Ay 21 – Winter 2008 – Final Exam

Distributed on March 13, due by 5 pm *sharp*, on Wednesday, March 19

The Rules: Closed book, closed notes, no web access, *closed everything* (except your minds) … but you can use tables of physical and astronomical constants or units (attached). You can use a pocket calculator, but not if it has display of formulas and such. You *cannot* discuss the problems with anyone until after everyone turns in their exams. The Honor System applies.

You have a maximum of 5 hours (it should take less) from the moment you start until the moment you finish. Please mark your exam with the start and stop times. You have to turn it in *in person*, either to the Prof or the TA, or the secretaries in rm. 211 Robinson.

There are 7 problems, with a maximum total score of 150 points. The exam counts towards 30% of your grade.

Please write legibly – it is in your own best interest. Good luck!

Problem 1 [27 points, 3 for each item]

Define or explain briefly (in a few sentences at most):

a. The main differences between dwarf ellipticals and regular ellipticals; draw diagrams if needed.
b. The Schechter luminosity function; draw a diagram; what are the typical values of its parameters?
c. The reionization era and why is it important, and at what redshifts did it happen?
d. Blazars (BL Lac's) and how they differ from other types of AGN; why are they so interesting?
e. Synchrotron emission and its origin; sketch its spectrum, label the axes.
f. Kerr black hole vs. Schwarzschild black hole; which one can produce a more luminous AGN, for the same accretion rate?
g. Unified models of AGN; do draw a picture.
h. Types of absorbers in quasar spectra, and how do they relate to galaxies?
i. The cosmic web, and how do we study it?

Problem 2 [20 points]

a. What is the Fundamental Plane of elliptical galaxies? [5 points]
b. Use the Virial Theorem to derive it (or an analogous relation), and state clearly your assumptions. [5 points]
c. What physical conditions must be satisfied in order to reproduce the observed slope and the small scatter? [5 points]
d. What are its main cosmological uses? [5 points]

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Problem 3 [10 points]

a. The Andromeda galaxy (M31) is about 700 kpc away. The velocity dispersion and the rotation speed are observed to increase in the center, so that the *lower limit* on the mean velocity of stars over and above the normal velocity dispersion in the bulge of M31 is $\langle V \rangle > 400$ km/s, at projected radii < 0.5 arcsec. Estimate the lower limit to the mass of the central MBH in M31. [5 points]

b. Consider a quasar with a SMBH of mass $M_{\mathrm{bh}} \sim 10^8 M_\odot$. The velocity half-width of the broad emission lines (BEL) is 5000 km/s in the quasar restframe. Estimate the physical radius of the BEL region. Can it be resolved with the HST (resolution ~ 0.1 arcsec) if it was in the Virgo cluster, 20 Mpc away? Can it be resolved with a VLBI experiment with a resolution of 1 microarcsec, if it is 100 Mpc away? [5 points]

Problem 4 [18 points]

Consider a quasar powered by an Eddington-limited accretion to a black hole, with an efficiency of 10%. Derive the function describing the evolution of the black hole’s mass and the quasar’s luminosity, and the numerical value of the characteristic time scale entering these formulas.

Problem 5 [30 points]

The observed energy density of the cosmic far-infrared background (CIRB) is $\boldsymbol{u}_{\mathrm{CIRB}} \approx 7 \times 10^{-15}$ erg cm^{-3}. Assume that the background is generated by a population of obscured starbursts with a mean luminosity $\langle L \rangle \approx 10^{12} L_\odot$, with all energy emerging in the mid/far IR, with a mean redshift $\langle z \rangle \approx 3$, each lasting on average $\Delta t \approx 3 \times 10^7$ yr.

a. How many such starbursts should there be in the entire observable universe, in order to account for the CIRB?

b. Estimate the comoving number density of their progeny at $z \sim 0$, and compare it with the estimated comoving number density of normal galaxies today. (Hint: estimate the total comoving volume of the observable universe first; an order-of-magnitude, reasoned estimate is OK.)

c. Assuming the yield of nuclear reactions of 7 MeV/nucleon, how much helium was generated by these starbursts? How much metals, assuming the ratio of helium to metals production by mass $\Delta Y / \Delta Z = 5$? If each of these starbursts was in a galaxy with a baryonic mass $M \approx 10^{11} M_\odot$, what is the mean metallicity of the resulting stars?

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Problem 6 [20 points]

The observed brightness of the cosmic X-ray background (CXRB) is $\nu I_\nu \approx 3\times10^{-10}$ W m^{-2} sr^{-1}.

a. Compute the corresponding volume energy density. Compare with the energy density of the CMB, assuming $T_{CMB} = 2.735°K$. [4 points]

b. It is estimated that on average there is a SMBH with $M_{bh} \sim 10^7 M_\odot$ per average ($L \sim L_*$) galaxy. Assuming that the efficiency of accretion in converting the rest mass into energy was ~ 10% (i.e., 90% ends in the SMBH, 10% is radiated away), and that the mean redshift of emission was $\langle z \rangle \sim 3$, compute the energy density today, generated by the making of these SMBHs. Compare it with the numbers for the CXRB and CMBR computed in (a). (Note: you will need to estimate the comoving density of L_* galaxies today.) [16 points]

Problem 7 [25 points]

Consider a quasar with $L_{bol} \approx 5\times10^{12} L_\odot$, at $z = 2$. Assume an Einstein – de Sitter universe with $H_0 = 50$ km/s/Mpc.

a. Compute the luminosity distance to the quasar. [5 points] (Note: if you don't know how to evaluate or derive this distance quickly, make a reasoned estimate for only 2 points credit, and move on.)

b. Compute its absolute and apparent bolometric magnitudes, assuming $M_{bol_\odot} = 4.8$ mag. (Note: this is the absolute magnitude, not the mass!) [5 points]

c. Assuming that 30% of the entire luminosity of this quasar is emitted in X-rays, compute the observed X-ray flux (in cgs or SI units). [5 points]

d. How many such quasars would it take to generate *all* of the observed CXRB? (See the brightness in the previous problem). Compute their projected surface density on the sky (number per deg^2), and compare with the numbers derived in problem 6. [10 points]

Physical Constants and Astronomical Data

New! Try my Physical Calculator. It is a JavaScript calculator with all of the constants below programmed into it.

Physical Constants

(converted to CGS units from the NIST Constant Index)

Name	Symbol	Number	Exp	CGS Units	Relative Error (ppm)
speed of light in a vacuum	c	2.99792458	10	$cm\ s^{-1}$	exact
Planck constant	h	6.6260755(40)	-27	erg s	0.60
	hbar	1.05457266(63)	-27	erg s	0.60
Gravitational constant	G	6.67259(85)	-8	$cm^3\ g^{-1}\ s^{-2}$	128
Electron charge	e	4.8032068(14)	-10	esu	0.30
Mass of electron	m_e	9.1093897(54)	-28	g	0.59
Mass of proton	m_p	1.6726231(10)	-24	g	0.59
Mass of neutron	m_n	1.6749286(10)	-24	g	0.59
Mass of hydrogen	m_H	1.6733	-24	g	--
Atomic mass unit	amu	1.6605402(10)	-24	g	0.59
Avagadro's number	N_A	6.0221367(36)	23		0.59
Boltzmann constant	k	1.380658(12)	-16	$erg\ k^{-1}$	8.5
Electron volt	eV	1.6021772(50)	-12	erg	~0.60
Radiation density constant	a	7.5646	-15	$erg\ cm^{-3}\ K^{-4}$	--
Stefan-Boltzmann constant	\sigma	5.67051(19)	-5	$erg\ cm^{-2}\ K^{-4}\ s^{-1}$	34
Fine structure constant	\alpha	7.29735308(33)	-3		0.045

Rydberg constant	$R_{\inf}$	2.1798741(13)	-11 erg	0.60

Note: a "--" in the error column means that I have not found a good source for that constant, so the value quoted is just an approximation

Astronomical Units/Data

NAME	SYMBOL	NUMBER	EXP	CGS UNITS
Astronomical unit	AU	1.496	13	cm
Parsec	pc	3.086	18	cm
Light year	ly	9.463	17	cm
Solar mass	$M_{\odot}$	1.99	33	g
Solar radius	$R_{\odot}$	6.96	10	cm
Solar luminosity	$L_{\odot}$	3.9	33	erg s^{-1}
Solar Temperature	$T_{\odot}$	5.780	3	K

NAME	MASS (g)		RADIUS (cm)		PERIOD (yr)		SEMI-MAJOR (AU)		ECCENTRICITY
Mercury	3.303	26	2.439	8	2.4085	-1	3.87096	-1	0.205622
Venus	4.870	27	6.050	8	6.1521	-1	7.23342	-1	0.006783
Earth	5.976	27	6.378	8	1.00004	0	9.99987	-1	0.016684
Mars	6.418	26	3.397	8	1.88089	0	1.523705	0	0.093404
Jupiter	1.899	30	7.140	9	1.18622	1	5.204529	0	0.047826
Saturn	5.686	29	6.000	9	2.94577	1	9.575133	0	0.052754
Uranus	8.66	28	2.615	9	8.40139	1	1.930375	1	0.050363
Neptune	1.030	29	2.43	9	1.64793	2	3.020652	1	0.004014
Pluto	1.	25	1.2	8	2.47686	2	3.991136	1	0.256695

A Few Conversion Factors

CGS	-->	SI	Multiply by
centimeter (cm)		meter (m)	10^{-2}
gram (g)		kilogram (kg)	10^{-3}
erg		Joule (J)	10^{-7}
dyne (dyn)		newton (N)	10^{-5}