Disk Galaxy Rotation Curves: Mass Component Contributions

Dark Matter dominates at large radii.

It cannot be concentrated in the disk, as it would make the velocity dispersion of stars too high.

Mass Budget of the Universe

In addition to dynamical measurements (e.g., rotation curves) there is a more comprehensive evidence for the existence of dark matter. Modern cosmological experiments give the following breakdown of the mass/energy budget:

~ 70% “dark energy” (aka quintessence, cosmological constant, vacuum energy …)
~ 30% matter, of which
  ~ 5% baryonic (from cosmic nucleosynthesis and CMBR) of which
  ~ 0.5% visible (stars, gas)

Since 5% < 30%, there is non-baryonic dark matter
Since 0.5% < 5%, there is baryonic dark matter

Possibilities for dark matter include:

- molecular hydrogen gas clouds
- very low mass stars / brown dwarfs
- stellar remnants: white dwarfs, neutron stars, black holes
- primordial black holes
- elementary particles, probably currently unknown - non-baryonic dark matter

The Milky Way halo probably contains some baryonic dark matter - brown dwarfs + stellar remnants accompanying the known population of low mass stars.

This uncontroversial component of dark matter is not enough - is the remainder baryonic or non-baryonic?

(From P. Armitage)
Non-Baryonic DM Candidates

- **Massive neutrinos**
  - Known to exist and to have mass, but how much?
- **Weakly Interacting Massive Particles (WIMPs)**
  - Not known to exist, but possible
  - A generic category, e.g., the neutralino = the least massive SUSY particle
  - Thermal relics from the Big Bang
  - Possible masses > 10 GeV
- **Axions**
  - Predicted in some versions of quantum chromodynamics
  - Originate in non-thermal processes
  - Could interact electromagnetically
- **Many other speculative possibilities …**

Baryonic DM Candidates

- **MAssive Compact Halo Objects (MACHOs)**
  - Very low mass stars, white dwarfs, neutron stars, black holes
    (produced post-nucleosynthesis, from baryons), brown dwarfs, interstellar comets, slushballs…
- **Cold molecular (H$_2$) gas clouds**
  - Would have to be compact, dense, low volume fill factor
  - Very hard to detect!
- **Warm/hot gas**, bound to galaxy groups
  - Leftover gas from IGM, never collapsed to galaxies
  - Virial temperatures $\sim 10^5 - 10^6$ K, corresponding to the velocity dispersions $\sim 300$ km/s
  - Very hard to detect! (ISM opaque to FUV/soft-X)

Laboratory Detection of Dark Matter Particles?

Gravitational Lensing as a Tool to Study Dark Matter

- Mass (visible or not) deflects light, in an achromatic fashion:
  - Background image is magnified by the lens (e.g., a MACHO). If the lens moves, the background source will brighten and then fade.
Expected Gravitational Microlensing Lightcurves:

The peak magnification depends on the lens alignment (impact parameter).

The event duration depends on the lens velocity.

How can we see MACHOs?

- Problem: a probability of a distant star being lensed is maybe $\sim 10^{-7}$ per year
- Solution: monitor $\sim 10^7$ stars simultaneously

Microlensing experiments

Several experiments have searched for microlensing events:

- toward the Galactic Bulge (lenses are disk or bulge stars)
- toward the Magellanic Clouds (lenses could be stars in the LMC / SMC, or halo objects)

MACHO (Massive Compact Halo Object):
- observed 11.9 million stars in the Large Magellanic Cloud for a total of 5.7 years.

OGLE (Optical Gravitational Lensing Experiment):
- ongoing experiment
- presently monitor 33 millions stars in the LMC, plus 170 million stars in the Galactic Bulge.

Microlensing observables:

Suppose that between us and the Magellanic Clouds there are a large number of dark, compact objects.

At any one time, we will see a clear lensing event (i.e. the background star will be magnified) if the line of sight passes through the Einstein ring of one of the lenses.

Previously derived the angular radius of the Einstein ring on the sky $\theta_E$. Area is $\pi \theta_E^2$. 

(From P. Armitage)
\[ \theta_E = \frac{2}{c} \sqrt{\frac{G M d_{LS}}{d_L d_S}} \]  
Single lens of mass M, at distance \( d_L \).  
Observer - source distance is \( d_S \), lens -  
source distance is \( d_{LS} (=d_S-d_L) \)

Probability that this lens will magnify a given source is:

\[ P \propto \theta_E^2 \propto \left( \frac{d_{LS}}{d_L d_S} \right) \times M \]  
directly proportional to the mass of the lens

Same is obviously true for a population of lenses, with total mass \( M_{\text{pop}} \) - just add up the individual probabilities. Conclude:

- measuring the fraction of stars that are being lensed at any one time measures the total mass in lenses, independent of their individual masses
- geometric factors remain - we need to know where the lenses are to get the right mass estimate.

(From P. Armitage)

\[ \tau = \frac{4}{\nu_L c} \sqrt{\frac{G M d_L d_{LS}}{d_S}} \]  
Time scale is proportional to the square root of the individual lens masses.

Put in numbers appropriate for disk stars lensing stars in the Galactic bulge:

- \( d_S = 8 \) kpc, \( d_L = d_{LS} = 4 \) kpc
- \( M = 0.3 \) \( M_{\odot} \)
- \( \nu_L = 200 \) km s\(^{-1} \)

\[ \tau = 40 \sqrt{\frac{M}{0.3 M_{\odot}}} \text{ days} \]

Weak dependence on mass is very convenient observationally, if we observe every night can detect:

- events with \( \tau \sim 1 \) day: \( M < \) Jupiter mass (\( 10^{-3} \) \( M_{\odot} \))
- events with \( \tau \sim 1 \) year: \( M \sim 25 \) Solar masses (e.g. stellar mass black holes)
- + everything in between…

(From P. Armitage)

No way to determine from a single image whether a given star is being magnified by lensing. Need a series of images to see star brighten then fade as the alignment changes:

![Position of Einstein ring when event stars](image1)

...when event ends

![Line of sight](image2)

Motion of lens

**Lensing time scale**: equals the physical distance across the Einstein ring divided by the relative velocity of the lens:

\[ \tau = \frac{2 d_L \theta_E}{\nu_L} \]

(From P. Armitage)

The First MACHO Event Seen in the LMC Experiment:

To date, several tens (or more) microlensing events have been detected by various groups.

![Figure 1. The first LMC microlensing candidate from the MACHO project. (Expanded view: 6 yr of constant data are...](image3)
For each event, there are only two observables:
- duration \( \tau \) - if we know the location of the lens along the line of sight this gives the lens mass directly
- peak amplification \( A \): this is related to how close the line of sight passes to the center of the Einstein ring

Define \( u = \frac{b}{d_L \theta_E} \)

\[
A = \frac{u^2 + 2}{uvu^2 + 4}
\]

Note: amplification tells us nothing useful about the lens.

Additionally, observing many events gives an estimate of the probability that a given source star will be lenses at any one time (often called the optical depth to microlensing). This measures the total mass of all the lenses, if their location is known.

(From P. Armitage)

What Are MACHOs?

Analysis of the LMC microlensing experiments suggest that MACHO masses are \( \sim 0.5 \, M_\odot \): too heavy for brown dwarfs.

Old halo white dwarfs?? (There are problems with that…)

(Alcock et al. 1997)

Dark Halo: Microlensing results

\(~20\%\) of the galactic halo is made of compact objects of \( \sim 0.5 \, M_\odot \)

**MACHO**: 11.9 million stars toward the LMC observed for 5.7 yr \( \Rightarrow \) 13-17 events \( \Rightarrow \) 8%-50% (C.L. 95%) of halo made of 0.15-0.9 \( M_\odot \) compact objects.

**EROS-2**: 17.5 million stars toward LMC for 2 yr \( \Rightarrow \) 2 events (+2 events from EROS-1) \( \Rightarrow \) less that 40% (C.L. 95%) of standard halo made of objects \( < 1 \, M_\odot \)

**Candidate MACHOs**:
- Late M stars, Brown Dwarfs, planets
- Primordial Black Holes
- Ancient Cool White Dwarfs

Limits for 95\% C.L. on the halo mass fraction in the form of compact objects of mass \( M \), from all LMC and SMC EROS data 1990-98 (Lassarre et al 2000). The MACHO 95\% C.L. accepted region is the hatched area, with the preferred value indicated by the cross (Alcock et al. 1997)

Based on the number and duration of MACHO events:

If the lenses are objects in the Galactic Halo

- 20\% of the mass of the Galactic halo (inferred from the Galactic rotation curve) is in the form of compact objects
- Typical mass is between 0.15 \( M_\odot \) and 0.9 \( M_\odot \)
- Idea that **all** the mass in the halo is MACHOs is definitely ruled out

One interpretation of these results is that the halo contains a much larger population of white dwarf stars than suspected.

This poses other problems: requires a major epoch of early star formation to generate these white dwarfs - but what about the corresponding metals?
Other authors suggest the lenses may not be our halo at all, but rather reside in the Magellanic Clouds. If correct, implies that none of the halo is in the form of planetary mass to \( \sim 10 \) \( M_{\text{sun}} \) compact objects.

Ambiguity in the distance to the lenses is the main problem. Can be resolved in a few special cases:

a) If distortions to the light curve caused by the motion of the Earth around the Sun can be detected (‘parallax events’)

b) If the lens is part of a binary system. Light curves produced by binary lenses are much more complicated, but often contain sharp spikes (‘caustic crossings’) and multiple maxima. Provide more information about the event.

One event seen toward the Small Magellanic Cloud was a binary event, and it is known to lie close to the SMC.

(From P. Armitage)

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**Spiral arms**

Defining feature of spiral galaxies - what causes them?

**Observational clues**

- Seen in disks that contain gas, but not in gas poor S0 galaxy disks.
- Defined by blue light from hot massive stars. Lifetime is \( \ll \) galactic rotation period.

When the sense of the galactic rotation is known, the spiral arms almost always trail the rotation.

(From P. Armitage)

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**Differential rotation**

First ingredient for producing spiral arms is differential rotation. For galaxy with flat rotation curve:

\[
V(R) = \text{constant}
\]

Angular velocity

\[
\Omega(R) = \frac{V}{R} \propto R^{-1}
\]

Any feature in the disk will be wrapped into a trailing spiral pattern due to differential rotation:

Tips of spiral arms point away from direction of rotation.

(From P. Armitage)
Due to differential rotation, stars near the galactic center don’t need to travel far to circle the galaxy, but stars further out can take a long time to go around. An initial line of stars will be drawn out into a spiral:

But this is not why galaxies have spiral arms!

Differential rotation is not enough to explain observed spiral structure. Again assuming a flat rotation curve:

\[ \Omega = \frac{V}{R} \]

\[ \frac{d\Omega}{dR} = -\frac{V}{R^2} \]

Two points in the disk, separated by \( \Delta R \) in radius, and initially at the same azimuth, will be sheared apart over time. After time \( t \), separated by an angle:

\[ \frac{d\Omega}{dR} \Delta R t \]

Return to the same azimuth (one wrapping up) after a time given by:

\[ \frac{d\Omega}{dR} \Delta R t = 2\pi \]

(From P. Armitage)

A spiral pattern will be wrapped up on a radial scale \( \Delta R \) after a time \( t \) given by:

\[ \frac{\Delta R}{R} = \frac{2\pi R}{Vt} \]

Using values appropriate for the Milky Way (\( R = 8.5 \) kpc, \( V = 200 \) km/s):

\[ \frac{\Delta R}{R} = 0.25 \left( \frac{1 \text{ Gyr}}{t} \right) \]

This is already a very tightly wrapped spiral. Spiral arms would make an angle of \( \sim 2 \) degrees to the tangent direction.

In real galaxies, arms are not so wrapped up:

• Sa spirals, \( \sim 5 \) degrees
• Sc spirals, around 10 to 30 degrees

Implies spiral pattern must be continually renewed.

(From P. Armitage)
Spiral Density Waves

- The orbits in spiral galaxies are not quite circles – they are ellipses. These ellipses are slightly tilted with respect to each other.

- Thus there are regions of slightly higher density than their surroundings. The higher density means higher gravity.

- Objects (such as a gas cloud) will be attracted to these regions and will drift towards them.

Properties of spiral arms can be explained if they are not rotating with the stars, but rather density waves:

- Spiral arms are locations where the stellar orbits are such that stars are more densely packed.
- Gas is also compressed, possibly triggering star formation and generating population of young stars.
- Arms rotate with a pattern speed which is not equal to the circular velocity - i.e. long lived stars enter and leave spiral arms repeatedly.
- Pattern speed is less than the circular velocity - partially alleviating the winding up problem.

In isolated disk, creation of a density wave requires an instability. Self-gravity of the stars and / or the gas can provide this.

Simplest case to consider is gas. Imagine a small perturbation which slightly compresses part of the disk:

- Self-gravity of the compressed clump will tend to compress it further.
- Extra pressure will resist compression.

If the disk is massive (strong self-gravity) and cold (less pressure support) first effect wins and develop spiral wave pattern.

- When the gas cloud collides with other gas clouds, stars will be formed. (This is where most of the galaxy’s star formation takes place.)
- Many of the stars will be faint, red main sequence stars, but some will be bright blue OB stars. These stars will continue to drift through the region.
- The OB stars don’t go far before they explode. The brightest (and bluest) of a galaxy’s stars will never be far from the spiral arm where they were born.
Spiral Density Waves

Since all the bright blue stars die before leaving the spiral arm, the spiral density waves must show up better at ultraviolet wavelengths.

Density Wave Theory Summary

- Spiral arms are waves of compression that move around the galaxy and trigger star formation
- Star formation will occur where the gas clouds are compressed
- Stars pass through the spiral arms unaffected
- This theory is successful in explaining the properties of spiral galaxies
- Two outstanding problems with it:
  1. What stimulates the formation of the spiral pattern?
  2. What accounts for the branches and spurs in the spiral arms?

Star Formation in Spiral Arms

- Spiral density wave creates spiral arms by the gravitational attraction of the stars and gas flowing through the arms
- Even if there was no star formation, there would be spiral arms - but star formation makes them more prominent
- Star formation can self-propagate in a differentially rotating disk, e.g., as supernova shocks compress neighboring molecular clouds
- Self-propagating star formation may be responsible for the branches and spurs in the spiral arms, or disks without evident spiral density waves

Morphological classification of galaxies

Edwin Hubble devised a scheme for classifying galaxies, based on their appearance, in his 1936 book *The Realm of the Nebulae*.

Elliptical galaxies

- **E0**: Elliptical galaxy
- **E1**: Elliptical galaxy

Spiral galaxies

- **S0**: Normal spirals
- **S0a**: Barred spirals
- **Sb**: Normal spirals
- **Sc**: Normal spirals
- **Sbc**: Barred spirals

(From P. Armitage)
Elliptical galaxies are smooth, usually round, and almost featureless. No spiral arms or prominent dust lanes. Generally lacking in cool gas, and hence few young blue stars (plenty of hot, X-ray gas, though).

**Ellipticity** is defined as:
\[
\varepsilon = 1 - \frac{b}{a}
\]

Elliptical galaxies are denoted \( E_n \), where:
\[
\frac{b}{a} = 1 - \frac{n}{10}
\]

A round elliptical is \( E_0 \), the most elongated ellipticals are of type \( E_7 \).

This classification is not very meaningful - the ellipticity does not correlate with any physically interesting quantity.

Spiral galaxies are named for their bright spiral arms, which are prominent due either to bright O and B stars (evidence for recent star formation), or to dust lanes.

Define two sequences of spiral galaxies:

<table>
<thead>
<tr>
<th>Sa</th>
<th>Sb</th>
<th>Sc</th>
<th>Sd</th>
<th>Irr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central bulge becomes less important</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Disk becomes more important</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spiral arms become more open and ragged</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irregulars e.g., Magellanic Clouds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Sba SBb SBc SBd

As above, except that these galaxies also have a central, linear **bar**, while the Sa, Sb... are unbarred

(From P. Armitage)

Transition class between ellipticals and spirals are the **S0 galaxies**, also called **lenticulars**.

S0 galaxies have a rotating disk in addition to a central elliptical bulge, but the disk lacks spiral arms or prominent dust lanes.

Lenticulars can also have a central **bar**, in which case they are labeled SB0.

They are probably “former” spirals which have somehow lost their gas (burned it out, or got swept).

Classification of real galaxies on Hubble’s **tuning fork** diagram

Obviously easiest to classify face-on spirals.

Hubble interpreted this diagram as an evolutionary sequence - this is not supported by more modern work.

(From P. Armitage)
Galaxy Properties and the Hubble Sequence

Hubble sequence turned out to be surprisingly robust: many, but not all, physical properties of galaxies correlate with the classification morphology:

- E
- S0
- Sa
- Sb
- Sc
- Sdm/Irr

- Pressure support ➔ Rotational support
- Passive ➔ Actively star forming
- Red colors ➔ Blue colors
- Hot gas ➔ Cold gas and dust
- Old ➔ Still forming
- High luminosity density ➔ Low lum. dens.

But, for example, masses, luminosities, sizes, etc., do not correlate well with the Hubble type: at every type there is a large spread in these fundamental properties.

Problems with traditional galaxy classification

Appearance of galaxies is strongly dependent on which wavelength the observations are made in.

- X-ray
- UV
- Visible
- Near-IR
- Far-IR

Note: large change in appearance between the UV and the near infrared images.

Galaxies look `clumpier' in the UV, and increasingly smooth as we go to the visible and longer wavelengths.

(From P. Armitage)

The Meaning of Galaxy Classification

- Galaxy morphologies and other properties reflect different formative and evolutionary histories
- Much can be explained by considering galaxies as composites made of two dominant visible components:
  1. Old, pressure supported bulges, where most of the star formation occurred early on
  2. Young(er), rotationally supported disks, where star formation happened gradually and is still going on
- Note that we do not involve in this the dominant mass component - the dark matter
  … and that spiral arms may be mainly ornamental …
Photometric properties of galaxies

Empirically, the surface brightness declines with distance from the center of the galaxy in a characteristic way for spiral and elliptical galaxies.

For spiral galaxies, need first to correct for:
  • Inclination of the disk
  • Dust obscuration
  • Average over spiral arms to obtain a mean profile

Corrected disk surface brightness drops off as:

\[ I(R) = I(0) e^{-R/h_R} \]

where \( I(0) \) is the central surface brightness of the disk, and \( h_R \) is a characteristic scale length.

Typical values for the scale length are:

\[ 1 \text{ kpc} < h_R < 10 \text{ kpc} \]

In practice, surface brightness at the center of many spiral galaxies is dominated by stars in a central bulge or spheroid. Central surface brightness of disk must be estimated by extrapolating inward from larger radii.

Elliptical galaxies

Surface brightness of elliptical galaxies falls off smoothly with radius. Measured (for example) along the major axis of the galaxy, the profile is normally well represented by the \( R^{1/4} \) or de Vaucouleurs law:

\[ I(R) = I_e e^{-kR^{1/4}} \]

where \( k \) is a constant. This can be rewritten as:

\[ I(R) = I_e e^{-7.67(R/R_e)^{0.25} -1} \]

where \( R_e \) is the effective radius - the radius of the isophote containing half of the total luminosity. \( I_e \) is the surface brightness at the effective radius. Typically, the effective radius of an elliptical galaxy is a few kpc.
Profile of elliptical galaxies can deviate from the $R^{1/4}$ law at both small and large radius.

Close to the center:
• Some galaxies have **cores** - region where the surface brightness flattens and is $\sim$ constant
• Other galaxies have **cusps** - surface brightness rises steeply as a power-law right to the center

A cuspy galaxy might appear to have a core if the very bright center is blurred out by atmospheric seeing.

HST essential to studies of galactic nuclei.

**From Morphology to Physics**

- Exploring correlations between various galaxy properties provides some insights into their physics and origins. Examples:

**Tully-Fisher relation** for disk galaxies: rotational velocity correlates with the total luminosity, $L \sim V^4$

**Fundamental Plane** for ellipticals: a set of bivariate correlations connecting many of their properties

- **The Galaxy luminosity function**

The luminosities of galaxies span a very wide range - most luminous ellipticals are $10^7$ more luminous than faintest dwarfs.

**Luminosity function**, $\Phi(L)$, describes the relative number of galaxies of different luminosities.

Definition: If we count galaxies in a representative volume of the Universe, $\Phi(L)dL$ is the number of galaxies with luminosities between $L$ and $L + dL$.

Identical to the definition of the stellar luminosity function.

Luminosity functions are easiest to measure in clusters of galaxies, where all the galaxies have the same distance.

- **The Schecter luminosity function**

A convenient approximation to the luminosity function was suggested by Paul Schecter in 1976:

$$\Phi(L)dL = n_* \left( \frac{L}{L_*} \right)^\alpha \exp \left( - \frac{L}{L_*} \right) \frac{dL}{L_*}$$

In this expression:
• $n_*$ is a normalization factor which defines the overall density of galaxies (number per cubic Mpc)
• $L_*$ is a characteristic galaxy luminosity. An $L_*$ galaxy is a bright galaxy, roughly comparable in luminosity to the Milky Way. A galaxy with $L < 0.1 \, L_*$ is a dwarf.
• $\alpha$ defines the 'faint-end slope' of the luminosity function. $\alpha$ is typically negative, implying large numbers of galaxies with low luminosities.
The Schecter function plotted for different faint-end slopes, \( \alpha = -0.5 \) (red), \( \alpha = -0.75 \) (green), \( \alpha = -1 \) (blue):

\( \phi(L) \) vs. \( L / L_\odot \)

Illustrative numbers:
- Related to mean galaxy density:
  \( n_\ast = 8 \times 10^{-3} \text{ Mpc}^{-3} \)
- Luminosity of galaxies that dominate light output:
  \( L_\ast = 1.4 \times 10^{10} L_\odot \)
- Lots of faint galaxies:
  \( \alpha = -0.7 \)

...where \( L_\odot = 3.9 \times 10^{33} \text{ erg s}^{-1} \) is the Solar luminosity.

The Schecter luminosity function

The Schecter function is:
- A fitting formula that does not distinguish between galaxy types
- As with the stellar mass function, parameters must be determined observationally:

Mass function of galaxies

For stars, measurements of the luminosity function can be used to derive the Initial Mass Function (IMF).

For galaxies, this is more difficult:
- Mass to light ratio \( (M/L) \) of the stellar population depends upon the star formation history of the galaxy.
- Image of the galaxy tells us nothing about the amount and distribution of the dark matter.

More difficult measurements are needed to try and get at the mass function of galaxies.

Significance of the Schecter function

Does the Schecter form of the luminosity function have any deeper significance - or just a good fitting function?

- Luminous galaxies become exponentially rarer at high enough luminosities. Similar to a general result in cosmology (confusingly, Press-Schecter theory) - the number density of massive objects (e.g. clusters) drops off exponentially at high masses.
- But remember that Schecter function is a composite which includes all types of galaxies. Luminosity function of any individual class of galaxies looks very different - irregular galaxies typically have much lower luminosities than ellipticals for example.