor involves an average of a volume $\Delta V$ which in the case of antiprotons and positrons would be a typical diffusion scale, and therefore could implicitly depend on particle kind and energy (see Fig. 1).

For gamma-ray observations, the enhancement should be computed within the line of sight cone, and therefore one may, for certain lines of sight, get very large boost factors, if e.g., these lines cross dense subhaloes (or regions, like the Galactic center, where
the influence of baryons could give an enhanced density through adiabatic contraction processes).

The computation of the boost factor in realistic astrophysical and particle physics scenarios is a formidable task, which has so far only been partially addressed. It may be anticipated that this will be one of the main problem areas of future indirect detection studies of dark matter. For direct detection, there is no corresponding enhancement of the scattering rate. However, the detailed small-scale structure of the local region of the dark matter halo may play a role.

1.2. Axions

Although at times not very much in focus of dark matter phenomenologists and experimentalists, the axion remains one of the earliest suggestions of a viable particle candidate for dark matter, and in fact one of the most attractive. This is not least due to the fact that its existence was motivated by solving the strong $CP$ problem in particle physics, and its possible role for dark matter comes as an extra bonus. A disadvantage in the cosmological context is, however, that the axion needed to solve the $CP$ problem only solves the dark matter problem for a small range of masses – thus some fine-tuning...
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mechanism seems to be needed. For a recent review of the field, see \[21\].

The original idea of Peccei and Quinn was to make the CP violating phase dynamical \[22\] by introducing a global symmetry, \(U(1)_{PQ}\), which is spontaneously broken. The Goldstone boson of this broken global symmetry is the axion, which however gets a non-zero mass from the QCD anomaly, which can be interpreted as a mixing of the axion field with the \(\pi\) and \(\eta\) mesons \[23, 24\].

The earliest attempts, using only the standard model particles but with an enlarged Higgs sector, were soon ruled out experimentally and the “invisible axion” was invented \[25, 26\] with a very high mass scale of symmetry breaking and with very massive fermions carrying \(PQ\) charge. This means that only a feeble strong or electromagnetic interaction leaks out to the visible sector through triangle loop diagrams.

The phenomenology of the axion is determined, up to numerical factors, by one number only - the scale \(f_a\) of symmetry breaking. In particular, the mass is given by

\[
m_a = 0.62 \text{ eV} \left(\frac{10^7 \text{ GeV}}{f_a}\right).
\]

A naive expectation, from, e.g., Grand Unified Theories, is that \(f_a\) is related to the unification scale \(\sim 10^{16} \text{ GeV}\), which would make the expected mass fall in the sub-\(\mu\)eV range. It turns out, however, that such a light axion would overclose the Universe and thus is not viable \[21\]. There are some hypothetical mechanisms in string theory \[27\], however, that could make the mass scale smaller.

The experimentally important coupling to two photons is due to the effective Lagrangian term

\[
L_{a\gamma\gamma} = \left(\frac{\alpha_{\text{em}}}{2\pi f_a}\right) \kappa \left(\vec{E} \cdot \vec{B}\right) a,
\]

where \(E\) is the electric field, \(B\) is the magnetic field and \(\kappa\) is a model-dependent parameter of order unity.

The axion has been gradually more and more constrained by laboratory searches, stellar cooling and the supernova dynamics to indeed be very light, \(m_a < 0.01 \text{ eV}\) \[28\]. It then couples so weakly to other matter \[29\] that it never was in thermal equilibrium in the early universe and it would behave today as cold dark matter. The window where axions are viable dark matter candidates is progressively getting smaller, but still there is an acceptable range between around \(10^{-5}\) and \(10^{-2}\) eV where axions pass all observational constraints and would not overclose the universe.

There is a considerable uncertainty in the relation between mass and relic density, depending on the several possible sources of axion production such as vacuum misalignment, emission from cosmic strings etc. For a recent discussion of the relic density of axions in various scenarios, see \[30\].

The coupling in Eq. (7) implies that resonant conversion between a galactic axion and an electric photon mode may take place in the presence of a strong magnetic field - not even the “invisible axion” may be undetectable \[31\], since the number density of these light particles in the Galaxy has to be enormous if axions are to make up the dark matter.
There are now a couple of experiments (for a recent review, see [32]) which have had the experimental sensitivity to probe, and so far rule out, parts of the interesting region.

There have recently been laboratory searches [33] for light axions emitted from processes in the Sun. Although not in a mass and coupling constant range directly relevant for dark matter, the exclusion region covered is quite impressive [33]. There could also be other interesting mechanisms, like axion-photon conversion, that could possibly influence cosmological measurements in interesting ways [34].

For the time being the axion remains undetected, but if it exists in the appropriate mass range it is still one of the prime candidates for the dark matter.

1.3. Inert Higgs

We now turn to massive particles, WIMPs, and start with one of the most minimal extensions of the Standard Model (for an earlier, even simpler one, see [35]). It was noted already in 1978 that a model with two Higgs doublets containing a discrete symmetry could contain a state, the lightest neutral scalar or pseudoscalar boson, which is stable [36]. Almost three decades later, the model reappeared [37] as a way to obtain improved naturalness with a Higgs that could be rather massive, larger than 300 GeV.

The possibility of one of the lighter neutral states in the enlarged Higgs sector to be the dark matter was also pointed out, and soon the basic properties of this “inert” Higgs candidate for dark matter were investigated [38]. It turns out that this model contains a dark matter candidate, a particle that does not couple directly to Standard Model fermions and is stable due to the discrete symmetry of the model (hence its relative inertness). Rates for indirect detection (i.e. the observation of products of pair annihilation in the halo [38], or in the Earth or Sun [39]) would then appear to be suppressed, unless its mass would be larger than the \( W \) mass. However, if it is just below the \( W \) mass, the virtual creation of a \( W \) pair which then converts to \( \gamma\gamma \) or \( Z\gamma \), would give rather spectacular rates for these observationally interesting line processes [40]. These would populate an energy region which is particularly favourable for detection in the Large Area Gamma-ray Telescope of the Fermi satellite [41]. The first results on dark matter searches from Fermi should appear soon, for estimates of its potential for dark matter detection made before launch, see [42].

The inert Higgs doublet model of dark matter is compatible with existing accelerator bounds [37, 43] and is an interesting, very minimal model with interesting phenomenology.

1.4. Sterile Neutrino

The neutrino was a favoured particle dark matter candidate in the period starting in the end of the 1970’s, with the first calculations of the relic density for massive neutrinos [44, 45, 46, 47]. Of the many candidates for non-baryonic dark matter proposed, neutrinos are often said to have the undisputed virtue of being known to exist. The
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direct mass limits from accelerators are not very useful for cosmological neutrinos, given the small mass differences measured in neutrino oscillation experiments. The only direct limit of relevance is the one on the electron neutrino, 2 eV \[48\], which taken together with mass differences inferred from neutrino observations can still allow 6 eV for the sum of neutrino masses. However, there are observational limits from cosmology on the mass range allowed for this sum, which are much more restrictive. From an analysis of the WMAP 5-year data \[13\] a bound of 0.63 eV can be derived on the sum \[49\]. If one is willing to also trust modelling of the Ly-\(\alpha\) forest, a significantly better limit can be obtained \[50\]. There has recently been progress in the rather difficult problem of treating the structure formation problem including light neutrinos in an accurate way \[51\].

Since the contribution to the dark matter density is

\[
\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{94 \text{ eV}},
\]

we see that light neutrinos (which would behave as hot, not cold, dark matter) can at most be roughly one tenth of the cold dark matter implied by the WMAP data.

There is a fundamental objection to having a massive but light neutrino (or in fact, any fermion) as the dominant constituent of dark matter on all scales where it is observationally needed. This has to do with the fact that neutrinos are spin-\(1/2\) particles obeying the Pauli exclusion principle. To make up the dark matter in dwarf galaxies (which are observed to be completely dominated by the dark matter component), neutrinos would have to be stacked together so tightly in phase-space that it is impossible to evade the Pauli principle. Quantitatively, Tremaine and Gunn found \[52\] that to explain the dark matter of a dwarf galaxy of velocity dispersion \(\sigma\) (usually of order 100 km/s) and core radius \(r_c\) (typically 1 kpc), the neutrino mass has to fulfill

\[
m_\nu \geq 120 \text{ eV} \left(\frac{100 \text{ km/s}}{\sigma}\right)^{1/4} \left(\frac{1 \text{ kpc}}{r_c}\right).
\]

Recently, this bound has been improved somewhat \[53\], and gives a lower limit of roughly 400 eV for the sterile neutrino mass.

Sterile neutrinos do not interact through standard weak interactions \[54\], but communicate with the rest of the neutrino sector through fermion mixing (for recent reviews of such models, see \[55, 56\]). They are limited by a variety of observational data \[57\], but it seems that, e.g, a region below 10 keV for mixing angles smaller than \(\sin^2 \theta \sim 10^{-10}\) is allowed. Again one has to impose some unknown tuning mechanism to match the WMAP data on relic density. A sterile neutrino of this mass would be warm dark matter, i.e., intermediate between hot and cold dark matter, and this would have some beneficial effects on some possible problems with the CDM scenario such as the absence of a predicted cusp in the central regions of some galaxies, or the lack of substructure in the form of dwarf galaxies bound to the Milky Way. Actually, the latter problem does not seem as serious now, as new data from the Sloan Digital Sky Survey and the Keck telescope have revealed a number of new, faint satellite galaxies \[58, 59\],
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and the mass dependence of the number distribution seems to agree well with CDM simulations [60].

In principle, one could also have a cold dark matter standard model neutrino of mass around 3 GeV, but this window was closed by the LEP experiment at CERN. The pioneering papers which worked out the dark matter phenomenology of such massive neutrinos (e.g., [44, 45, 46, 47]) were important, however, since they showed that a weakly interacting, massive particle (“WIMP”) could serve as cold dark matter with the required relic density.

To conclude this section about neutrinos, it seems that it is very plausible that they make up some of the dark matter in the universe (given the experimental results on neutrino oscillations), but most of the dark matter is probably of some other form. Particle physics offers several other promising candidates for this.

1.5. Supersymmetric Dark Matter

Supersymmetry has since its invention [61] fascinated a generation of theoretical physicists, and motivated many experimentalists like those now involved in CERN’s LHC project, likely to produce data on multi-TeV proton-proton collisions next year.

Supersymmetry is an ingredient in many superstring theories which attempt to unite all the fundamental forces of nature, including gravity. In most versions of the low-energy theory there is, to avoid, for example, excessive baryon number violating processes, a conserved multiplicative quantum number, R-parity:

\[ R = (-1)^{3(B-L)+2S}, \] (10)

where \( B \) is the baryon number, \( L \) the lepton number and \( S \) the spin of the particle. This implies that \( R = +1 \) for ordinary particles and \( R = -1 \) for supersymmetric particles. This means that supersymmetric particles can only be created or annihilated in pairs in reactions of ordinary particles. It also means that a single supersymmetric particle can only decay into final states containing an odd number of supersymmetric particles. In particular, this makes the lightest supersymmetric particle stable, since there is no kinematically allowed state with negative R-parity which it can decay to. In fact, this is similar to the discrete parity mentioned for the inert Higgs model. It seems that most (but not all) models for dark matter have to rely on a similar discrete symmetry (which in the simplest case is just a \( Z_2 \) symmetry), so maybe one should nowadays, when the possibility of explaining dark matter is one of the main motivations when constructing new particle physics models, generically introduce a multiplicative discrete “\( D \)-symmetry” (with \( D \) standing for Dark) with

\[ D = 1 \quad \text{Standard Model Sector} \] (11)

\[ D = -1 \quad \text{New Particle Sector} \] (12)

Since this is a multiplicative quantum number, it means that particles in the \( D = -1 \) sector can only be pair-annihilated or -produced, and the lightest particle with \( D = -1 \) is stable. If it is electrically neutral, it is then a dark matter candidate.
Thus, pair-produced neutralinos $\chi$ in the early universe which left thermal equilibrium as the universe kept expanding should have a non-zero relic abundance today. If the scale of supersymmetry breaking is related to that of electroweak breaking, $\chi$ will be a WIMP and $\Omega_\chi$ will be of the right order of magnitude to explain the non-baryonic cold dark matter. It would indeed appear as an economic solution if two of the most outstanding problems in fundamental science, that of dark matter and that of the unification of the basic forces, would have a common element of solution - supersymmetry.

The idea that supersymmetric particles could be good dark matter candidates became attractive when it was realised that breaking of supersymmetry could be related to the electroweak scale, and that, e.g., the supersymmetric partner of the photon (the photino) would couple to fermions with electroweak strength [62]. Then most of the phenomenology would be similar to the (failed) attempts to have multi-GeV neutrinos as dark matter. After some early work along these lines [63, 64, 65, 66, 67], the first more complete discussion of the various possible supersymmetric candidates was provided in [68], where in particular the lightest neutralino was identified as perhaps the most promising one.

A disadvantage of a full supersymmetric model (even making the particle content minimal, the Minimal Supersymmetric Standard Model, MSSM) is that the number of free parameters is excessively large - of the order of 100. Therefore, most treatments have focused on constrained models, such as minimal supergravity (mSUGRA) models [69], where one has the opportunity to explain electroweak symmetry breaking by radiative corrections caused by running from a unification scale down to the electroweak scale (for a detailed analysis of dark matter in mSUGRA models, see [70]).

1.6. Supersymmetric particles

Let us now focus on the lightest supersymmetric particle, which if $R$-parity is conserved, should be stable. In some early work, a decaying photino [63] or a gravitino [64] were considered, but for various reasons [68] the most natural supersymmetric dark matter candidate was decided to be the lightest neutralino $\chi$. In fact, especially the decaying gravitino option has recently been revived with considerable interest [71]. In view of the need for very large boost factors to explain the new PAMELA [9] and ATIC [10] data, decaying gravitino scenarios may provide an alternative [72].

Returning to the neutralino $\chi$, it is a mixture of the supersymmetric partners of the photon, the $Z$ and the two neutral $CP$-even Higgs bosons present in the minimal extension of the supersymmetric standard model (see, e.g., [73]). It has gauge couplings and a mass which for a large range of parameters in the supersymmetric sector imply a relic density in the required range to explain the observed $\Omega_M h^2 \sim 0.1$. As we will see, its couplings to ordinary matter also means that its existence as dark matter in our galaxy’s halo may be experimentally tested. For an extensive review of the literature on supersymmetric dark matter up to mid-1995, see Ref. [4]. Some improvements were
discussed in [5], and the most recent, rather full discussion can be found in [6, 8].

The phenomenology has not changed very much since these reviews. The neutralino remains a very promising candidate, with possibilities for discovery in direct detection [74] and in various channels of indirect detection [75].

Here we just point out two recent developments. It was noticed in [19] that the indirect process of annihilation to $\gamma\gamma$ and $Z\gamma$ [19], although generally with too small branching ratios to be observable, has a remarkable behaviour as the annihilating particles are either electroweak doublets (pure higgsinos) or triplets (pure gauginos). Namely, the cross section tends to a constant value proportional to $1/m_W^2$ as could be expected on dimensional grounds. This means that the unitarity limit [76]

$$\sigma_{\text{unitarity}} < \frac{4\pi}{vm_X^2}$$

will eventually be violated at very high masses. This led Hisano, Matsumoto and Nojiri [77] to investigate the behaviour of the amplitude near that limit. They discovered that including perturbatively higher order corrections, they would get a value slightly higher than that found in [19], but more importantly, unitarity was restored. A crucial step forward was then taken in [78] by non-perturbatively summing up in the ladder approximation to all orders the attractive $t$-channel exchange diagrams. The result is a zero-energy bound state for some particular dark matter masses and typical galactic velocities. The appearance of the bound state makes the cross section increase two to three orders of magnitude, compared to that when velocities were corresponding to the freeze-out temperature $T \sim m_X/20$ (corresponding to $v \sim 0.3 - 0.4$).

This phenomenon, thus discovered in [78], and verified in [79, 80, 81, 82] gives the possibility of very strong $\gamma$-ray signals for particular masses (usually in the TeV region). It is analogous to what happens for positronium near bound state thresholds, as originally discussed by Sommerfeld [83]. It is today the well-known “Sommerfeld enhancement” of the annihilation rate, and may be a generic phenomenon. Of course, supersymmetric TeV particles interacting through standard model gauge bosons may have difficulty to give the required relic density, unless one tolerates some fine-tuning, as is explicitly done in “split SUSY” models [84]. The Sommerfeld enhancement is today extensively discussed in connection with the surprising new results on the high positron flux at high energies, see later. If Sommerfeld enhancement is active for positrons, one would also expect large, perhaps detectable, signals in radio waves and gamma-rays [86, 88, 89]. Rather important bounds on the enhancement follow from the early structure formation and effects on the diffuse gamma-ray background or the cosmic microwave background, especially if there is no saturation of the effect at very small velocities [90].

Another example of a recent development of supersymmetric dark matter phenomenology (although its history goes back to the late 1980’s [91]) is the change of helicity structure caused by QED radiative corrections to low-velocity annihilation.
Positron and $\gamma$ spectra from Internal Bremsstrahlung

Figure 2. The $\gamma$-ray (solid) and positron (dashed) spectrum obtained by internal bremsstrahlung of the highly suppressed lowest order process $\chi\chi \rightarrow e^+e^-$. Here $x = E_\gamma/m_\chi$ or $E_{e^+}/m_\chi$, and the $t$-channel fermion, from which the radiation originates, is assumed to be 5% heavier than the neutralino.

This has recently been proposed as a method to detect an otherwise undetectable leptonic dark matter candidate [92]. It has also been applied to MSSM and mSUGRA models, and found to be very important [93], causing sometimes large boosts to the highest energy end of the $\gamma$-ray spectrum. In particular, it has been shown to increase the potential for $\gamma$-ray detection from dwarf satellite galaxies [94].

A Majorana fermion (as many dark matter candidates are) suffers a helicity suppression for S-wave annihilation [95], such that the amplitude contains a factor of fermion mass $m_f$, meaning that, e.g., the $e^+e^-$ final state is highly suppressed. However, by emitting a photon from an internal ($t$-channel) charged leg, which only costs a factor of $\alpha_{em}/\pi$, the helicity suppression may be avoided. The effect will be that these radiative corrections, instead of as usual being a percent of the lowest order process, may instead give enhancement factors of several thousand to million times the suppressed lowest order, low-velocity, rate [91]. The resulting spectra will have a characteristic very sharp drop at the endpoint $E_\gamma = m_\chi$ of both the $\gamma$-ray and positron spectrum, see Fig. 2.

1.7. Kaluza-Klein Particles in Universal Extra Dimensions

The lightest KK particle (LKP) is an interesting, viable particle dark matter candidate arising from extra dimensional extensions of the standard model (SM) of particle physics. It appears in models of universal extra dimensions (UED) [96] (for the first proposal of having TeV sized extra dimensions see [98]), where all SM fields propagate in the higher dimensional bulk, and is stable due to conserved KK parity, a remnant of KK mode number conservation. This again an example of a $D$-symmetry analogous to the $R$-parity of supersymmetry, meaning the the LKP is a WIMP. Contrary to the case
of supersymmetry however, the unknown parameter space is quite small and will be scanned entirely by, e.g., LHC.

Consider the simplest, five dimensional model with one UED compactified on an $S^1/Z_2$ orbifold of radius $R$. All Standard Model fields are then accompanied by a tower of KK states; at tree-level, the $n$th KK mode mass is given by

$$m^{(n)} = \sqrt{(n/R)^2 + m_{\text{EW}}^2},$$

where $m_{\text{EW}}$ is the corresponding zero mode mass. The LKP can be shown to be the first KK mode $B^{(1)}$ of the weak hypercharge gauge boson. The spectrum and the $B^{(1)}$ relic density were first computed in [99]. Depending on the exact form of the mass spectrum and the resulting coannihilation channels, the limit from WMAP [13] of $\Omega_{\text{CDM}}h^2 = 0.1131 \pm 0.0034$ corresponds to a mass of the dark matter candidate $B^{(1)}$ between roughly 0.5 to 1 TeV. Collider measurements of electroweak observables give a current constraint of $R^{-1} > 0.3$ TeV, whereas LHC should probe compactification radii up to 1.5 TeV (for a review of the detailed phenomenology of KK dark matter, see [100]).

The most interesting aspect of KK dark matter is that it provides an example of a spin-1 dark matter candidate. This means that the helicity structure of the matrix elements for annihilation will change, in particular the explicit factor of fermion mass that appears in the $s$-wave matrix element for slow Majorana particles annihilating in the halo is not present. This means that new interesting direct annihilation modes will appear, such as $\nu \bar{\nu}$, or $e^+e^-$, which are usually severely suppressed for neutralino annihilation, for example. The LKP was early recognized [101] as a potentially important source of positrons, and was used to investigate the HEAT excess [102], which now has been superseded by the much more convincing PAMELA positron excess [9]. In fact, in the recent ATIC paper claiming evidence for a possible peak around 600 GeV perhaps related to dark matter [10], a KK model was shown as an example of a model that would give a good fit to the shape of the electron plus positron spectrum (although there the normalization was fitted to an arbitrary, high value). This model has recently been revisited by Hooper and Zurek [103], who conclude that a boost factor of the order of several hundred is needed, which may be difficult to explain using current models of the dark matter halo. They point out, however, that such a high boost factor need not necessarily conflict with other data at present.

### 1.8. Models with Enhanced Annihilation Rate

We have already mentioned the dramatic change of focus that has taken place in the dark matter community, triggered by the new positron data. In the late summer of 2008, the first of these exciting new data were reported from the PAMELA satellite, designed to measure the content of antimatter in the cosmic rays. The results were first communicated at conferences, and later a paper was put on the arXiv [9]. An unexpectedly high ratio of positrons over electrons was measured, in particular in the region between 10 and 100 GeV, where previously only weak indications of an excess
had been seen [103]. This new precision measurement of the cosmic ray positron flux, which definitely disagrees with a standard background [105] has opened up a whole new field of speculations about the possible cause of this positron excess. As mentioned, a similar, difficult to explain excess, a “bump”, in the electron plus positron spectrum was reported by the balloon experiment ATIC [10]. Simultaneously, other data from PAMELA indicate that the antiproton flux is in agreement with standard expectations [106].

There are a variety of astrophysical models proposed for the needed extra primary component of positrons, mainly based on having nearby pulsars as a source [107]. Although pulsars with the required properties like distance, age, and energy output are known to exist, it turns out not to be trivial to fit both ATIC and PAMELA with these models (see, for example, [108, 109]). For this and other reasons, the dark matter interpretation, which already had been applied to the much more uncertain HEAT data [110] has been one of the leading hypotheses (the list of relevant papers is already too long to be displayed here; for a partial list of selected papers, see [111]).

It was clear from the outset that to fit the PAMELA and ATIC positron data with a dark matter model a high mass is needed (reflecting the bump at around 600 GeV of ATIC). However, since the local average dark matter density is well-known to be around 0.3 - 0.4 GeV/cm$^3$, the number density decreases as $1/M_X$ with $M_X$ the mass of the annihilating particle. This means that with $\langle \sigma v \rangle = 3 \cdot 10^{-26}$ cm$^3$/s, which is the standard value of the annihilation rate in the halo for thermally produced WIMPs (see Eq. (2)), the rate of positrons, even for a contrived model which annihilates to $e^+e^-$ with unit branching ratio is much too small to explain the measured result.

To a good approximation, the local electron plus positron flux for such a model is given by, assuming an energy loss of $10^{-16} E^2$ GeVs$^{-1}$ (with $E$ in GeV) from inverse Compton and synchrotron radiation,

$$E^3 \frac{d\phi}{dE} = 6 \cdot 10^{-4} E \left( \frac{1 \text{ TeV}}{M_X} \right)^2 \theta(M_X - E) B_{\text{tot}} \text{ m}^{-2} \text{s}^{-1} \text{sr}^{-1} \text{GeV}^2,$$

which means that the boost factor $B_{\text{tot}} \sim 200$ (see Eq. (5) for the definition of $B_{\text{tot}}$) for a 600 GeV particle, that may otherwise explain the ATIC bump. Similar boost factors seem to be generic, also for supersymmetric models giving $e^+e^-$ through internal bremsstrahlung [112].

Returning to Eq. (5), we see that such a boost factor can be given by a large inhomogeneity which has to be very local, since positrons and electrons of several hundred GeV do not diffuse very far before losing essentially all their energy, see Fig. 1. Although not excluded [113] (see, however, [114]), this would seem to be very extremely unlikely in most structure formation scenarios. Therefore, most models rely on the second factor in Eq. (5), i.e., the Sommerfeld enhancement factor. This means arguably also a non-negligible amount of fine-tuning of the mass spectrum, in particular also for the degeneracy between the lightest and next-to-lightest particle in the new sector. For a detailed discussion of the required model-building, see [51]. Similar fine-tuning is needed
for the decaying dark matter scenario, where the decay rate has to be precisely tuned to give the measured flux. Since the antiproton ratio seems to be normal according to the PAMELA measurements [106], the final states should be mainly leptons (with perhaps intermediate light new particles decaying into leptons). For an interesting such model, which may in fact contain an almost standard axion, see [115].

It seems that at present it is possible to construct models of the Sommerfeld enhanced type [85, 86, 87] which do marginally not contradict present data [88, 89, 116]. We will soon, however, be presented with high precision data from the Fermi satellite [41], both for γ-rays up to 300 GeV and for the summed electron and positron spectrum up to a TeV. Also, PAMELA and ATIC are processing further data that will soon be made public. It will be interesting to see whether this will give enough information to decide the answer to the question that at the moment is hovering in the air: Has dark matter already been detected?

Acknowledgements

This work was supported by the Swedish Research Council (VR). I wish to thank colleagues at the new Oskar Klein Centre in Stockholm and elsewhere for stimulating discussions, the Galileo Galilei Institute for Theoretical Physics in Florence for the hospitality, and the INFN for partial support during completion of this work.

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Dark Matter Candidates

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