

Ay 122 - Fall 2004

Lecture 6

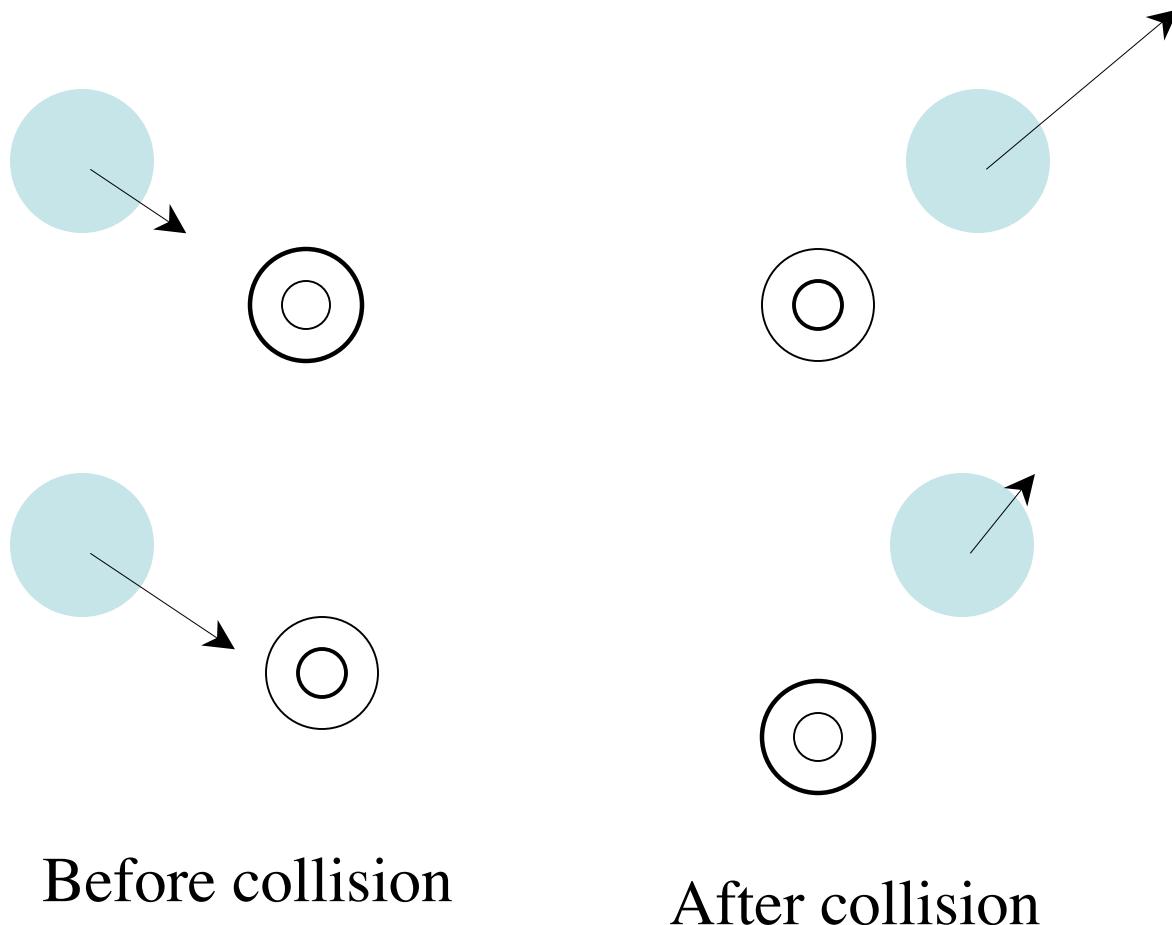
(given by Tony Travouillon)

**Stellar atmospheres,
classification of stellar spectra**

(Many slides c/o Phil Armitage)

Formation of spectral lines:

1. excitation



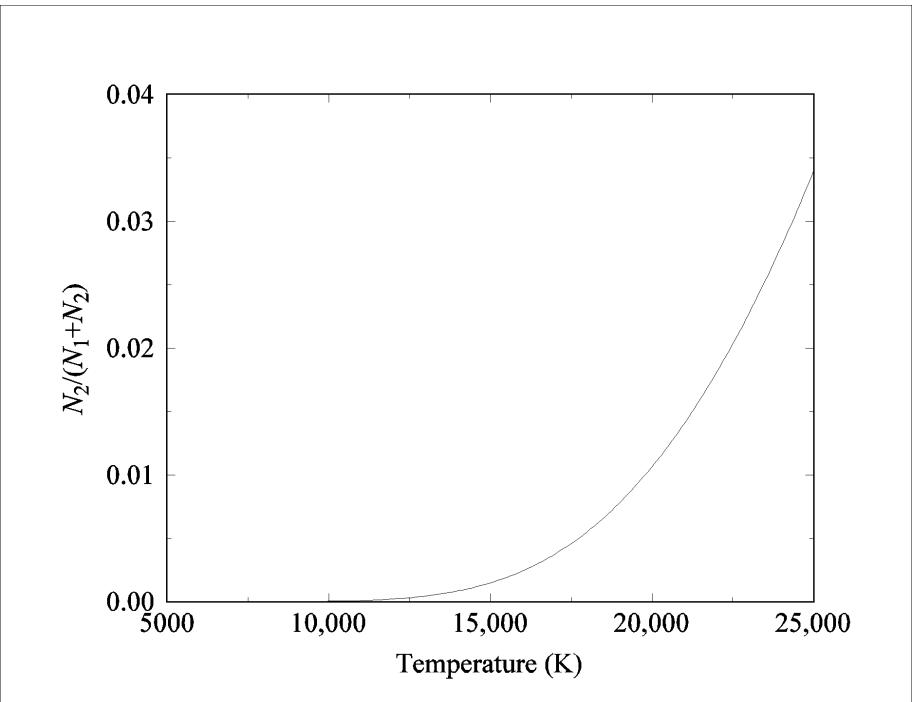
Two key questions:

- In what orbital are electrons most likely to be found?
- What are the relative numbers of atoms in various stages of ionization?

Formation of spectral lines: 1.excitation

From Maxwell-Boltzmann distribution function (statistical mechanics), we can generate the ratio of electrons in different excitation state (boltzmann equation):

$$\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-(E_b - E_a)/kT}$$



Where g is the degeneracy of each level:

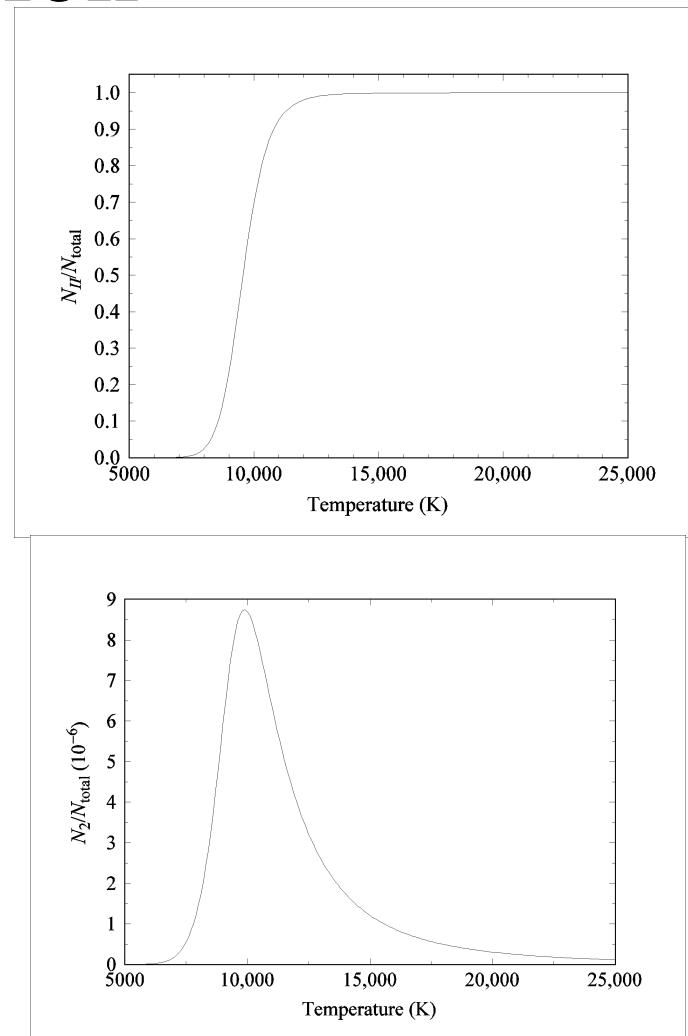
$$g_n = 2n^2$$

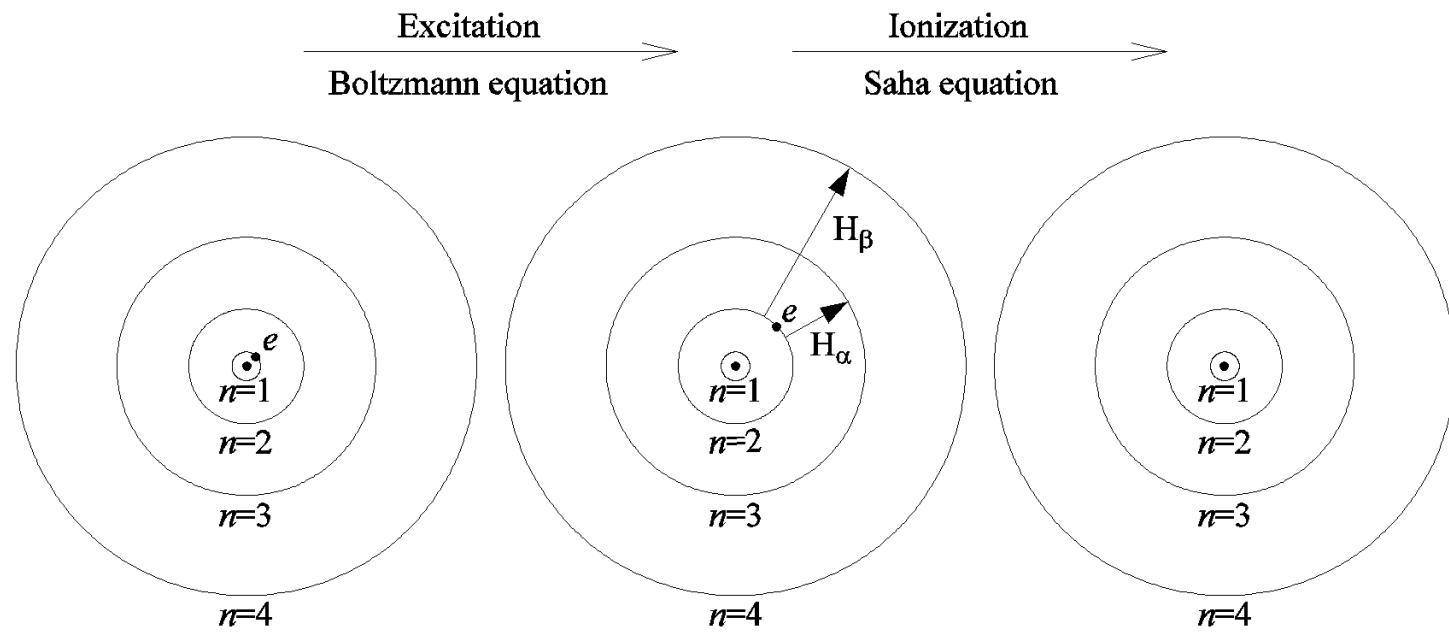
Formation of spectral lines: 2.ionization

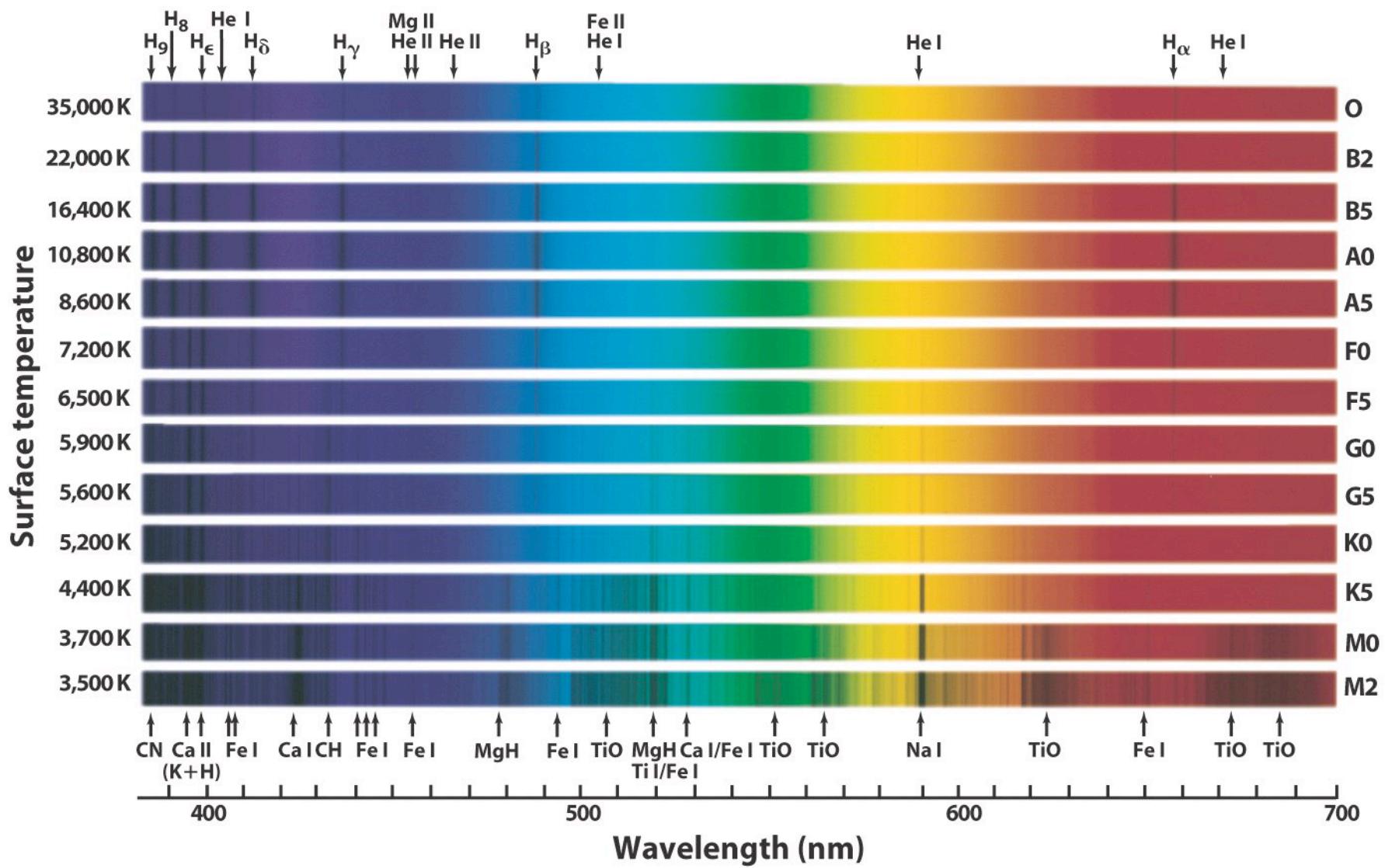
When the temperature become high enough, ionization becomes important, as it can no longer participate in line emission or absorption. When the gas is ionized, electrons and positive ions will recombine. When rates of ionization and recombination are equal, the gas is in ionization equilibrium.

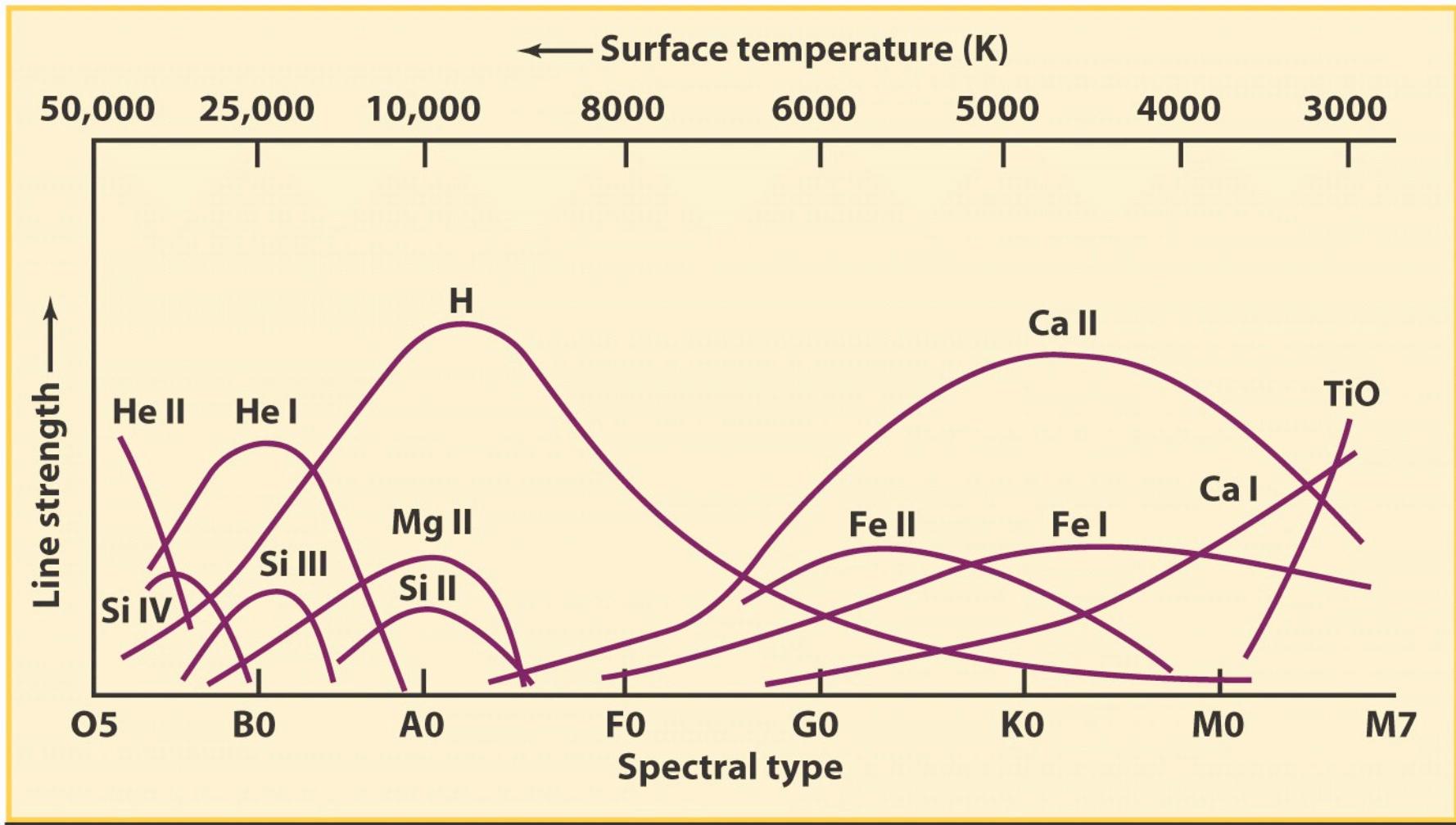
Saha's equation:

$$\frac{n_e n(X_{r+1})}{n(X_r)} = \frac{2g_r}{g_{r+1}} \left(\frac{2\pi m_r k T_k}{h^2} \right)^{3/2} e^{-[E_i/kT_k]}$$









The spectral class and type of a star is directly related to its surface temperature: O stars are the hottest and M stars are the coolest

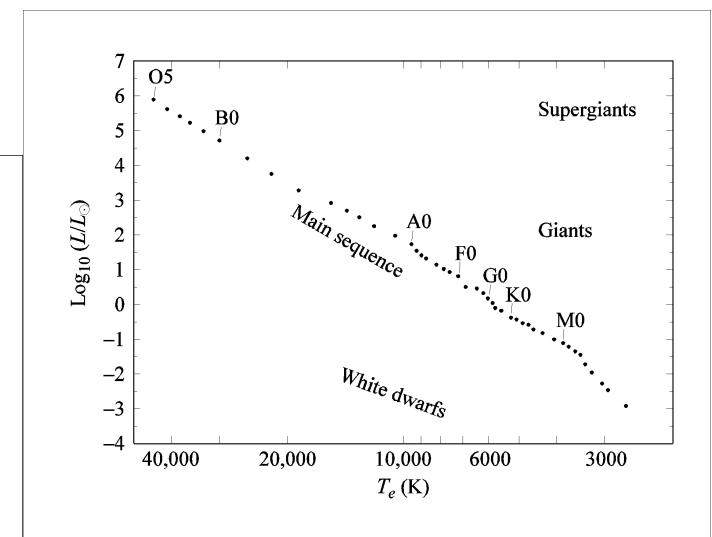
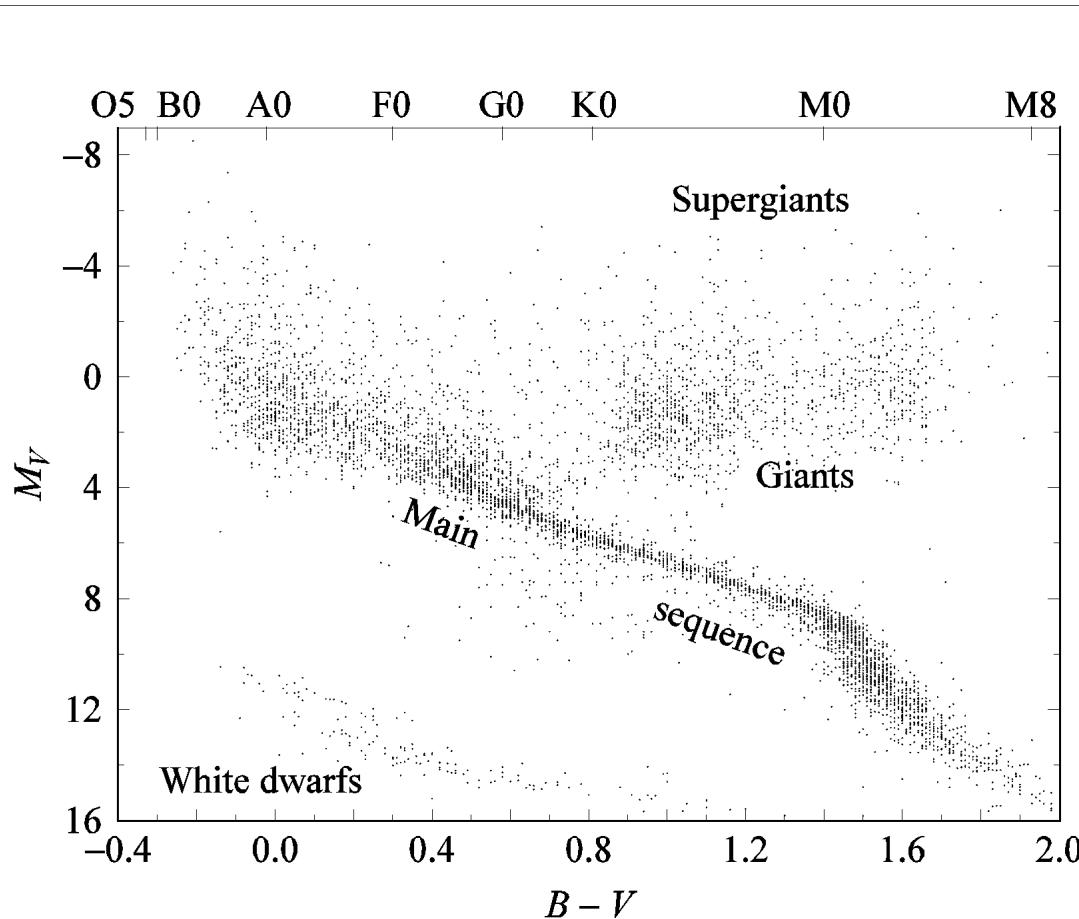
table 19-2

The Spectral Sequence

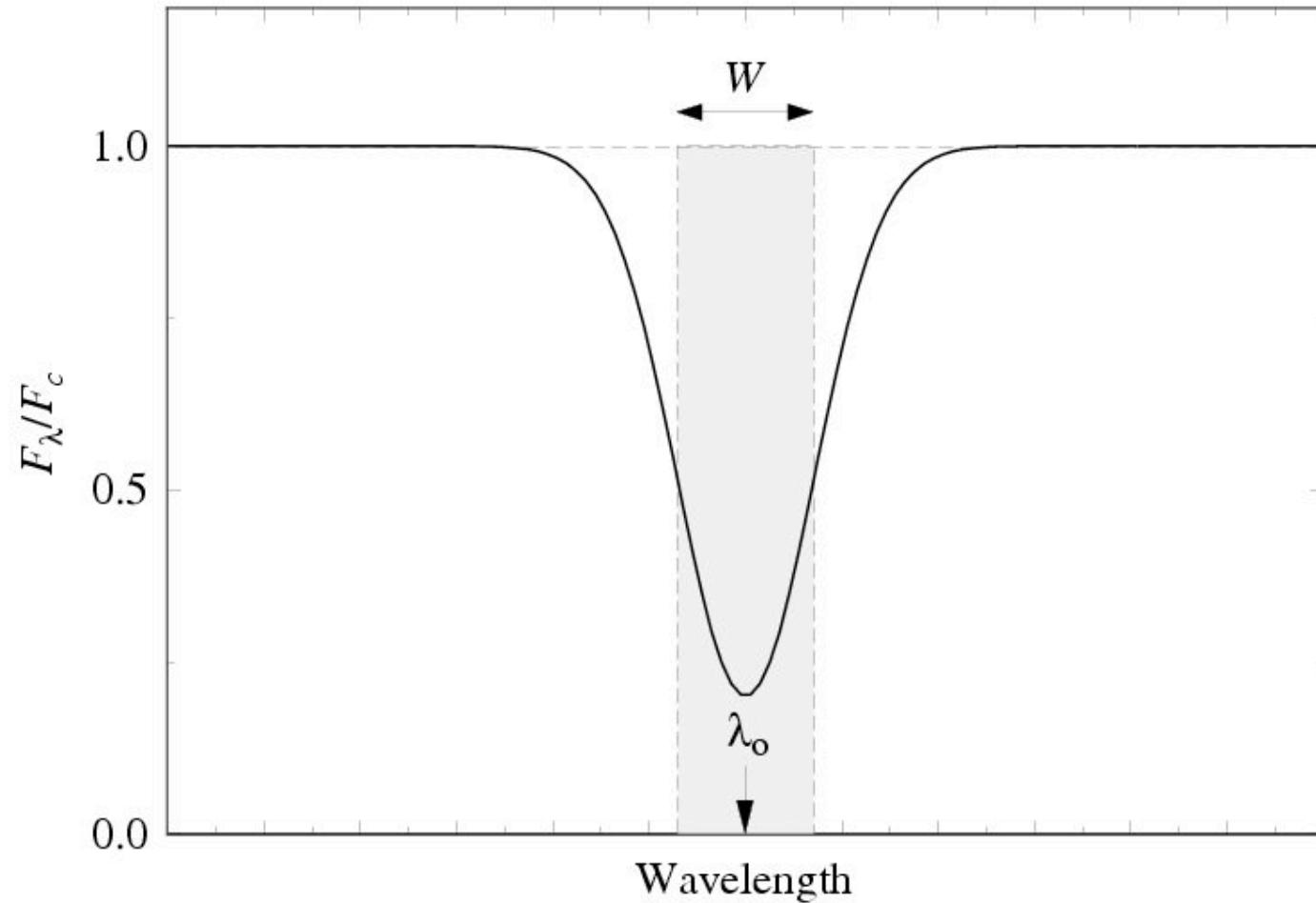
Spectral class	Color	Temperature (K)	Spectral lines	Examples
O	Blue-violet	30,000–50,000	Ionized atoms, especially helium	Naos (ζ Puppis), Mintaka (δ Orionis)
B	Blue-white	11,000–30,000	Neutral helium, some hydrogen	Spica (α Virginis), Rigel (β Orionis)
A	White	7500–11,000	Strong hydrogen, some ionized metals	Sirius (α Canis Majoris), Vega (α Lyrae)
F	Yellow-white	5900–7500	Hydrogen and ionized metals such as calcium and iron	Canopus (α Carinae), Procyon (α Canis Minoris)
G	Yellow	5200–5900	Both neutral and ionized metals, especially ionized calcium	Sun, Capella (α Aurigae)
K	Orange	3900–5200	Neutral metals	Arcturus (α Boötis), Aldebaran (α Tauri)
M	Red-orange	2500–3900	Strong titanium oxide and some neutral calcium	Antares (α Scorpii), Betelgeuse (α Orionis)
L	Red	1300–2500	Neutral potassium, rubidium, and cesium, and metal hydrides	Brown dwarf Teide 1
T	Red	below 1300	Strong neutral potassium and some water (H_2O)	Brown dwarf Gliese 229B

- Most brown dwarfs are in even cooler spectral classes called L and T
- Unlike true stars, brown dwarfs are too small to sustain thermonuclear fusion

The Hertzsprung-Russell Diagram



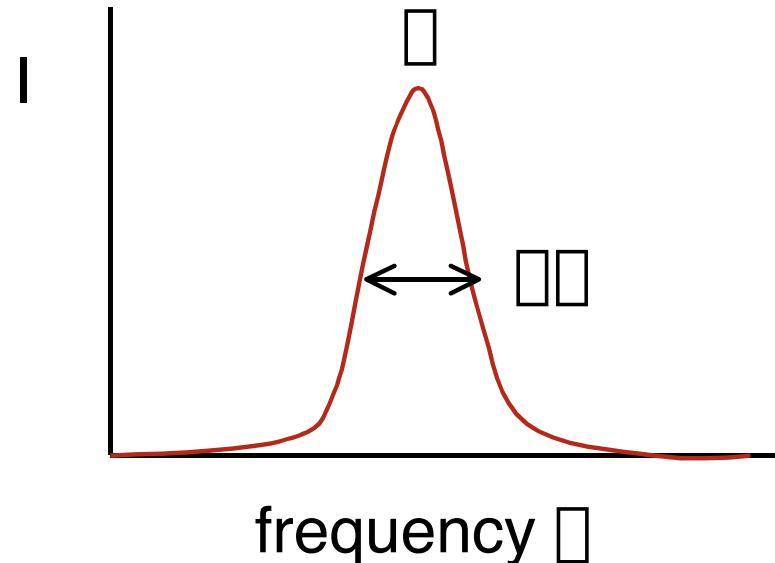
Line broadening



Broadening of spectral lines

An individual atom making a transition between energy levels emits one photon with a well-defined energy / frequency.

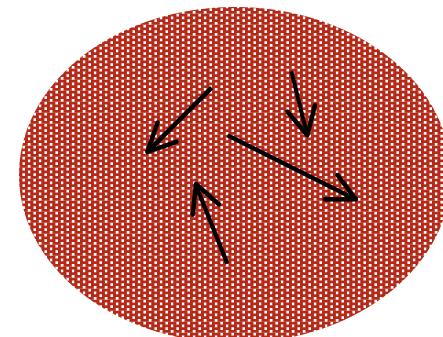
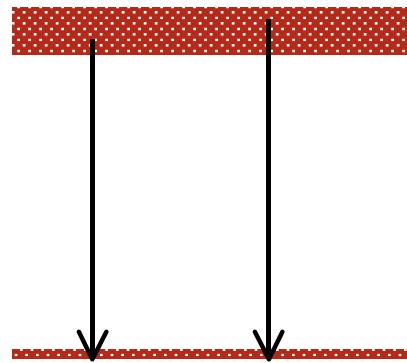
However, profiles of real spectral lines are not infinitely narrow.



e.g. for an emission line, width of the spectral line $\square\square$ could be defined as the full width at half the maximum intensity of the line.

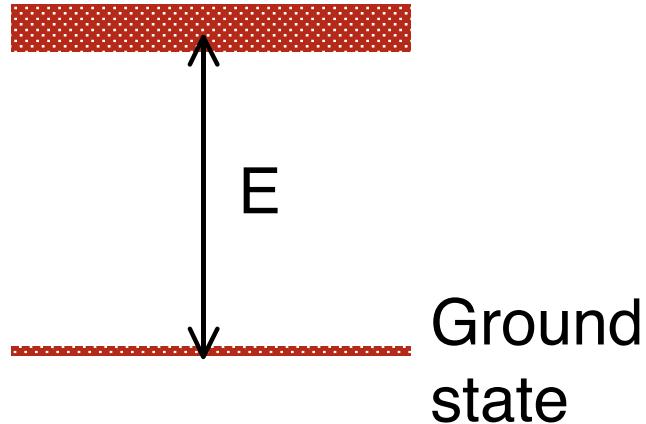
Details of definition don't matter - important to see what causes lines to have finite width.

Two basic mechanisms:



- 1) Energy levels themselves are not infinitely sharp:
emitted photons have a range of frequencies
- 2) Atoms in the gas are moving relative to the observer:
observed photons don't have the same frequency as the emitted photons because of the Doppler effect.

Natural linewidth



Consider excited state with energy E above the ground state.

Electrons in excited state remain there for average time \bar{t} before decaying to ground state.

Uncertainty principle: energy difference between states is uncertain by an amount ΔE given by:

$$\Delta E \bar{t} \approx \frac{\hbar}{2\Delta t}$$

But since $E = \hbar\bar{t}$, $\Delta E = \hbar\Delta t$ $\rightarrow \Delta E \approx \frac{1}{2\Delta t}$

Broadening due to this effect is called the natural linewidth.

Natural linewidth sets absolute minimum width of spectral lines. However, normally very small - other effects dominate.

e.g. for hydrogen n=2 to n=1 transition (Lyman α transition) the lifetime is of the order of 10^{-9} s.

Natural linewidth is $\sim 10^8$ Hz.

Compare to frequency of transition: $\frac{\Delta\nu}{\Delta t} \approx 10^{17}$

In **astrophysical** situations, other processes will often give much larger linewidths than this.

Collisional broadening

In a dense gas, atoms are colliding frequently. This effectively reduces the lifetime of states further, to a value smaller than the quantum mechanical lifetime.

If the frequency of collisions is ν_{col} , then expect to get a collisional linewidth of about $\Delta\nu \propto \nu_{\text{col}}$.

Frequency of collisions increases with density - expect to see broader lines in high density regions as compared to low density ones.

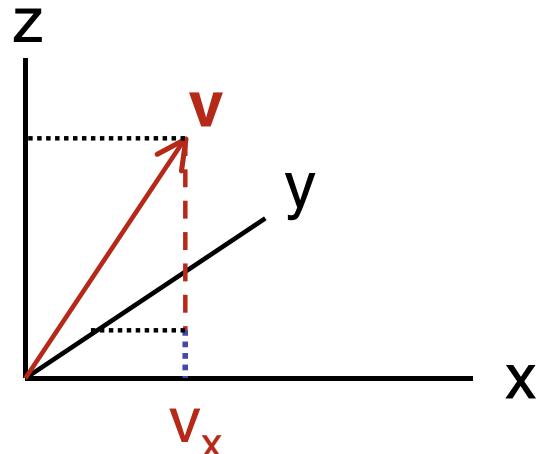
e.g. a main sequence star (small radius) has a higher density at the photosphere than a giant of the same surface temperature. Spectral lines in the main sequence star will be broader than in the giant.

Doppler or thermal broadening

Atoms in a gas have random motions that depend upon the temperature. For atoms of mass m , at temperature T , the typical speed is obtained by equating kinetic and thermal energy:

$$\frac{1}{2}mv^2 = kT \quad k = \text{Boltzmann's const}$$

Number of atoms with given speed or velocity is given by **Maxwell's law**. Need to distinguish between forms of this law for speed and for any one velocity component:



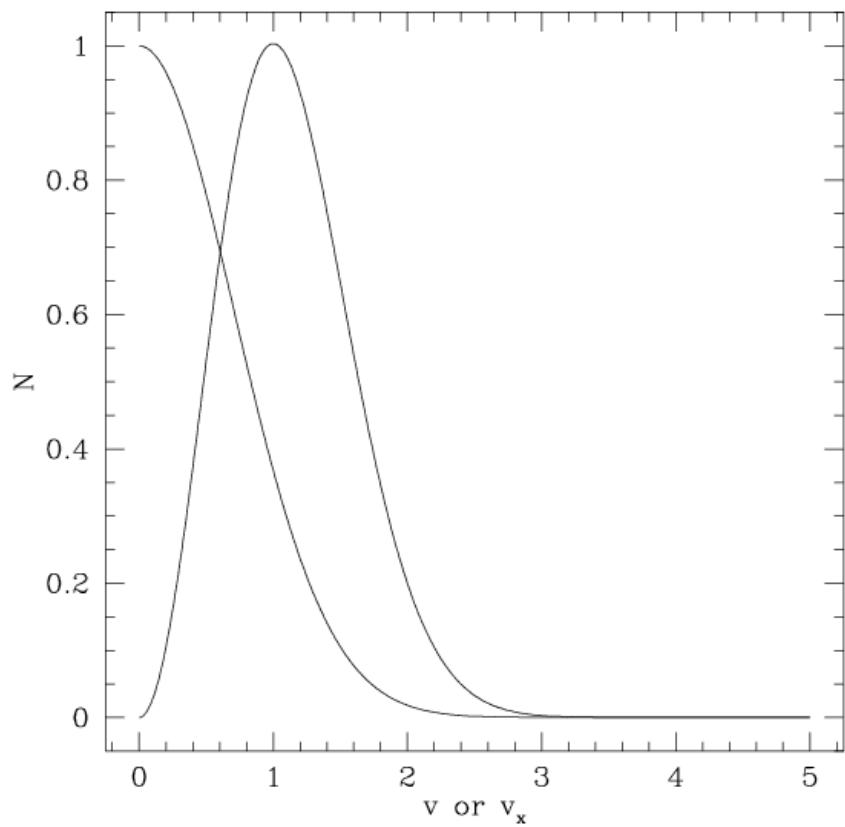
$$|\mathbf{v}|^2 = v_x^2 + v_y^2 + v_z^2$$

Distribution of *one component* of the velocity, say v_x , is relevant for thermal broadening - only care about motion along line of sight.

For one component, number of atoms dN within velocity interval dv_x is given by:

$$dN(v_x) \propto \exp\left(-\frac{mv_x^2}{2kT}\right) dv_x$$

Distribution law for speeds has extra factor of v^2 :



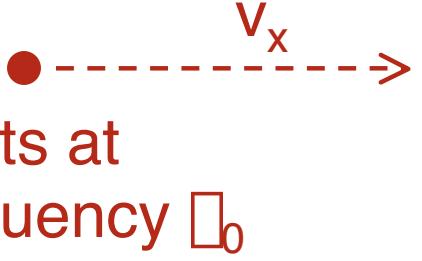
$$dN(v) \propto v^2 \exp\left(-\frac{mv^2}{2kT}\right) dv$$

Most probable speed:

$$v_{peak} = \sqrt{\frac{2kT}{m}}$$

Average speed:

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3kT}{m}}$$


Emits at frequency ν_0
Observed at frequency ν

Consider atom moving with velocity v_x along the line of sight to the observer.

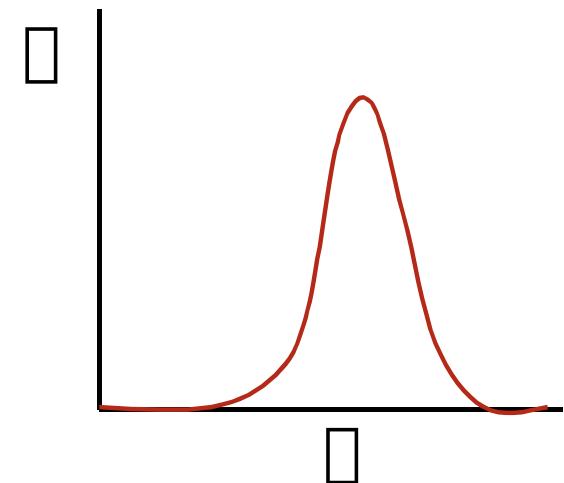
Doppler shift formula:
$$\frac{\nu - \nu_0}{\nu_0} = \frac{v_x}{c}$$

Combine this with the thermal distribution of velocities:

$$f(\nu) = \frac{1}{\pi \nu_D \sqrt{2}} \exp \left[-\frac{(\nu - \nu_0)^2}{(2 \nu_D)^2} \right]$$

...where the **Doppler width** of the line:

$$\nu_D = \frac{\nu_0}{c} \sqrt{\frac{2kT}{m}}$$



If the gas also has large-scale (i.e. not microscopic) motions due to turbulence, those add to the width:

$$\frac{\Delta \sigma_D}{\Delta \sigma_0} = \frac{c}{m} \sqrt{\frac{2kT}{m}} + v_{turb}^2 \sqrt{\frac{1}{m}}$$

v_{turb} is a measure of the typical turbulent velocity (note: really need same velocity distribution for this to be strictly valid).

Some numbers for hydrogen:

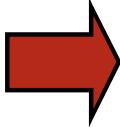
$$\frac{\Delta \sigma_D}{\Delta \sigma_0} \approx 4.3 \times 10^{05} \sqrt{\frac{T}{10^4 \text{ K}}}$$

larger than natural linewidth

$$\frac{\Delta \sigma_D c}{\Delta \sigma_0} \approx 13 \sqrt{\frac{T}{10^4 \text{ K}}} \text{ km s}^{-1}$$

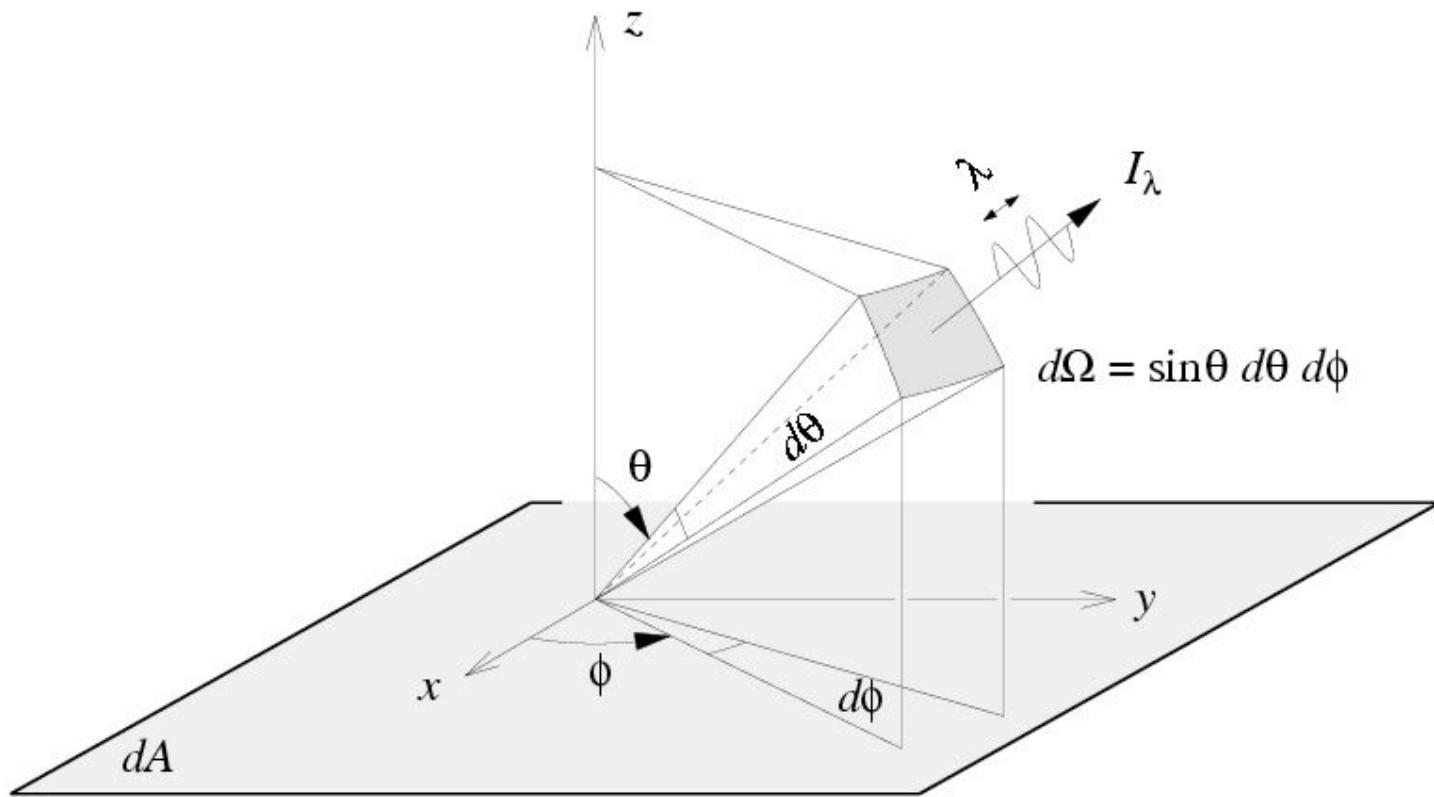
measured in velocity units,
comparable to the sound
speed in the gas

Summary:

- **Strength** of different spectral lines depends upon the abundance of different elements, and on the excitation / ionization state (described in part by the Boltzmann formula).
 - **Width** of spectral lines depends upon:
 - *Natural linewidth* (small)
 - *Collisional linewidth* (larger at high density)
 - *Thermal linewidth* (larger at higher temperature)
-  High quality spectrum gives information on composition, temperature and density of the gas.

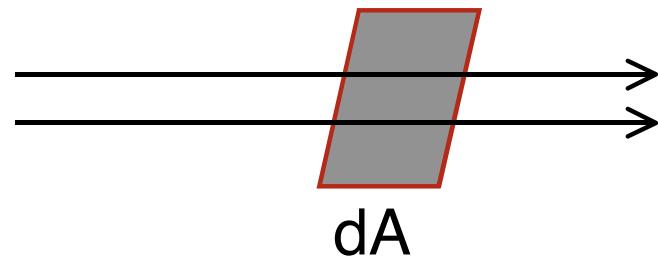
c.f. 'Modern Astrophysics' section 8.1: more on thermal broadening, Boltzmann law, and Saha equation (version of Boltzmann law for ionization).

Stellar Atmosphere

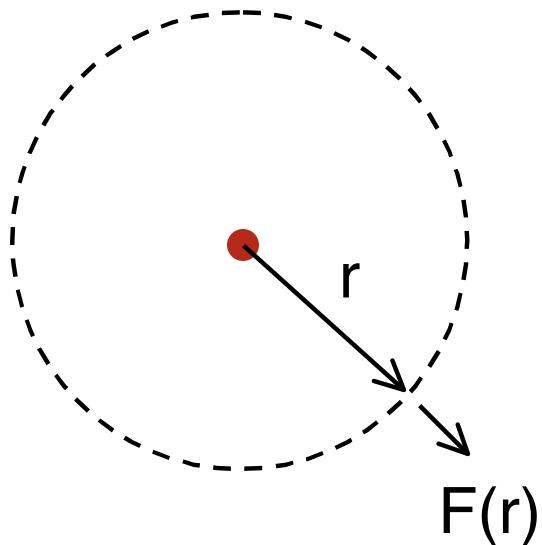


Flux

Consider a small area dA , exposed to radiation for a time dt . Energy passing through the area is $F.dA.dt$, where F is the **energy flux** (units $\text{erg s}^{-1} \text{cm}^{-2}$).



Unless the radiation is **isotropic** (same in all directions), F will depend on orientation of dA .



Spherically symmetric steady source of luminosity L . Energy conservation:

$$L = 4\pi r^2 F(r)$$

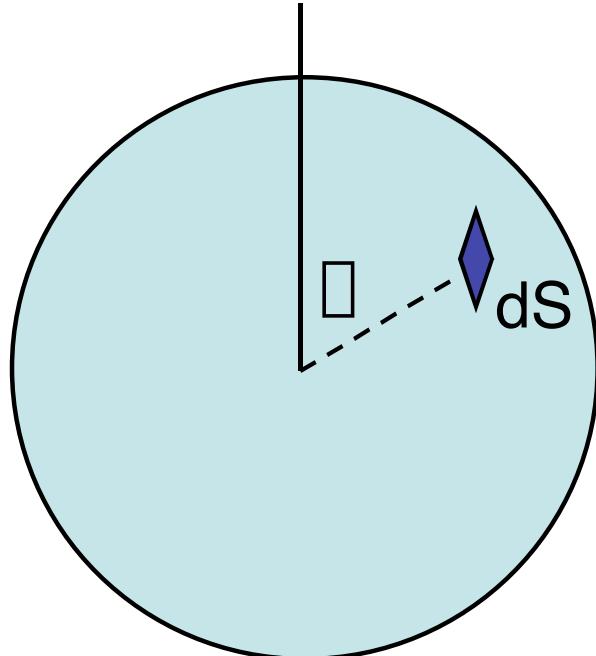
$$F(r) = \frac{L}{4\pi r^2}$$

Inverse square law.

The equation of radiative transfer

How does the intensity of radiation change in the presence of emission and / or absorption?

Definition of solid angle and steradian



Sphere radius r - area of a patch dS on the surface is:

$$dS = r d\Omega \quad r \sin \theta d\Omega = r^2 d\Omega$$

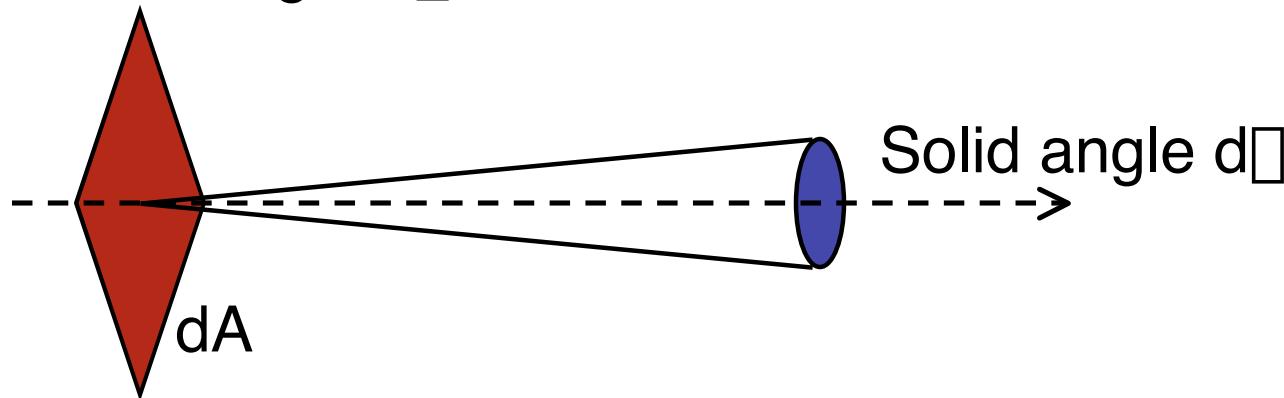
$d\Omega$ is the solid angle subtended by the area dS at the center of the sphere.

Unit of solid angle is the steradian.

4 π steradians cover whole sphere.

Definition of the specific intensity

Construct an area dA normal to a light ray, and consider all the rays that pass through dA whose directions lie within a small solid angle $d\Omega$.



The amount of energy passing through dA and into $d\Omega$ in time dt in frequency range $d\nu$ is:

$$dE = I_\nu dA dt d\nu d\Omega$$



Specific intensity of the radiation.

Compare with definition of the flux: specific intensity is very similar except it depends upon direction and frequency as well as location.

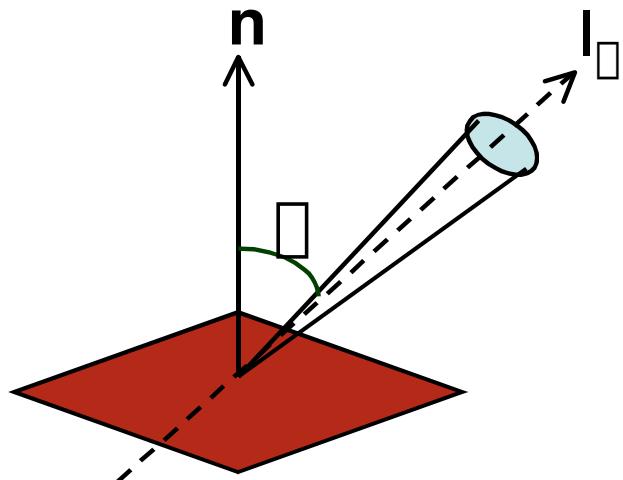
Units of specific intensity are: erg s⁻¹ cm⁻² Hz⁻¹ steradian⁻¹

Same as F_{\square}

Another, more intuitive name for the specific intensity is **brightness**.

Simple relation between the flux and the specific intensity:

Consider a small area dA , with light rays passing through it at all angles to the normal to the surface \mathbf{n} :



If $\theta = 90^\circ$, then light rays in that direction contribute **zero** net flux through area dA .

For rays at angle θ , foreshortening reduces the effective area by a factor of $\cos(\theta)$.

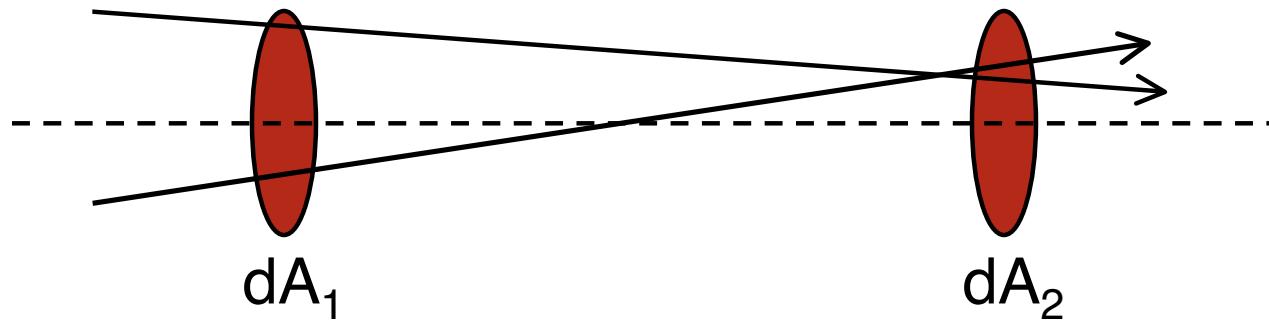
Hence, net flux in the direction of \mathbf{n} is given by integrating (the specific intensity $\times \cos \theta$) over all solid angles:

$$F_\theta = \int I_\theta \cos \theta d\Omega$$

Note: to actually evaluate this need to express $d\Omega$ in terms of $d\theta$ and $d\phi$ as before.

How does specific intensity change along a ray

If there is no emission or absorption, specific intensity is just **constant** along the path of a light ray. Consider any two points along a ray, and construct areas dA_1 and dA_2 normal to the ray at those points. How much energy is carried by those rays that pass through **both** dA_1 and dA_2 ?



$$\left. \begin{aligned} dE_1 &= I_{\square 1} dA_1 dt d\square_1 \\ dE_2 &= I_{\square 2} dA_2 dt d\square_2 \end{aligned} \right\} \text{ where } d\square_1 \text{ is the solid angle subtended by } dA_2 \text{ at } dA_1 \text{ etc}$$

The same photons pass through both dA_1 and dA_2 , without change in their frequency. Conservation of energy gives:

$$dE_1 = dE_2 \quad \text{- equal energy}$$

$$d\Omega_1 = d\Omega_2 \quad \text{- same frequency interval}$$

Using definition of solid angle, if dA_1 is separated from dA_2 by distance r :

$$d\Omega_1 = \frac{dA_2}{r^2}, \quad d\Omega_2 = \frac{dA_1}{r^2}$$

Substitute:

$$I_{\Omega 1} dA_1 dt d\Omega_1 = I_{\Omega 2} dA_2 dt d\Omega_2 \quad \text{dE}_1 = \text{dE}_2$$

$$I_{\Omega 1} dA_1 dt d\Omega_1 \frac{dA_2}{r^2} = I_{\Omega 2} dA_2 dt d\Omega_2 \frac{dA_1}{r^2} \quad \text{d}\Omega_1 = \text{d}\Omega_2$$

$$\underline{I_{\Omega 1} = I_{\Omega 2}}$$

Conclude: specific intensity remains the same as radiation propagates through free space.

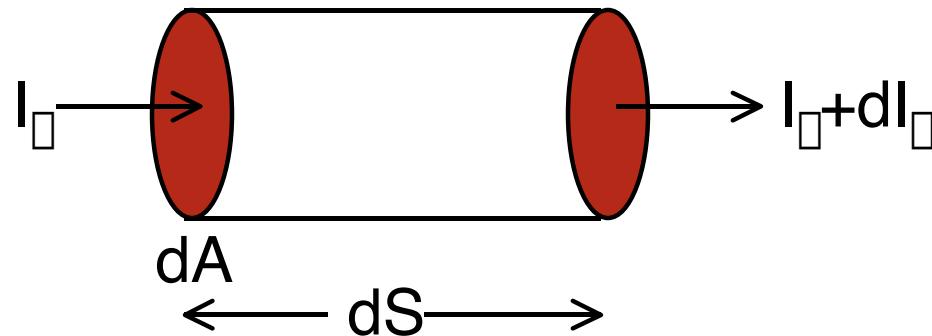
Justifies use of alternative term ‘brightness’ - e.g. brightness of the disk of a star remains same no matter the distance - **flux** goes down but this is compensated by the light coming from a smaller area.

If we measure the distance along a ray by variable s , can express result equivalently in differential form:

$$\frac{dI_{\perp}}{ds} = 0$$

Emission

If the radiation travels through a medium which is itself emitting radiation, that will add to the energy:



Spontaneous **emission coefficient** is the amount of energy emitted per unit time, per unit solid angle, per unit frequency interval, and per unit volume:

$$dE = j_{\perp} dV d\Omega dt d\Omega$$

In going a distance ds , beam of cross-section dA travels through a volume $dV = dA \times ds$.

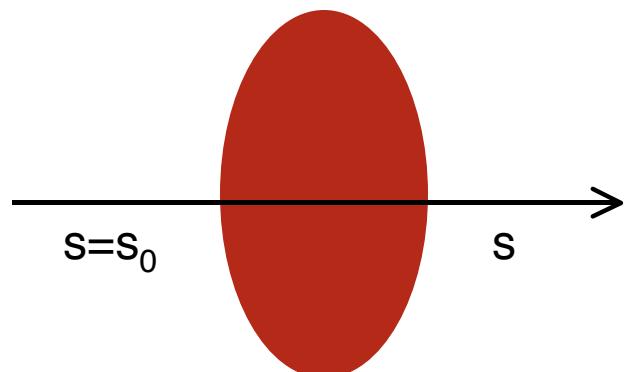
Change (increase) in specific intensity is therefore:

$$dI_{\square} = j_{\square} ds$$

Equation of radiative transfer for pure emission becomes:

$$\frac{dI_{\square}}{ds} = j_{\square}$$

If we know what j_{\square} is, can integrate this equation to find the change in specific intensity as radiation propagates through the gas:

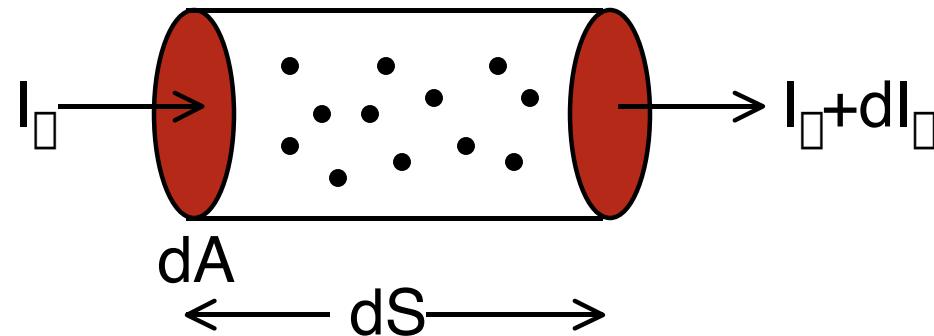


$$I_{\square}(s) = I_{\square}(s_0) + \int_{s_0}^s j_{\square}(s') ds'$$

i.e. add up the contributions to the emission all along the path.

Absorption

If the radiation travels through a medium which absorbs (or scatters) radiation, the energy in the beam will be reduced:



Number density of absorbers (particles per unit volume) = n
Each absorber has cross-sectional area = σ (units cm^2)

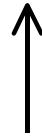
If beam travels through ds , total area of absorbers is:

$$\text{number of absorbers} \times \text{cross - section} = n dA ds \sigma$$

Fraction of radiation absorbed = fraction of area blocked:

$$\frac{dI_{\perp}}{I_{\perp}} = \frac{n dA ds \mu_{\perp}}{dA} = \mu_{\perp} ds$$

$$dI_{\perp} = \mu_{\perp} n ds I_{\perp}$$



absorption coefficient (units cm^{-1})

Can also write this in terms of mass:

$$\mu_{\perp} = \rho \mu_{\parallel}$$

μ_{\parallel} is called the mass absorption coefficient or the **opacity**.

Opacity has units of $\text{cm}^2 \text{ g}^{-1}$ (i.e. the cross section of a gram of gas).

Example: Thomson scattering

A free electron has a cross section to radiation given by the Thomson value:

$$\sigma_T = 6.7 \times 10^{-25} \text{ cm}^2$$

...independent of frequency. The opacity is therefore:

$$\kappa = \frac{n}{m_e} \sigma_T = N_A \sigma_T = 0.4 \text{ cm}^2 \text{ g}^{-1}$$

If the gas is pure hydrogen
(protons and electrons only)

(note: really should distinguish between absorption and scattering, but don't need to worry about that here...)

Equation of radiative transfer for pure absorption. Rearrange previous equation:

$$\frac{dI_{\perp}}{ds} = -\kappa_{\perp} I_{\perp}$$

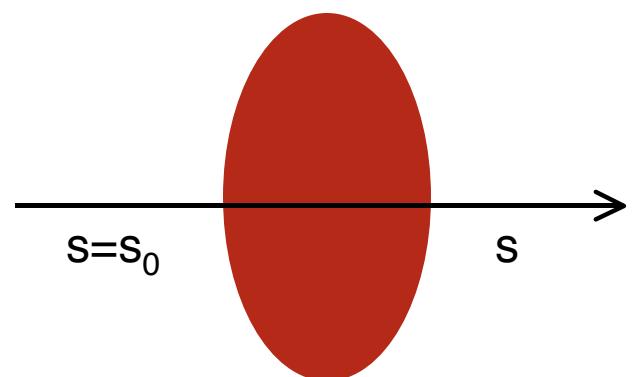
Different from emission because depends on how much radiation we already have.

Integrate to find how radiation changes along path:

$$\int_{s_0}^s \frac{dI_{\perp}}{I_{\perp}} = -\kappa_{\perp} \int_{s_0}^s ds$$

$$[\ln I_{\perp}]_{s_0}^s = -\kappa_{\perp} \int_{s_0}^s ds$$

$$I_{\perp}(s) = I_{\perp}(s_0) e^{-\kappa_{\perp} \int_{s_0}^s ds}$$



e.g. if the absorption coefficient is a constant (example, a uniform density gas of ionized hydrogen):

$$I_{\nu}(s) = I_0 e^{-\kappa_{\nu} s}$$

Specific intensity after distance s

Initial intensity

Radiation exponentially absorbed with distance

Radiative transfer equation with both absorption and emission:

$$\frac{dI_{\nu}}{ds} = -\kappa_{\nu} I_{\nu} + j_{\nu}$$

absorption

emission

Optical depth

Look again at general solution for pure absorption:

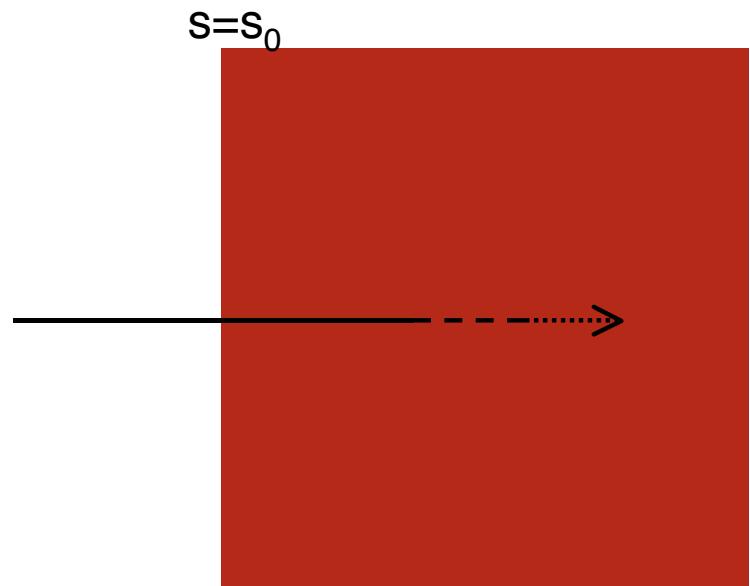
$$I_{\nu}(s) = I_{\nu}(s_0) e^{-\int_{s_0}^s \kappa_{\nu}(s') ds'}$$

Imagine radiation traveling into a cloud of absorbing gas, exponential defines a scale over which radiation is attenuated.

When:

$$\int_{s_0}^s \kappa_{\nu}(s') ds' = 1$$

...intensity will be reduced to
1/e of its original value.



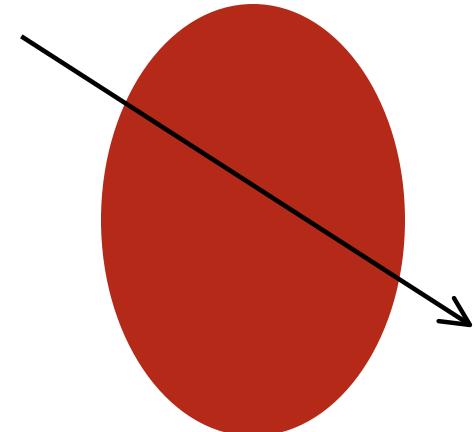
Define **optical depth** τ as:

$$\tau(s) = \int_{s_0}^s \tau(ds)$$

or equivalently $d\tau = \tau ds$

A medium is **optically thick** at a frequency ν if the optical depth for a typical path through the medium satisfies:

$$\tau \geq 1$$



Medium is said to be **optically thin** if instead:

$$\tau < 1$$

Interpretation: an optically thin medium is one which a typical photon of frequency ν can pass through without being absorbed.

Can rewrite the radiative transfer equation using the optical depth as a measure of ‘distance’ rather than s :

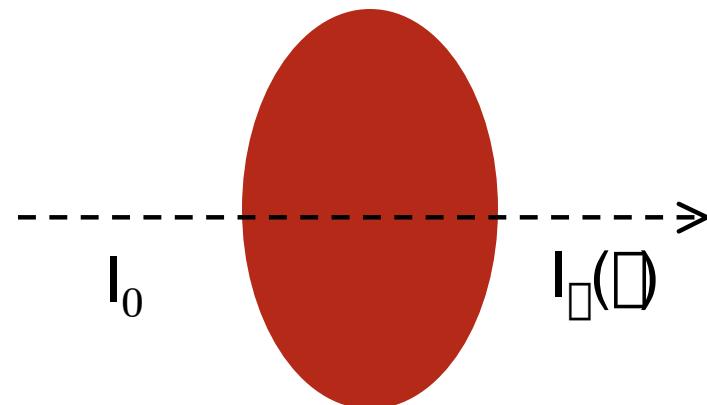
$$\frac{dI_{\perp}}{ds} = \kappa_{\perp} I_{\perp} + j_{\perp}$$

$$\frac{dI_{\perp}}{\kappa_{\perp} ds} = \kappa_{\perp} I_{\perp} + \frac{j_{\perp}}{\kappa_{\perp}} \quad \leftarrow \text{divide by the absorption coefficient}$$

$$\frac{dI_{\perp}}{d\tau_{\perp}} = \kappa_{\perp} I_{\perp} + S_{\perp}$$

...where $S_{\perp} = j_{\perp} / \kappa_{\perp}$ is the **source function**. An alternative and sometimes more convenient way to write the equation.

Use result we derived last time - consider radiation passing through a hot cloud of gas in thermal equilibrium:



Found:

$$I(\omega) = I_0 e^{-\kappa \omega} + B(\omega)(1 - e^{-\kappa \omega})$$

Suppose no intensity entering the cloud, $I_0 = 0$. If the cloud is very optically thin:

$$e^{-\kappa \omega} \approx 1 - \kappa \omega$$

$$I(\omega) = B(\omega)(1 - 1 + \kappa \omega) = \kappa \omega B(\omega)$$

Optical depth is related to the absorption coefficient via:

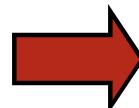
$$\tau_\nu = \kappa_\nu s \quad (\text{for constant } \kappa)$$

Means that:

$$I_\nu = B_\nu - \kappa_\nu B_\nu$$

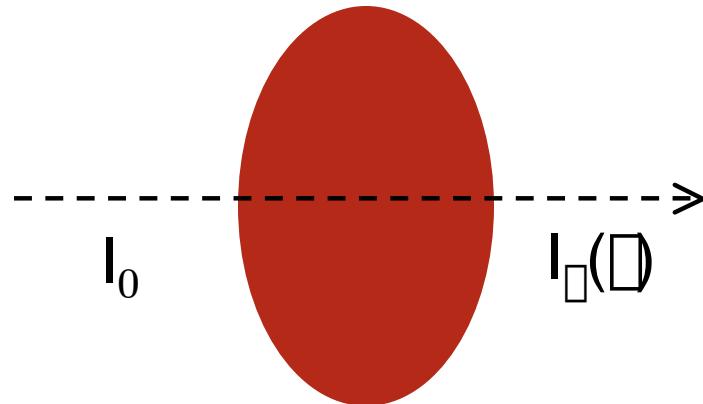
Intensity is large at frequencies where the absorption coefficient is large.

For a hot gas, absorption coefficient is large at the frequencies of the spectral lines.



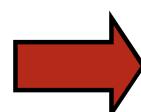
For an optically thin medium such as a nebula, expect an emission line spectrum with large intensity at the frequencies where κ_ν is large.

Summary: Friday's class



Used solution: $I_B(\Delta_B) = I_0 e^{\Delta_B} + B_B(1 - e^{\Delta_B})$

For small optical depth: $I_B = \Delta_B B_B$ $\Delta_B B_B$

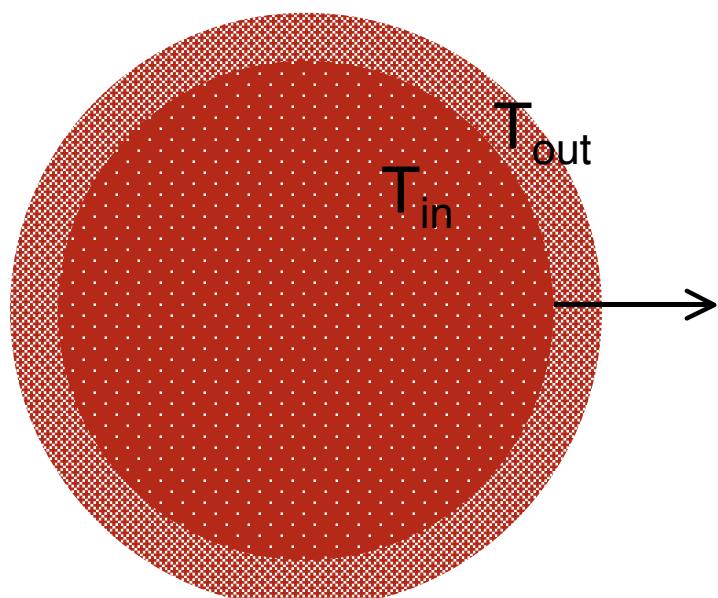


For an optically thin medium such as a nebula,
expect an emission line spectrum with large
intensity at the frequencies where Δ_B is large.

Next, consider an optically thick source:

- Already shown that in the interior, radiation will be described by the Planck function.
- Radiation escaping from the source will be modified because the temperature (and thus the Planck function) **varies** along the path.

Example: model a star using a two layer model:



Radiation starts from the inner layer as blackbody radiation at temperature T_{in} .

Escapes through an atmosphere of optical depth τ and temperature T_{out} .

Use same solution as before to describe change in intensity of the radiation:

$$I_{\nu}(\tau) = I_0 e^{-\tau} + B_{\nu}(1 - e^{-\tau})$$

Valid provided that all the gas is in thermal equilibrium (LTE).

Assume that optical depth of outer layer is small and use approximate expansion for the exponential as before:

$$I_{\nu}(\tau) = B_{\nu}(T_{in})e^{-\tau} + B_{\nu}(T_{out})[1 - e^{-\tau}]$$

$$I_{\nu}(\tau) = B_{\nu}(T_{in})[1 - \tau] + B_{\nu}(T_{out})\tau$$

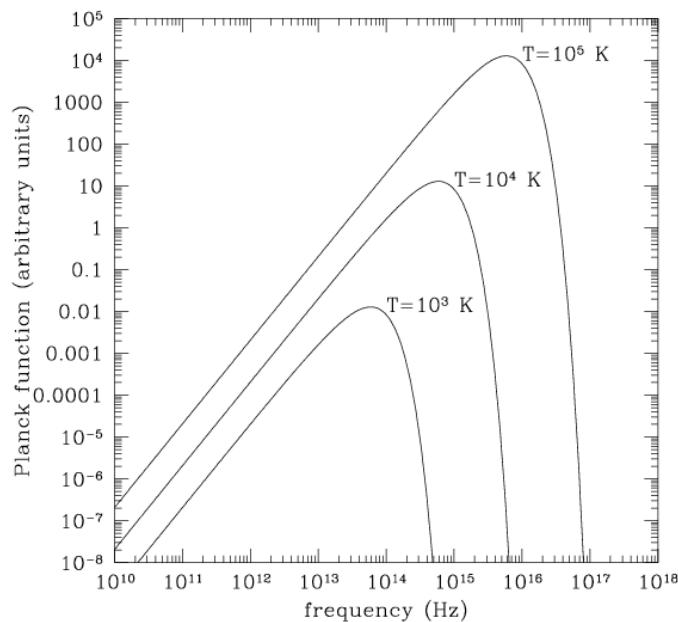
$$I_{\nu}(\tau) = B_{\nu}(T_{in}) + \tau[B_{\nu}(T_{out}) - B_{\nu}(T_{in})]$$

$$I_{\nu}(\square) = B_{\nu}(T_{in}) + \square [B_{\nu}(T_{out}) - B_{\nu}(T_{in})]$$

Initial radiation intensity

Change in intensity caused by the outer layer. Depends upon frequency.

Recall that intensity of blackbody radiation increases at **all** frequencies as the temperature goes up.



Sign of the second term depends upon whether $B_{\nu}(T_{out})$ is larger or smaller than $B_{\nu}(T_{in})$ - i.e. on whether $T_{out} > T_{in}$.

$$I_{\nu}(\nu) = B_{\nu}(T_{in}) + \nu [B_{\nu}(T_{out}) - B_{\nu}(T_{in})]$$

1) $T_{out} > T_{in}$: second term is positive:

Escaping intensity is **larger** at frequencies where ν is greatest (frequencies corresponding to spectral lines).
Expect emission lines on top of the continuum.

2) $T_{out} < T_{in}$: second term is negative:

Escaping intensity is **reduced** at frequencies where ν is greatest (frequencies corresponding to spectral lines).
Expect absorption lines superimposed on the continuum.

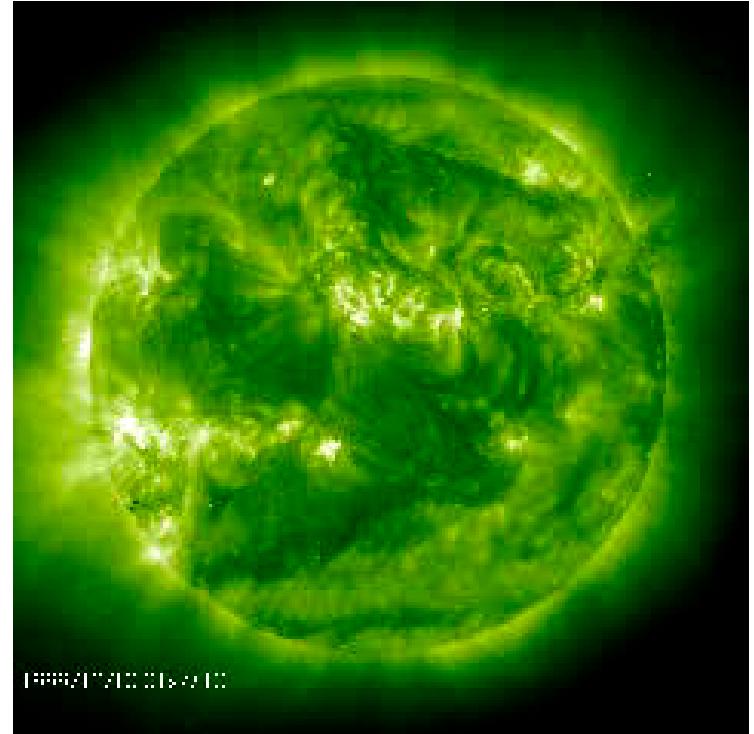
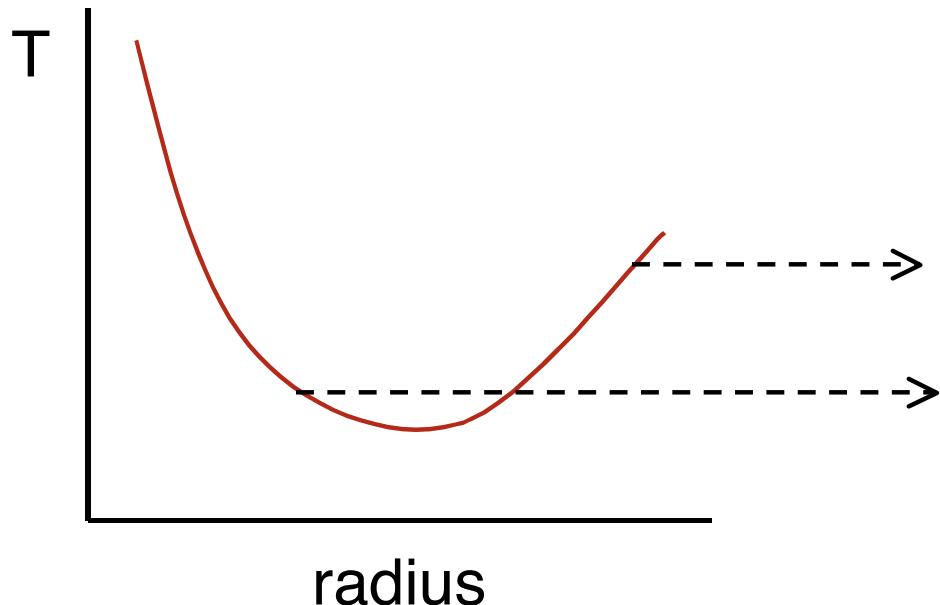
For the Sun, temperature near the optical photosphere **decreases** outward (as it must since energy transport is from the center to the outside).

In second regime: $T_{\text{out}} < T_{\text{in}}$

Expect to see an **absorption spectrum**, as observed:



Note: see strong UV and X-ray emission from the Solar corona, so obviously the temperature there is much hotter than that of the photosphere...



UV radiation comes from region where T increasing, so emission line spectrum

Optical radiation comes from region where T decreasing, so absorption spectrum