Ay 21 – Winter 2008 – Midterm Exam

Distributed on Feb. 8, due by 5:30 pm on Wednesday, Feb. 13

The Rules: Closed book, closed notes, no web access, *closed everything* (except your minds) … but you can use tables of physical and astronomical constants or units. You can use a pocket calculator, but not if it has display of formulas and such. You *cannot* discuss the problems with anyone until after everyone turns in their exams.

You have a maximum of 5 hours (it should take less) from the moment you start until the moment you finish. Please mark your exam with the start and stop times. You have to turn it in *in person*, either to the Prof or the TA, or the secretaries in rm. 211 Robinson.

Please write legibly – it is in your own best interest. Good luck!

Problem 1 [32 points, 4 for each item]

Define or explain briefly (in a few sentences at most):

a. The distinction between commoving and proper coordinates?
b. You and an alien astronomer in a galaxy which for you is at $z = 1$ observe a quasar which for you is at $z = 2$ (along the same line of sight); what is the redshift of the quasar from the viewpoint of your alien colleague?
c. List at least 3 distinct methods to measure the H_0, and their principal advantages and disadvantages.
d. What is the K-correction, and what does it depend on?
e. WIMPs and MACHOs?
f. What is the Einstein radius, and the difference between strong and weak lensing?
g. Cosmological constant vs. quintessence, and what is that parameter $\boldsymbol{w}$?
h. The recombination epoch, its redshift, approx. age of the universe at the time?

Problem 2 [45 points]

Assume a spatially flat universe, with $\Omega_{total} = \Omega_{matter} = 1$, and that $H_0 = 70$ km/s/Mpc.

a. Derive the formula for the angular diameter distance $D_A(z)$ in this cosmological model, and for the age of the universe $t(z)$. [10 points]
b. Compute the proper size in Mpc of the particle horizon diameter at the time of recombination, at $z = 1100$ (hint: from how far would light have traveled at that time in the history of the universe?) [5 Points]
c. Compute the corresponding angular size as observed now, if $\Omega_{total} = \Omega_{matter} = 1$ [5 points]
d. Same as (a), but for the empty universe with $\Omega_{total} = 0$. [10 Points]
e. Same as (b), for the empty universe model. [5 Points]
f. Same as (c), for the empty universe model. [5 Points]
g. Comment on these results and the actual observed angular scale (~ 0.9°) for the first Doppler peak of the CMBR fluctuations. [5 Points]

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Problem 3 [18 points total]

The Cosmic Neutrino Background is expected to have the number density (in neutrinos per cm^3) nearly equal to (actually 9/11 of) the number density of photons in CMBR.

a. Estimate the CMBR photon number density today, assuming T_{CMBR} = 2.7° K, and from that the relict neutrino number density today [5 points].

b. Estimate the number of these relict neutrinos passing through your body every second; state your assumptions [5 points].

c. How massive would these neutrinos have to be (in eV) in order to account for all of the dark matter (state your choices of the relevant cosmological parameters) [8 points].

Problem 4 [40 points total]:

Consider a typical disk galaxy like the Milky Way, with a flat rotation curve with V_{circ} = 220 km s^{-1}, and a halo extending out to R_{max} = 100 kpc. Assume that we live in an Einstein – de Sitter universe with $\Omega_m = \Omega_0 = 1$, and $h = 0.5$.

a. Derive the formula for the free-fall time (of the collapsing object itself) as a function of the object's mass M, and the initial radius R [10 points]

b. What is the total mass of this galaxy? [5 points]

c. If it formed via dissipationless collapse, what was the free-fall time? How does it compare with the orbital period at the galaxy's edge today? (hint: what is the ratio of the radius today to the initial radius?) [5 points]

d. What are the present density and the age in this universe? [10 points]

e. Assuming that the halo is virialized today, at what redshift did it start collapsing? [10 points]

f. How old was the universe then? [5 points]

Problem 5 [15 points total]:

What would be the form of the galaxy 2-point correlation function if:

a. Galaxies were distributed uniformly in space? [5 points]

b. All galaxies were on sheets/walls? [5 points]

c. All galaxies were in filaments? [5 points]

(Maximum total score for this exam = 150 points. The exam counts towards 20% of your grade.)

Physical Constants and Astronomical Data

New! Try my Physical Calculator. It is a JavaScript calculator with all of the constants below programmed into it.

Physical Constants

(converted to CGS units from the NIST Constant Index)

Name	Symbol	Number	Exp	CGS Units	Relative Error (ppm)
speed of light in a vacuum	c	2.99792458	10	$cm\ s^{-1}$	exact
Planck constant	h	6.6260755(40)	-27	erg s	0.60
	hbar	1.05457266(63)	-27	erg s	0.60
Gravitational constant	G	6.67259(85)	-8	$cm^3\ g^{-1}\ s^{-2}$	128
Electron charge	e	4.8032068(14)	-10	esu	0.30
Mass of electron	m_e	9.1093897(54)	-28	g	0.59
Mass of proton	m_p	1.6726231(10)	-24	g	0.59
Mass of neutron	m_n	1.6749286(10)	-24	g	0.59
Mass of hydrogen	m_H	1.6733	-24	g	--
Atomic mass unit	amu	1.6605402(10)	-24	g	0.59
Avagadro's number	N_A	6.0221367(36)	23		0.59
Boltzmann constant	k	1.380658(12)	-16	$erg\ k^{-1}$	8.5
Electron volt	eV	1.6021772(50)	-12	erg	~0.60
Radiation density constant	a	7.5646	-15	$erg\ cm^{-3}\ K^{-4}$	--
Stefan-Boltzmann constant	\sigma	5.67051(19)	-5	$erg\ cm^{-2}\ K^{-4}\ s^{-1}$	34
Fine structure constant	\alpha	7.29735308(33)	-3		0.045

Rydberg constant	$R_{\inf}$	2.1798741(13)	-11	erg	0.60

Note: a "--" in the error column means that I have not found a good source for that constant, so the value quoted is just an approximation

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Astronomical Units/Data

NAME	SYMBOL	NUMBER	EXP	CGS UNITS
Astronomical unit	AU	1.496	13	cm
Parsec	pc	3.086	18	cm
Light year	ly	9.463	17	cm
Solar mass	$M_{\odot}$	1.99	33	g
Solar radius	$R_{\odot}$	6.96	10	cm
Solar luminosity	$L_{\odot}$	3.9	33	erg s^{-1}
Solar Temperature	$T_{\odot}$	5.780	3	K

NAME	MASS (g)		RADIUS (cm)		PERIOD (yr)		SEMI-MAJOR (AU)		ECCENTRICITY
Mercury	3.303	26	2.439	8	2.4085	-1	3.87096	-1	0.205622
Venus	4.870	27	6.050	8	6.1521	-1	7.23342	-1	0.006783
Earth	5.976	27	6.378	8	1.00004	0	9.99987	-1	0.016684
Mars	6.418	26	3.397	8	1.88089	0	1.523705	0	0.093404
Jupiter	1.899	30	7.140	9	1.18622	1	5.204529	0	0.047826
Saturn	5.686	29	6.000	9	2.94577	1	9.575133	0	0.052754
Uranus	8.66	28	2.615	9	8.40139	1	1.930375	1	0.050363
Neptune	1.030	29	2.43	9	1.64793	2	3.020652	1	0.004014
Pluto	1.	25	1.2	8	2.47686	2	3.991136	1	0.256695

A Few Conversion Factors

CGS	-->	SI	Multiply by
centimeter (cm)		meter (m)	10^{-2}
gram (g)		kilogram (kg)	10^{-3}
erg		Joule (J)	10^{-7}
dyne (dyn)		newton (N)	10^{-5}
