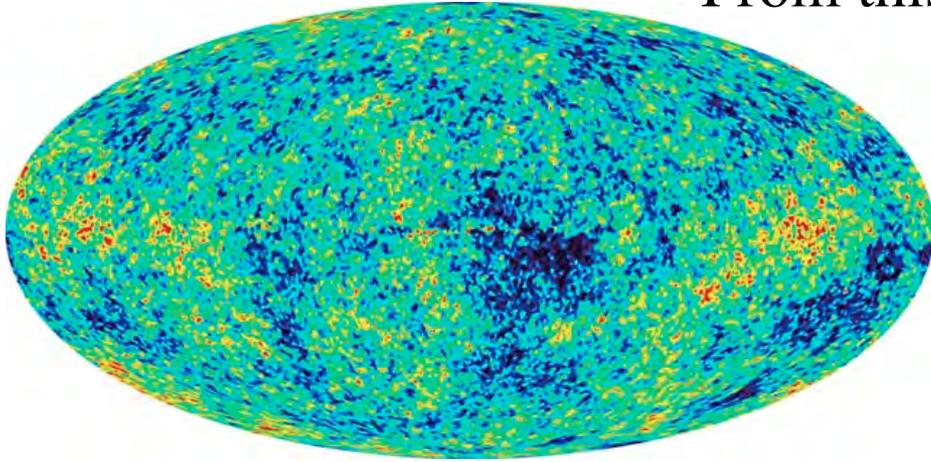


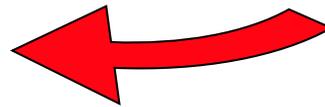
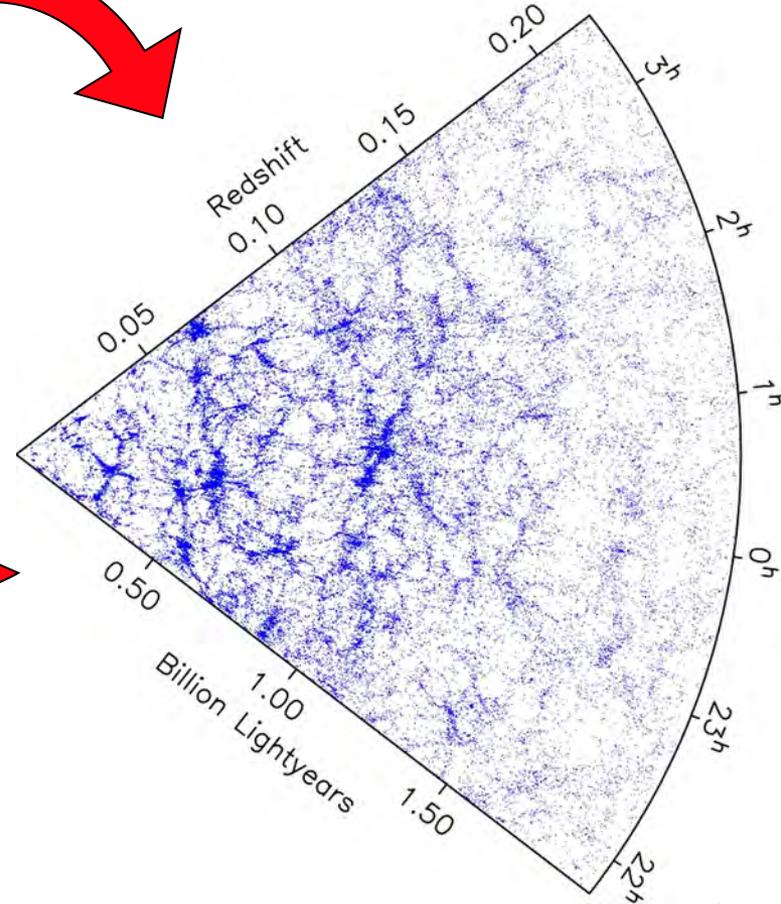
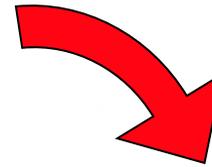
Large Scale Structure: Formation and Growth

Structure Formation and Evolution

From this ($\Delta\rho/\rho \sim 10^{-6}$)



to this
($\Delta\rho/\rho \sim 10^{+2}$)



to this
($\Delta\rho/\rho \sim 10^{+6}$)



Origin of Structure in the Universe

- Origin and evolution of the structure in the universe (galaxies, large-scale structures) is a central problem in cosmology
- Structure is generally thought to arise through *a growth of density perturbations* which originate in the early universe
 - We think they came from quantum fluctuations in the scalar field that caused inflation and were then amplified by the exponential inflation of the universe
- What do we know about the early structure formation?
 - We see CMB fluctuations with $\delta T/T \sim 10^{-6} \sim \Delta\rho/\rho$, since radiation and baryons are coupled before recombination
 - High- z objects: We observe galaxies and quasars at $z > 10$. A galaxy requires an overdensity of $\sim 10^6$ relative to the mean
- Can we get to such large density enhancements required for galaxies, clusters, etc., by evolving the small fluctuations we see in the CMB?

The Astrophysics of Structure Formation

GRAVITATIONAL INSTABILITY

QUANTUM DENSITY FLUCTUATIONS

INFLATION

PRIMORDIAL SPECTRUM OF FLUCTUATIONS

DARK MATTER

DAMPING

EXPANSION OF UNIVERSE

FORMATION OF DEEP GRAVITATIONAL POTENTIAL WELLS
DARK MATTER HALOES

DISSIPATIVE
GAS DYNAMICS

Messy!
Must be
done
numerically

VIRIALIZATION

RADIATIVE COOLING

FEEDBACK PROCESSES
supernovae, stellar winds

STAR FORMATION

Linear Growth of Density Fluctuations

How do fluctuations in density evolve with time? For simplicity, consider a flat, matter-dominated universe with $\Omega_m = 1$ (a good approx. for the early times, and we can also ignore Λ).

The Friedmann equation is:
(Note: using R instead of a)

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2} = 0$$

Now consider a small area with a slight overdensity of matter, it will evolve slightly differently:

$$H^2 - \frac{8}{3}\pi G\rho' = -\frac{k}{R^2}$$

Subtract the two equations:

$$-\frac{8}{3}\pi G(\rho' - \rho) = -\frac{k}{R^2}$$

Rewrite this as:

$$\rho' - \rho = -\frac{3k}{8\pi GR^2}$$

Linear Growth of Density Fluctuations

We can define δ as the *fractional overdensity*:

$$\delta \equiv \left(\frac{\rho' - \rho}{\rho} \right) = -\frac{3k}{8\pi G R^2 \rho}$$

But note that:

$$\delta \sim \frac{1}{R^2 \rho} \sim \frac{1}{R^2 R^{-3}} \sim R$$

Since $R \sim (1+z)^{-1}$, $\delta \sim (1+z)^{-1}$

And this is the *linear growth of density fluctuations*

Evolution of the density contrast between two redshifts is then:

$$\frac{\delta_f}{\delta_i} = \frac{(1+z)_i}{(1+z)_f}$$

The overdense region behaves as a slightly denser Friedman model

Linear Growth of Density Fluctuations

Let's try it from

recombination to $z = 10$: $\delta_f = 10^{-5} (1100+1) / (10+1) \approx 0.001$

Oops! It should be $\sim 10^6$ for galaxies - now what?

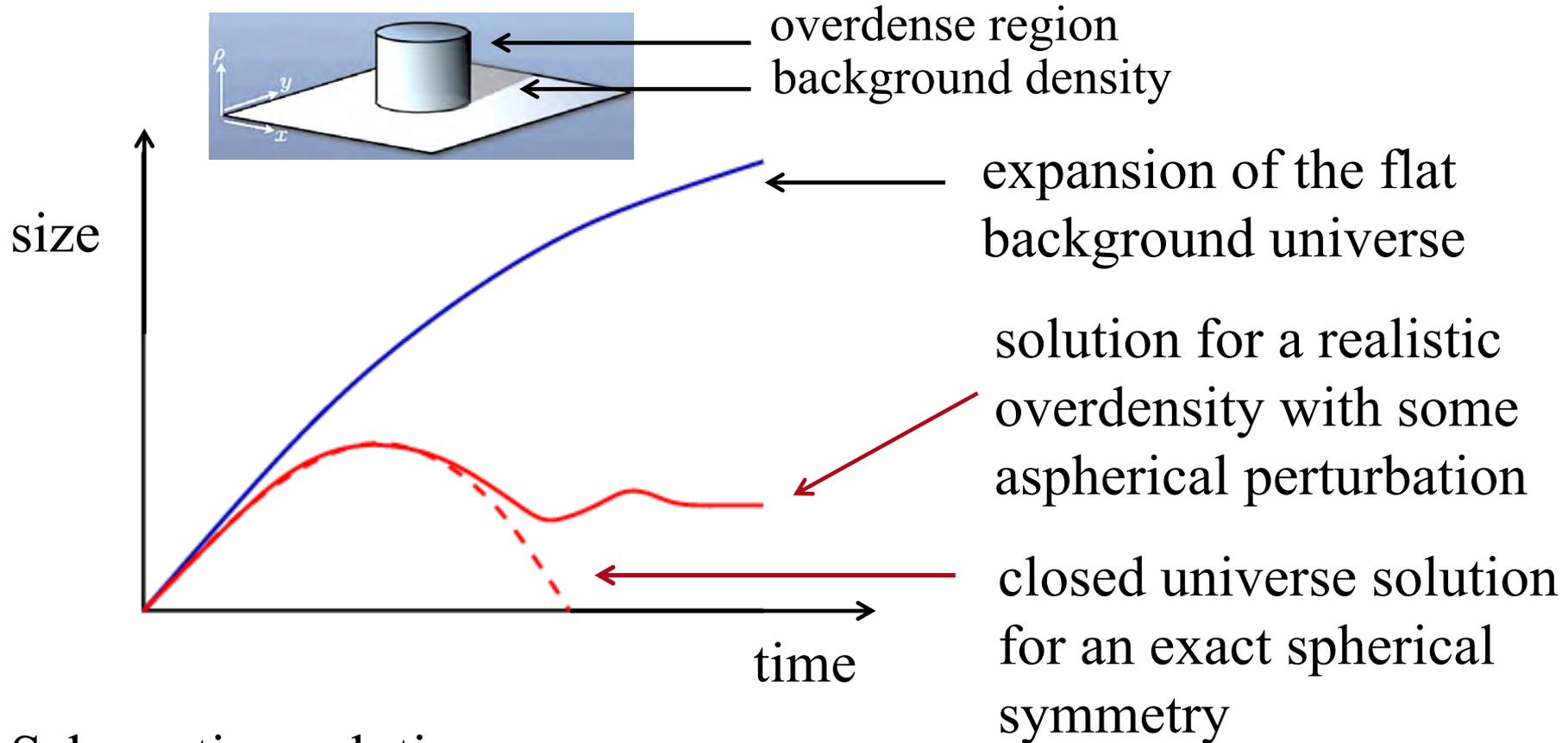
We need to start with larger fluctuations to get galaxies, etc.

We neglected the effects of *non-baryonic dark matter*. Before recombination, the radiation prevented the baryonic fluctuations of collapsing, but fluctuations in non-baryonic dark matter can start growing much earlier!

The fluctuations we see in the CMB (in the baryonic matter) are sitting on top of much stronger fluctuations in the non-baryonic matter. Once recombination occurs, the baryons can fall into the dark matter concentrations to form galaxies ...

We also neglected the effects of energy dissipation that can occur for the baryonic matter, and that would lead to higher densities

Evolution of the “Spherical Top-Hat Model”



Schematic evolution:

- Density contrast grows as universe expands
- Perturbation “turns around” at $R = R_{turn}$, $t = t_{turn}$
- If exactly spherical, collapses to a point at $t = 2 t_{turn}$
- Realistically, bounces and **virializes** at radius $R = R_{virial}$

Evolution of the Spherical Top-Hat Model

We can use the virial theorem to derive the final radius of the collapsed perturbation. Let perturbation have mass M , kinetic energy $\langle KE \rangle$, and gravitational potential energy $\langle PE \rangle$:

Virial theorem: $\langle PE \rangle + 2 \langle KE \rangle = 0$ (in equilibrium)

Energy conservation: $\langle PE \rangle + \langle KE \rangle = \text{const.}$

$$-\frac{GM^2}{R_{\text{turn}}} = -\frac{GM^2}{R_{\text{virial}}} + \langle KE \rangle = -\frac{GM^2}{2R_{\text{virial}}}$$

Potential energy at turnaround \nearrow

Conclude: $R_{\text{virial}} = \frac{1}{2} R_{\text{turn}}$

\nwarrow Potential energy when virialized

Note: this assumes *no* energy dissipation!

Evolution of the Spherical Top-Hat Model

We can apply the solution for a closed universe to calculate the final overdensity:

Friedmann equation: $\dot{a}^2 = \frac{A^2}{a} - kc^2$

where $A^2 = \left(\frac{8\pi G}{3}\right)\rho_0 a_0^3$

Solutions:

$$a = \left(\frac{3A}{2}\right)^{2/3} t^{2/3}$$

$k = 0$ solution derived previously

$$a = \frac{1}{2} \frac{A^2}{c^2} (1 - \cos \Psi)$$

$$t = \frac{1}{2} \frac{A^2}{c^3} (\Psi - \sin \Psi)$$

} Parametric solution for a closed, $k = 1$ universe (see the derivation elsewhere, e.g., in Ryden's book), but you don't need to know this

Evolution of the Spherical Top-Hat Model

Now we can calculate the turnaround time for the collapsing sphere by finding when the size of the small closed universe has a maximum:

$$\frac{da}{d\Psi} = \frac{1}{2} \frac{A^2}{c^2} \sin \Psi = 0 \quad \rightarrow \quad \Psi = 0, \pi, \dots$$

At turnaround, $\Psi = \pi$,
which corresponds to time

$$t_{turn} = \frac{1}{2} \frac{A^2}{c^3} \pi$$

The scale factors of the
perturbation and of the
background universe are:

$$a_{sphere} = \frac{1}{2} \frac{A^2}{c^2} (1 - \cos \pi) = \frac{A^2}{c^2}$$

$$a_{background} = \left(\frac{3A}{2} \right)^{2/3} t_{turn}^{2/3} = \left(\frac{3\pi}{4} \right)^{2/3} \frac{A^2}{c^2}$$

Evolution of the Spherical Top-Hat Model

The density contrast at turnaround is therefore:

$$\frac{\rho_{sphere}}{\rho_{background}} = \left(\frac{a_{sphere}}{a_{background}} \right)^{-3} = \frac{9\pi^2}{16}$$

- At the time when the collapsing sphere virialized, at $t = 2 t_{turn}$:
- Its density has *increased* by a factor of 8 (since $R_{turn} = 2 R_{vir}$)
 - Background density has *decreased* by a factor of $(2^{2/3})^3 = 4$

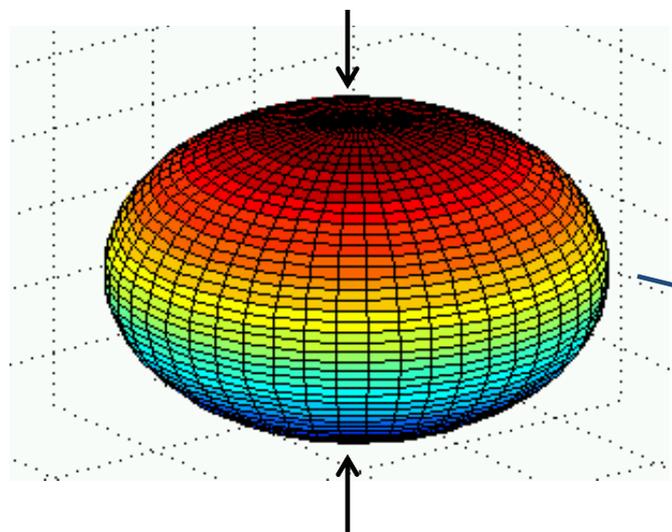
A collapsing object virializes when its density is greater than the mean density of the universe by a factor of $18 \pi^2 \sim 180$ (and this is about right for the large-scale structure today)

First objects to form are small and dense (since they form when the universe is denser). These later merge to form larger structures: **“bottom-up” structure formation**

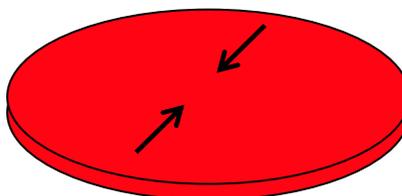
Non-Spherical Collapse

Real perturbations will not be spherical. Consider a collapse of an ellipsoidal overdensity:

The expansion turns into collapse along the shortest axis first ...



Then along the intermediate axis



Then along the longest axis



- Perturbation first forms a “**pancake**”
- Then forms **filaments**
- Then forms **clusters**

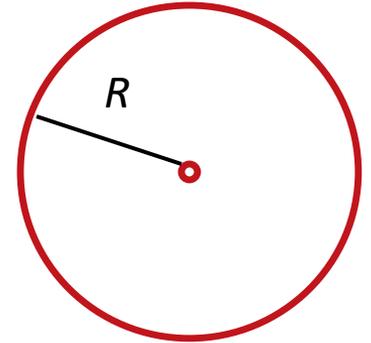
This kinds of structures are seen both in numerical simulations of structure formation and in galaxy redshift surveys

How Long Does It Take?

Dissipationless gravitational collapse timescale is on the order of the **free-fall time**, t_{ff} :

The outermost shell has acceleration $g = GM/R^2$
It falls to the center in:

$$t_{ff} = (2R/g)^{1/2} = (2R^3/GM)^{1/2} \approx (2/G\rho)^{1/2}$$



Thus, low density lumps collapse more slowly than high density ones. More massive structures are generally less dense, take longer to collapse. For example:

$$\text{For a galaxy: } t_{ff} \sim 240 \text{ Myr } (R/50\text{kpc})^{3/2} (M/10^{12}M_{\odot})^{-1/2}$$

$$\text{For a cluster: } t_{ff} \sim 7.5 \text{ Gyr } (R/5\text{Mpc})^{3/2} (M/10^{15}M_{\odot})^{-1/2}$$

So, we expect that galaxies collapsed early (at high redshifts), and that clusters are still forming now. This is as observed!

Quantifying the Density Field

- Consider the overall fluctuating density field as a superposition of waves with different wavelengths, phases, and amplitudes
- Then we can take a Fourier transform and measure the **Power** on different scales, expressed either as wavelengths l or frequencies or wave numbers $k = 1 / l$
- *Density fluctuations field* :
$$\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}}$$
- *Fourier Transform of density field* :
$$\delta_k = \sum \delta e^{-i\mathbf{k} \cdot \mathbf{r}}$$
- *Its Power Spectrum* :
$$P(k) = \left\langle |\delta_k|^2 \right\rangle$$

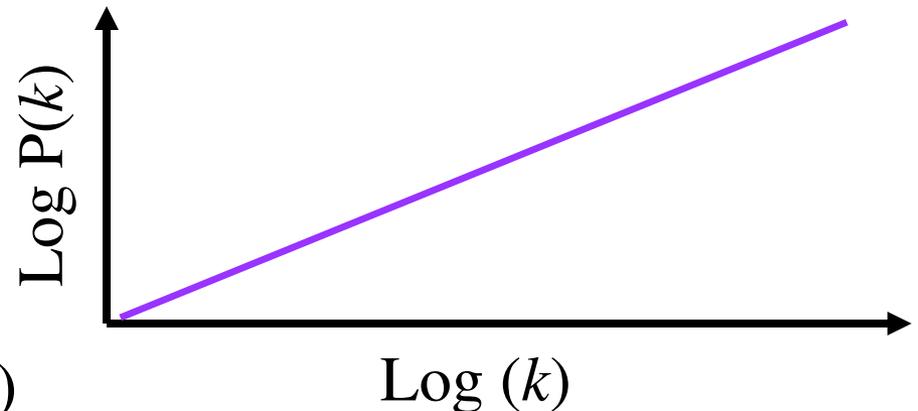
It measures the power of fluctuations on a given scale k
Sometimes one uses the “mass scale”; $M \sim k^{-3(3+n)/4}$

Density Fluctuation Spectrum

- A common assumption is that the fluctuations have the same amplitude $\delta \sim 10^{-4}$ when they enter the horizon
- This gives a *scale-free* or *Harrison-Zeldovich* spectrum
- Such spectrum is also predicted in the inflationary scenario
- More generally, it can be a power-law:

$$P(k) = \langle |\delta_k|^2 \rangle \propto k^n$$

(for Harrison-Zeldovich, $n = 1$)



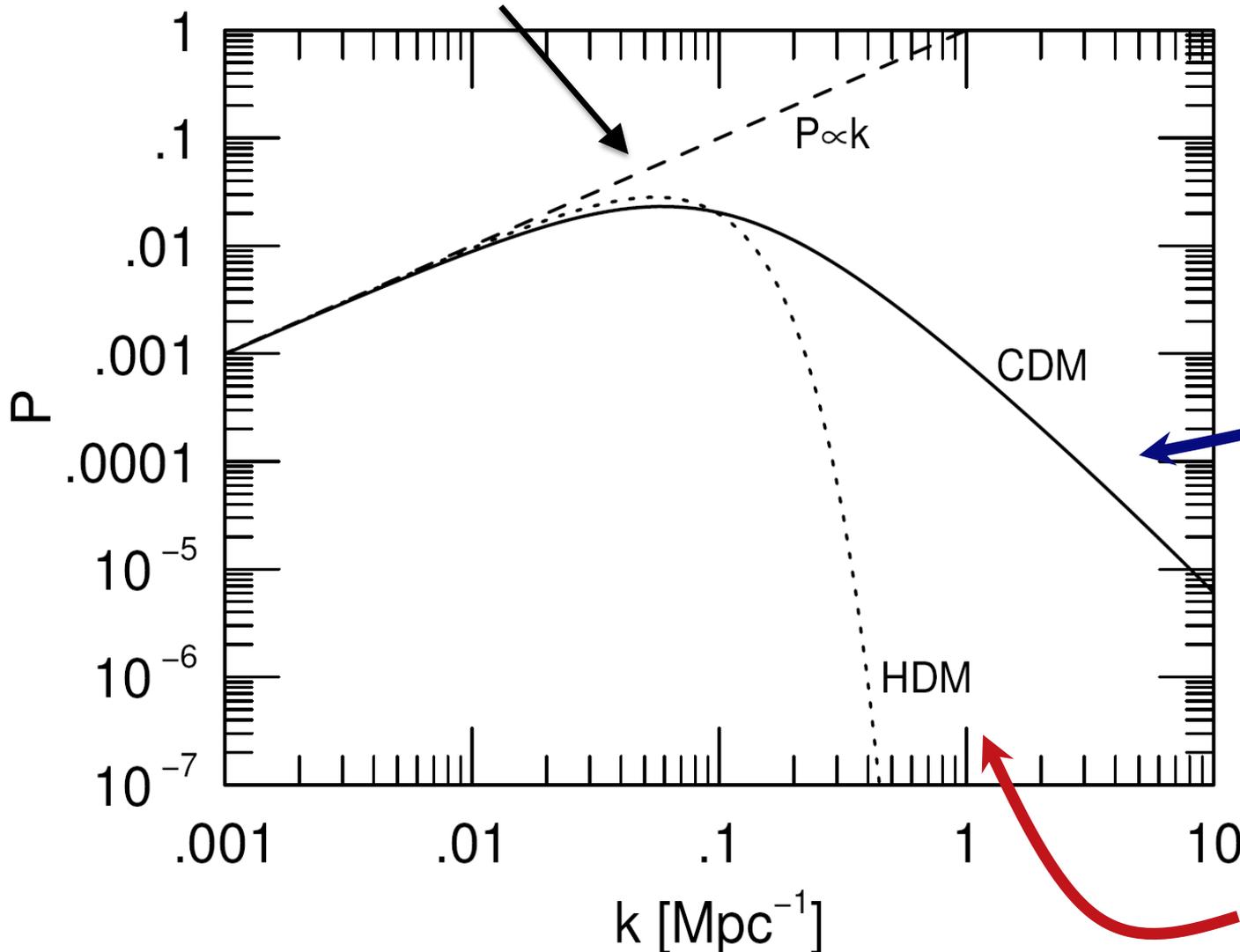
- The *fluctuations grow under self-gravity*, so the power increases (if all scales grow equally, the spectrum just shifts up in *log*), but *they can be also erased or damped* through various processes - the shape of the spectrum changes

Dark Matter and Damping of Fluctuations

- *Different types of dark matter form structure differently*
- Baryonic dark matter is coupled to radiation, so it does not help in forming structure prior to the recombination
- Fluctuations can be erased or damped by *sound waves* (this is also called the Meszaros effect). This is important for slowly moving DM particles, i.e., cold dark matter (CDM)
- They can be erased by *free streaming* of relativistic particles, i.e., hot dark matter (HDM); diffusion of photons, which then “drag along” the baryons in the radiation-dominated era, does the same thing (this is also called the Silk damping)
- Thus *HDM vs. CDM make very different predictions* for the evolution of structure in the universe!
- In any case, *the smaller fluctuations are always erased first*

Damping of Fluctuations

The primordial (H-Z) spectrum



Sound waves diminish the strength of small scale fluct' s for the CDM case

Relativistic streaming and photon diffusion erase them completely for the HDM case

Structure Formation in the CDM Scenario

- CDM density fluctuations exist on all scales, large and small
- Small lumps collapse first, big things collapse later. Larger overdensities will incorporate smaller ones as they collapse, via **merging**
- Structure forms early with CDM, and it forms “**bottom-up**”
Galaxies form early, before clusters, and clusters are still forming now
- This picture is known as *hierarchical structure formation*
- This closely matches what we observe
- It also produces the right kind of CMB fluctuations
- Thus, while we know that massive neutrinos (which would constitute HDM) do exist, *most of the dark matter must be cold!*

The Growth of Fluctuations

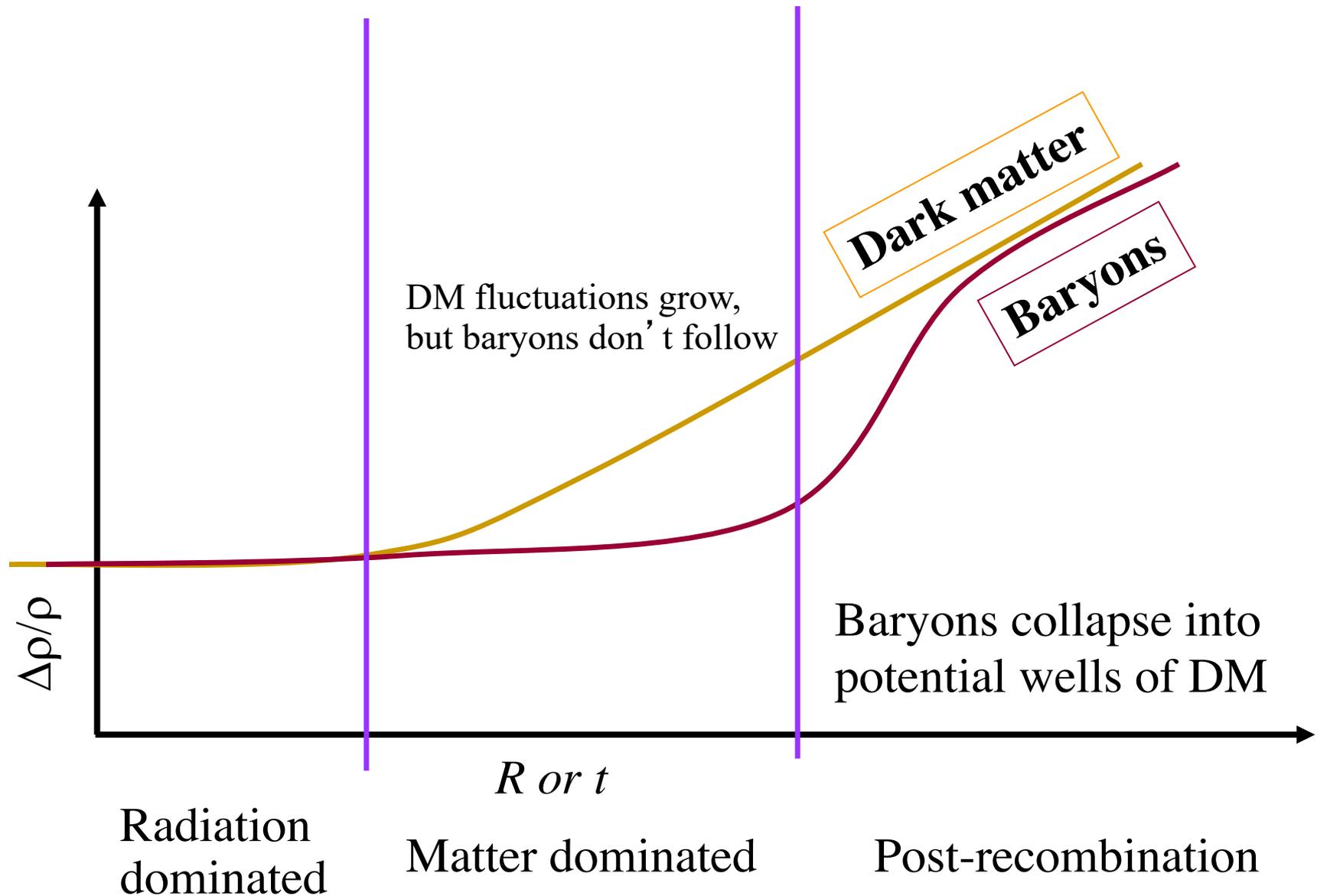
- Prior to matter-radiation equality perturbations are prevented from growing due to *radiation pressure*
- *Pressure opposes gravity* effectively for all wavelengths below the Jeans Length:

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

where c_s is the speed of sound, and the equation of state is: $c_s^2 = \frac{\partial P}{\partial \rho}$

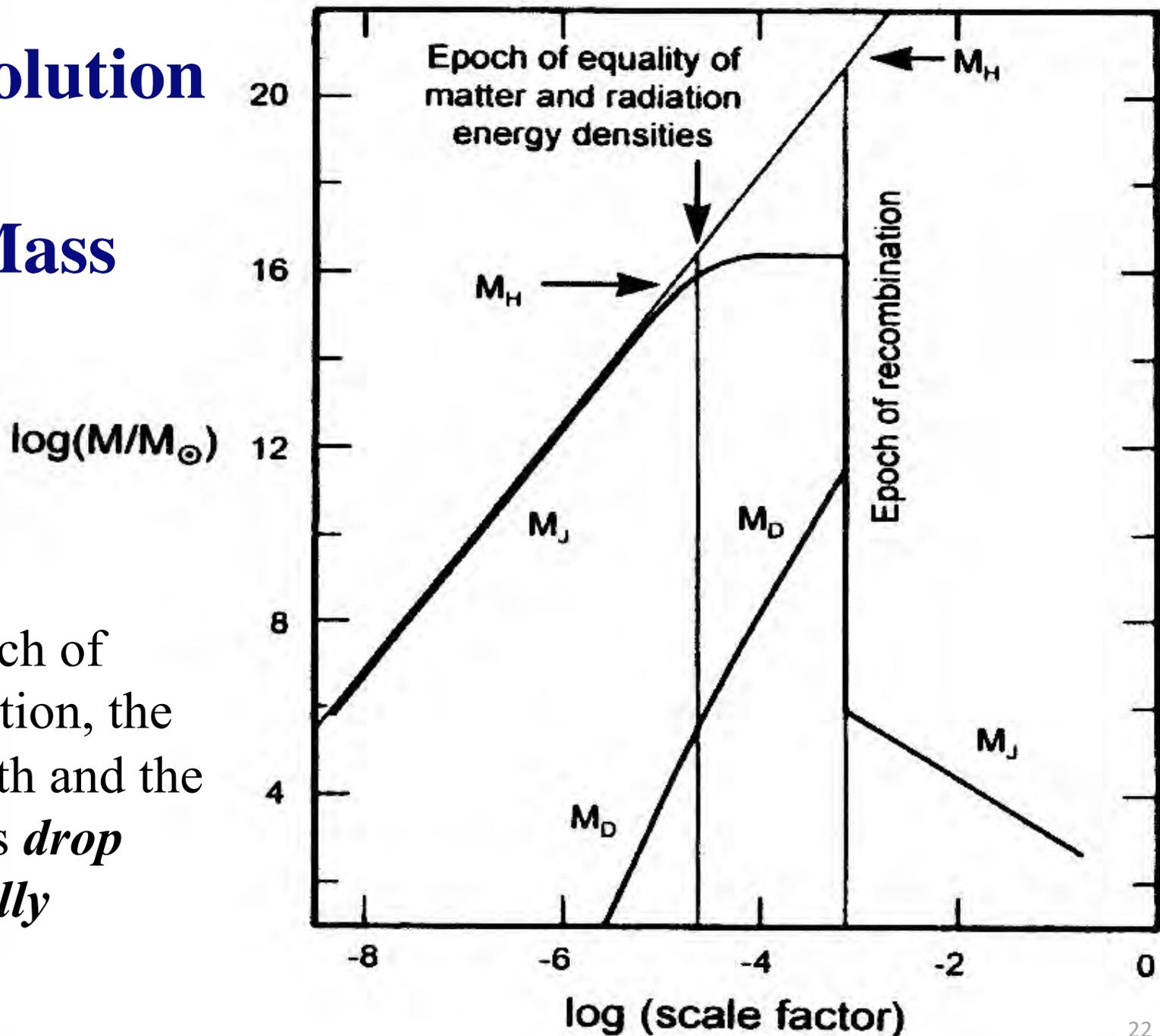
- *Jeans Length* is the scale at which sound waves can cross an object in about the time for gravitational collapse
- *In a radiation dominated universe, Jeans Length is close to the horizon size*
- At matter-radiation equality the *sound speed starts to drop*, and fluctuations can *grow*

The Evolution of Fluctuations



The Evolution of the Jeans Mass

At the epoch of recombination, the Jeans length and the Jeans mass *drop dramatically*

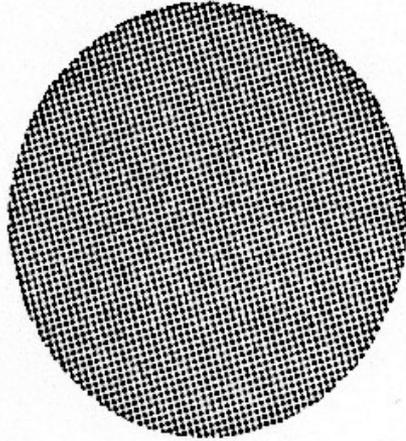


Post-Recombination Universe

- Fluctuations continue to grow, and soon enter the *non-linear regime*, at progressively ever larger scales
- This evolution can then only be followed *numerically*
- Once the gas infalling into the DM potential wells (dark halos) is compressed enough, *dissipative effects* begin to play a significant role (shocks, star formation, feedback). These are difficult to simulate numerically!
- *Energy dissipation accelerates collapse and leads to higher densities which cannot be achieved by the dissipationless collapse* (a factor of 8...). This is often called “*cooling*”
- A good mechanism is inverse Compton scattering of CMB photons on hot electrons; this process is effective only at $z < 100$ or so, since the CMB is too hot at higher z 's

Two Modes of Contraction

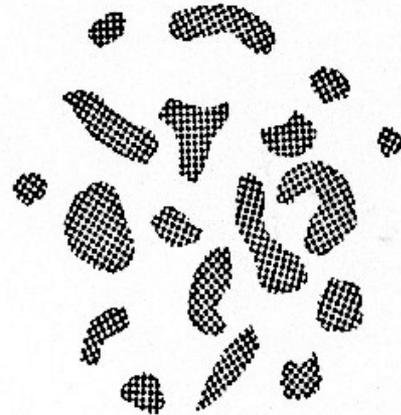
$$t_{\text{cool}} > t_{\text{free fall}}$$



quasi-static contraction
at $T \approx T_{\text{virial}}$

or

$$t_{\text{cool}} < t_{\text{free fall}}$$



cooling, free fall collapse
and fragmentation

Fluctuations which cool faster than they fall together under gravity alone are subject to Jeans instability and fragmentation

That is what separates galaxies (smaller, cooling important) from the large-scale structure (larger, cooling not important)

Cooling and Structure Formation

Define the *cooling time* as:

$$t_{cool} = E / |dE/dt| = 3 n k T / [n^2 \Lambda(T)]$$

where n is the particle number density, k is the Boltzmann constant, and $\Lambda(T)$ is a cooling rate function (*not* cosmological constant!) which depends on the chemical composition

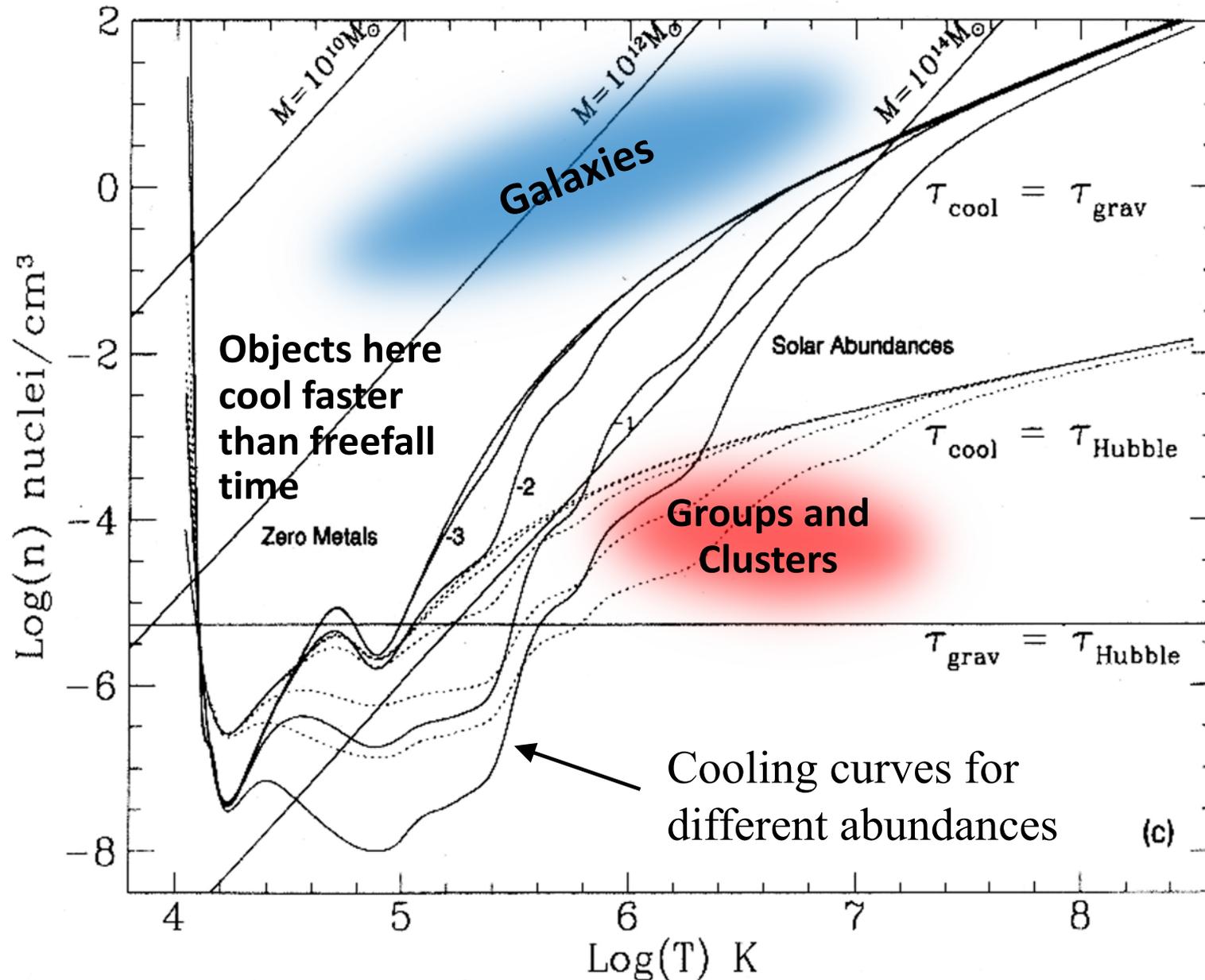
The *key question* is the relation between the free-fall time t_{ff} , cooling time, t_{cool} , and the Hubble time, t_H :

If $t_{cool} < t_{ff}$ the cooling dominates the contraction, objects collapse faster, to smaller radii and higher densities; and v.v.

If $t_{cool} > t_H < t_{ff}$ objects cannot form

The position of objects in the “cooling diagram” plane $\{T, n\}$ thus determines their fate! Note also that $M_{Jeans} = f(T, n)$

The Cooling Diagram

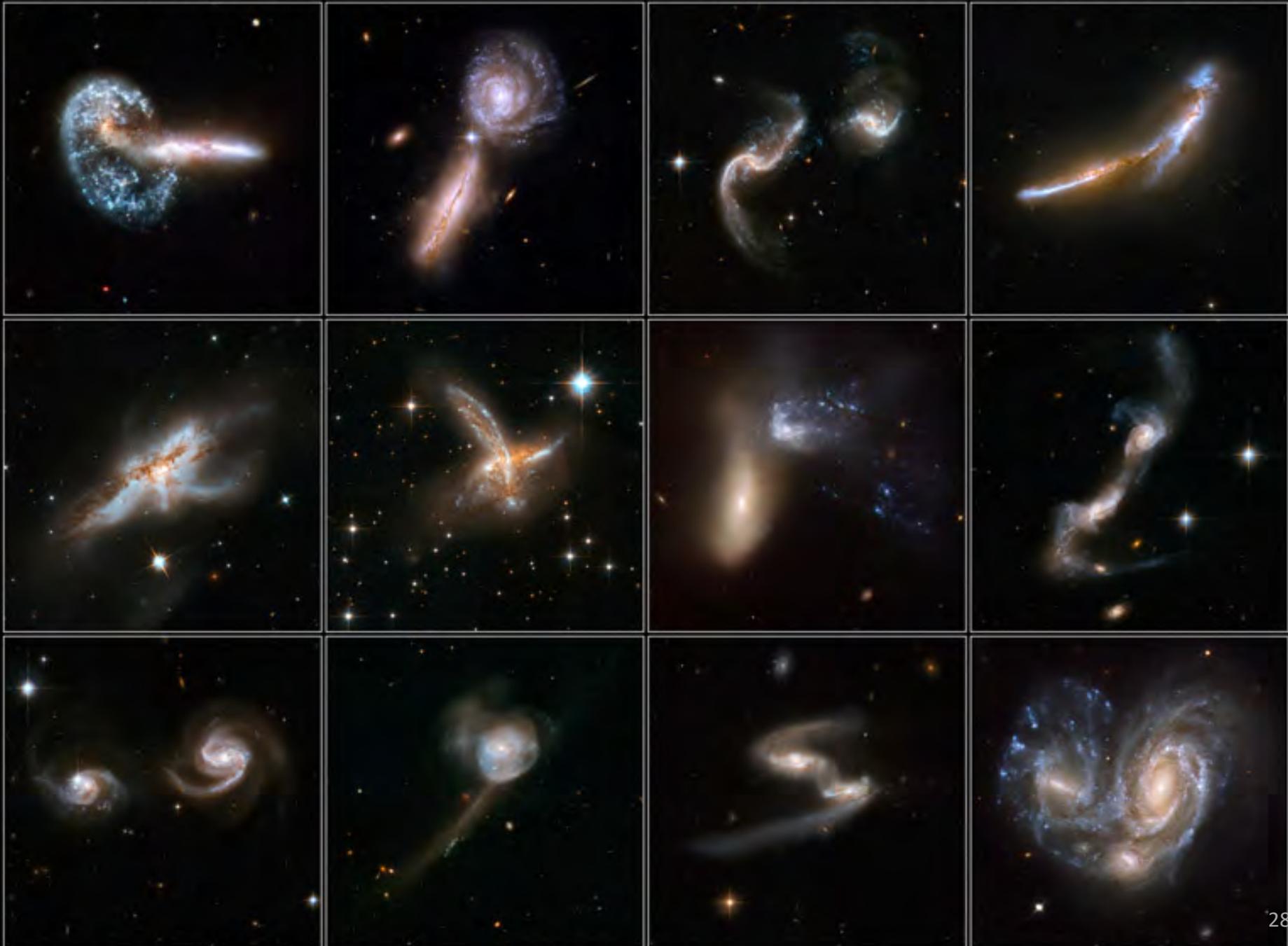


Numerical Simulations in Cosmology

- Many problems in astrophysics generally *do not allow analytic solution*. A full 3D numerical treatment is required
- For the Newtonian gravity, there are no exact analytical solutions beyond the 2-body (Kepler) problem
- Numerical simulations thus play a key role in cosmology
- In cosmology know both the governing equations (Newtonian gravity, cosmological expansion) *and* can assume the initial conditions (primordial density field)
- Pure gravity simulations are dissipationless; adding gas and radiation makes the problem much harder, due to dissipation, hydrodynamical effects, radiation transfer...
- The output of simulations can be compared to observations

Interacting Galaxies

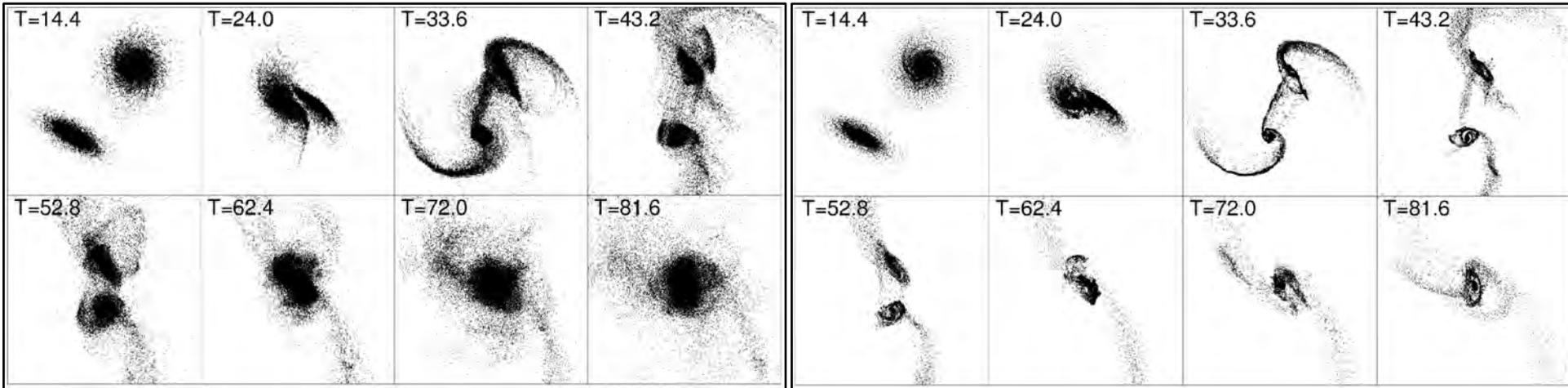
Hubble Space Telescope • ACS/WFC • WFPC2



Numerical Simulations of Galaxy Mergers

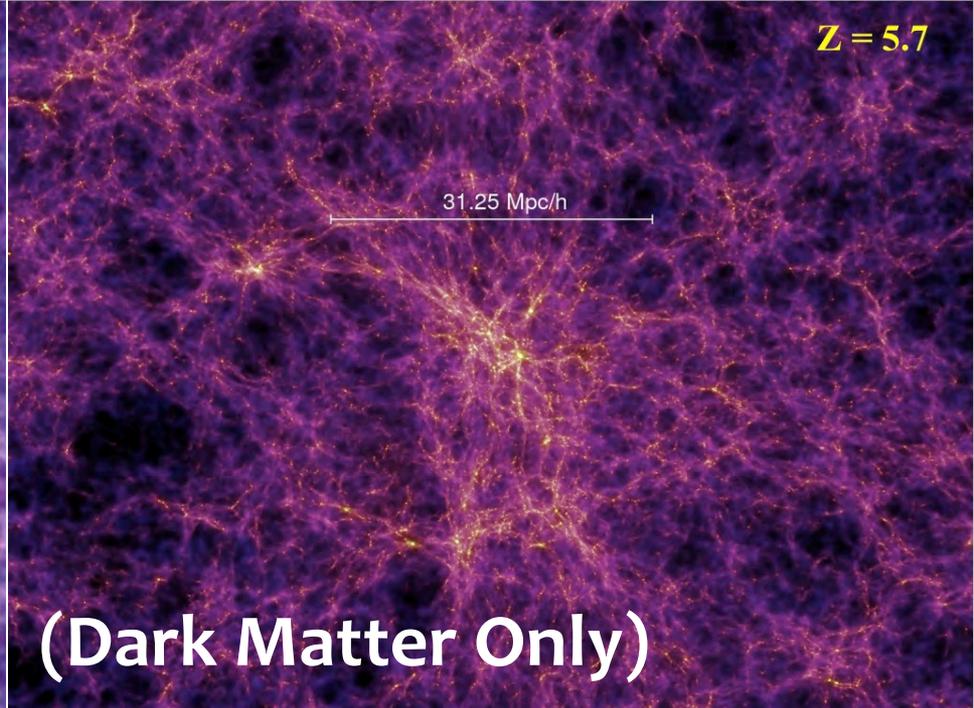
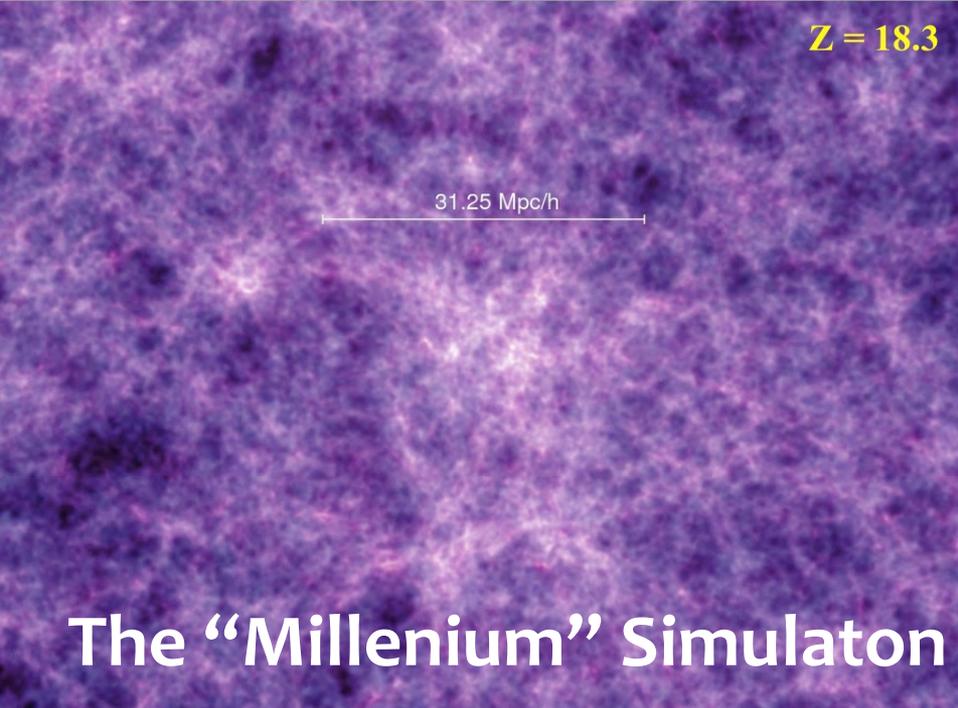
Stars and DM (Dissipationless)

Gas (Dissipative)

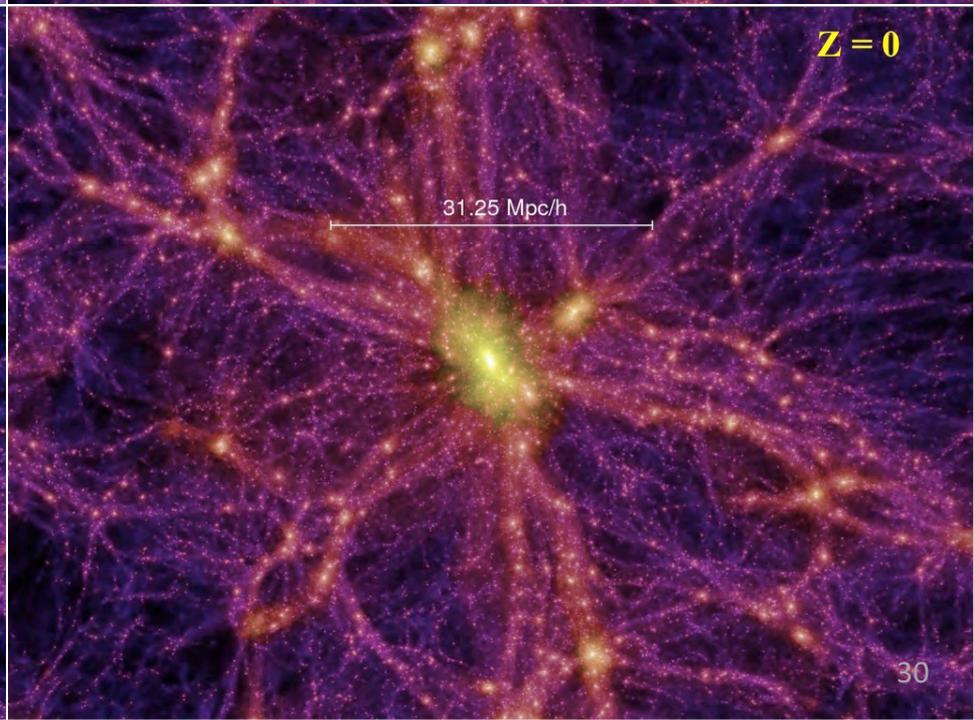
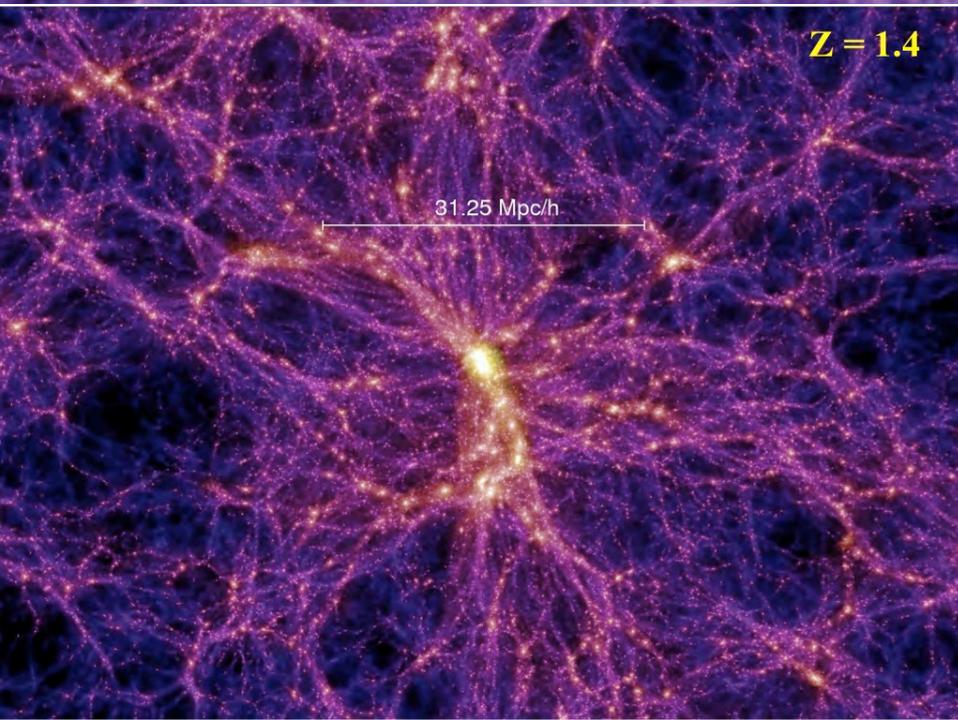


Simulations by Barnes & Hernquist

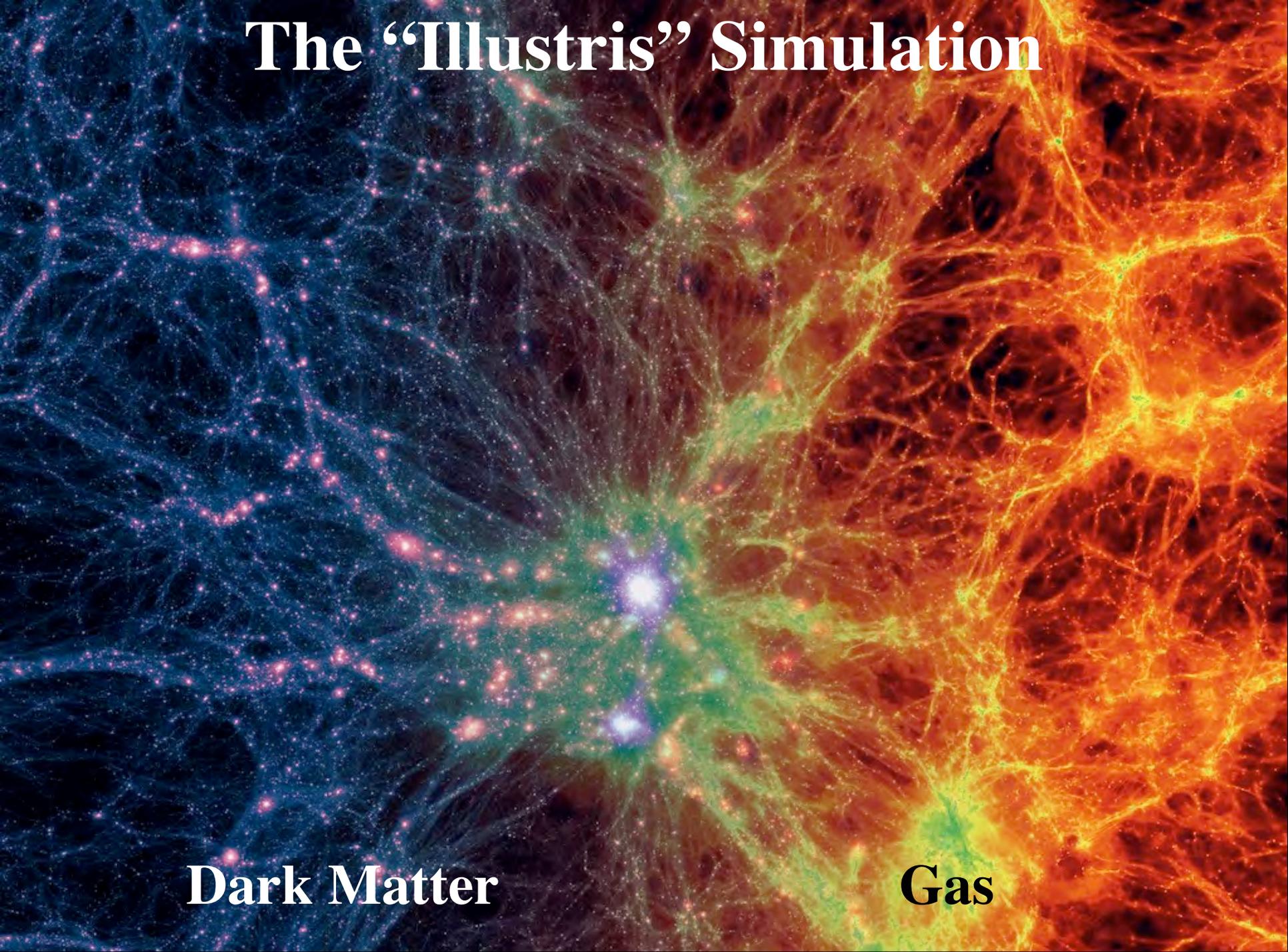
- Merge two nearly equal mass disk galaxies (or a lot of smaller galaxies); in a few dynamical times, *the remnant looks just like an elliptical galaxy*
- In the same merger, *gas quickly loses energy (since it is dissipative), and sinks towards the center*, where it can fuel a starburst, or an AGN if a massive black hole is present



The “Millenium” Simulaton (Dark Matter Only)



The “Illustris” Simulation

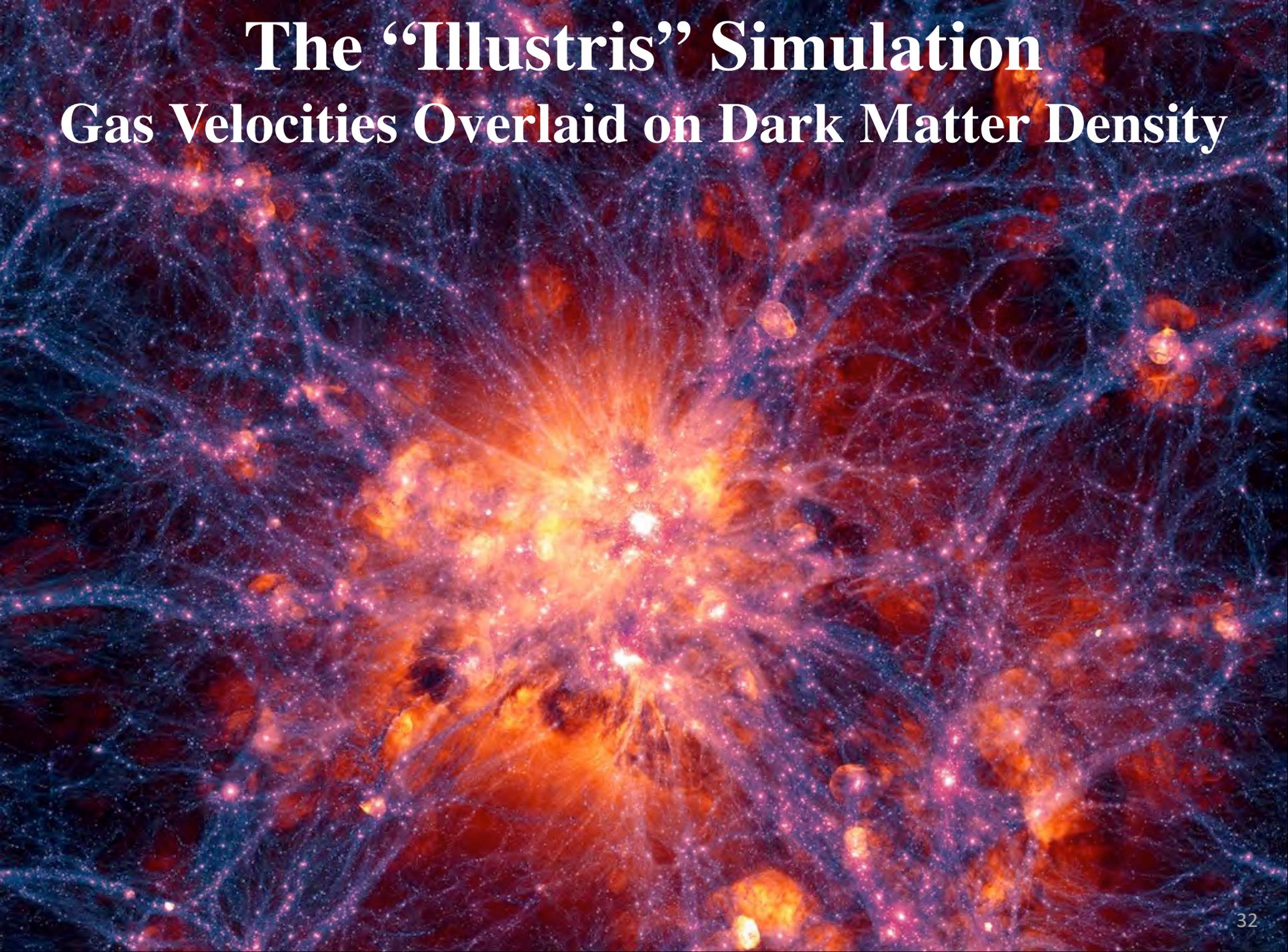


Dark Matter

Gas

The “Illustris” Simulation

Gas Velocities Overlaid on Dark Matter Density



Structure Formation Theory: A Summary of the Key Ideas (1)

- Structure grows from *initial density perturbations* in the early universe, via gravitational infall and hierarchical merging
- *Initial conditions* described by the primordial density (Fourier power) spectrum $P(k)$, often assumed to be a power-law, e.g., $P(k) \sim k^n$, $n = 1$ is called a Harrison-Zeldovich spectrum
- *Dark matter (DM) plays a key role*: fluctuations can grow prior to the recombination; after the recombination, baryons fall in the potential wells of DM fluct' s (proto-halos)
- *Damping mechanisms erase small-scale fluctuations*; how much, depends on the nature of the DM: HDM erases too much of the high-freq. power, *CDM fits all the data*
- Collapse occurs as blobs \rightarrow sheets \rightarrow filaments \rightarrow clusters

Structure Formation Theory: A Summary of the Key Ideas (2)

- *Pure (dissipationless) gravitational infall leads to overdensities of ~ 200* when the virialization is complete
- *Free-fall time scales* imply galaxy formation early on ($t_{ff} \sim$ a few $\times 10^8$ yrs), clusters are still forming ($t_{ff} \sim$ a few $\times 10^9$ yrs)
- *Characteristic mass for gravitational instability is the Jeans mass*; it grows before the recombination, then drops precipitously, from $\sim 10^{16} M_{\odot}$, to $\sim 10^5 M_{\odot}$
- *Cooling is a key concept:*
 - *Galaxies cool faster than the free-fall time*: formation dominated by the *dissipative* processes, achieve *high densities*
 - *Groups and clusters cool too slowly*: formation dominated by *self gravity*, *lower densities* achieved
 - The cooling curve separates them