Cosmic Microwave Background
Edward Wright

From
Encyclopedia of Astronomy & Astrophysics
P. Murdin

© IOP Publishing Ltd 2006

ISBN: 0333750888

Institute of Physics Publishing
Bristol and Philadelphia

Downloaded on Tue Jan 31 17:07:39 GMT 2006 [127.0.0.1]
The cosmic microwave background (CMB) is the blackbody radiation left over from the origin of the universe. This radiation was emitted before any astronomical objects such as stars, galaxies or quasars existed and is now observed to be like the thermal emission from a black object with a temperature of 2.7 K (degrees above absolute zero).

**Blackbody Radiation** is emitted by an isothermal object that absorbs all incident radiation. Its spectrum, or amount of power at different frequencies, depends only on its temperature and is given by the Planck function. The measured CMB spectrum agrees with the spectrum of a blackbody to within 50 parts per million, and the temperature of the blackbody that best matches the sky is measured to be 2.725±0.002 K. The energy density of this radiation, which peaks at wavelengths near 1 mm, is larger than any other cosmic radiation field. Its existence was predicted in 1947 and 1948 by Gamow, Alpher and Herman as part of their model for the formation of the elements during a hot dense phase in the early history of the universe. The properties of the CMB rule out almost all cosmological models except for these hot Big Bang Theory models.

Observations of the spectrum of the CMB show that less than 60 parts per million of the energy in the CMB was generated later than 2 months after the creation of the universe and thus indicate that the universe started in a very hot and dense state. The anisotropy of the CMB—the difference in the temperature of the blackbody radiation from point to point on the sky—reveals the structure of the universe at the time of recombination, a few hundred thousand years after the big bang, when the free electrons and protons of the cosmic plasma combined into transparent hydrogen and helium gas (see also Universe: Thermal History).

**History**

After the prediction of the CMB in the Gamow, Alpher and Herman model for element formation, the difficulties this model had in creating elements heavier than lithium led to a general neglect of the prediction, even though the development of radar during WW II and the invention of the Dicke radiometer provided the tools necessary for the direct detection of the CMB. The first indirect evidence for the CMB was the rotational excitation of the interstellar cyanogen (CN) radical seen in 1939 by McKellar, but this observation was not connected to Gamow’s prediction. The ‘effective discovery’ of the CMB was made by Penzias and Wilson in 1965, as they systematically tried to identify all the sources of noise in a very sensitive antenna used by Bell Labs for early communication satellite experiments. In every direction of sky they saw a constant level of excess noise, and neither instrumental noise nor atmospheric emission could account for this emission. Ironically, a group led by Dicke himself was building a radiometer to look for the CMB when they heard of the Penzias and Wilson discovery. Many groups made measurements of the intensity of the CMB at different wavelengths and quickly showed that the spectrum was close to a blackbody. However, these ground-based measurements of the CMB spectrum still allowed for possible deviations of 10% or more from a blackbody spectrum at millimeter wavelengths.

Searches for the anisotropy of the CMB led to a tentative detection of a dipole pattern by Conklin in 1969 which was confirmed by Henry in 1971, Corey and Wilkinson in 1976 and Smoot et al in 1977. A dipole pattern has two poles, with one side of the sky hotter than average, peaking at a hot ‘pole’, and the other side cooler than average, giving a cold ‘pole’. This dipole pattern is a measure of the motion of our solar system at 370 km s⁻¹ relative to the observable universe. After the dipole pattern was subtracted out of the observed temperature map, any anisotropy was less than the sensitivity of these early experiments, which was a few 100 parts per million.

Since the formation of clusters of galaxies by gravitational collapse requires the existence of density contrasts of 1 part per thousand when the universe became transparent, a concerted search for anisotropy beyond the dipole pattern was needed. An alternative model for the formation of large-scale structure assumed that giant explosions would push the matter in the universe into giant shells, but these explosions would add energy to the CMB at late times in a way that would make the spectrum deviate from a simple blackbody law. The Cosmic Background Explorer (COBE) project (figure 1), proposed in 1974 but launched in 1989, carried an instrument to search for spectral distortions, the Far-InfraRed Absolute Spectrophotometer (FIRAS), and an instrument to search for anisotropy, the Differential Microwave Radiometers (DMR).

The announcement in 1990 that the CMB spectrum measured by COBE showed no deviations from a blackbody led to a standing ovation at an American Astronomical Society meeting. The detection of non-dipole anisotropy by COBE was described as the ‘discovery of the century, if not of all time’ by Stephen Hawking in 1992.

Based on the absence of distortions in the spectrum and the presence of non-dipole anisotropy, the large-scale structure of the universe appears to have been generated by gravitational forces acting on Dark Matter. The extremely small initial density perturbations that produced the gravitational forces were created during the first picosecond after the big bang. Thus the properties of the CMB have provided a wealth of information about Cosmology.
Spectrum of the CMB
The best measurement of the spectrum of the CMB was made by the FIRAS instrument on COBE. This spectrometer compared the sky with a very good blackbody and thus made a very sensitive measurement of deviations from a Planck function. The temperature of the blackbody that best matched the sky spectrum was 2.725 K with an uncertainty of only ±2 mK. Figure 2 shows the spectrum of the CMB, the best fitting Planck function and an expanded view of the difference between the sky and the blackbody. The CMB spectrum is remarkably close to the blackbody.

The number density of photons as a function of redshift is given by

\[ n_\gamma = \frac{8\pi (kT_0/hc)^3 \Gamma(3) \zeta(3)(1+z)^3}{(1+z)^4} \]

which has a current \((z = 0)\) value of 401.5±1 cm\(^{-3}\). The number densities of baryons—heavy particles that contain three quarks, like the proton or neutron—and electrons are about 2 billion times lower. The specific heat of the radiation field is thus a billion times larger than the specific heat of the ordinary matter, and when in the early universe the radiation and matter interact strongly the temperature of the matter is forced to be equal to the temperature of the radiation field.

The expansion of the universe produces a redshift that preserves the blackbody character of the radiation. When the universe was 0.1% of its current size, the photons of the blackbody radiation field had 1000 times more energy than they do now. Since this time the wavelengths of the photons have increased by a factor of 1000 while the size of the universe has also increased by a factor of 1000. This ratio of final to initial wavelength is used to define \(z\), the redshift, via \(1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}}\). Thus, when the universe was 0.1% of its current size, the color temperature deduced from the photon energy was 1000 times higher than \(T_0 = 2.725\) K. However, the number density of the photons was \(10^9\) times higher than the current number density, and the energy density was \(10^{12}\) times higher than the current energy density. This is exactly the energy density proportional to \(T^4\) law seen for blackbodies. Thus once a blackbody radiation field is produced the expansion of the universe acts in a way that preserves the blackbody character of the radiation, with only the temperature changing.

However, if the observed redshift were caused by a tired light effect, that only changes the energy of the photons, then the number density of photons in the CMB does not change with redshift. Thus the number density now would be too high for the mean energy of the
photons by a factor of \((1 + z_0)^3\), where \(z_0\) is the redshift for the origin of the CMB. Thus, if the Planck function is

\[
B_\nu(T) = \frac{2k\nu^3}{c^2} \frac{1}{\exp(h\nu/kT) - 1}
\]

the spectrum of the CMB in a tired light model would be \(I_\nu = (1 + z_0)^3 B_\nu(T_0)\). The FIRAS data require that the factor in front of \(B_\nu\) must be unity to within 100 parts per million. Thus in a tired light model the CMB must have been produced, and the universe must have been opaque, at a redshift \(z_0 < 0.000\,033\). However, discrete radio sources are seen emitting millimeter wave radiation from \(z < 4\), so the blackbody nature of the CMB rules out tired light models for the cosmic redshift.

The universe now produces a mixture of starlight and infrared radiation that is far from a blackbody. Thus the blackbody radiation that we see now must have been produced during an earlier epoch. This is a very direct proof of the evolution of the universe and shows that the steady state model is wrong. For times earlier than a few thousand years after the big bang \((z > 3 \times 10^4)\), electron scattering \((e + \gamma \rightarrow e + \gamma)\) can drive the spectrum of the CMB toward a thermal distribution, but this interaction cannot change the number density of photons. A month or two after the big bang the electron scattering rate is so high that the rare double-photon Compton scattering process, \(e + \gamma \rightarrow e + \gamma + \gamma\), has a high enough rate to guarantee a blackbody spectrum that both is thermal and has the correct number density of photons. Any energy added to the CMB earlier than 1 month after the big bang will be converted to a blackbody spectrum with a higher temperature. Energy added between 1 month and a few thousand years after the big bang will be converted into a thermal Bose–Einstein distribution of photons with a non-zero chemical potential \(\mu\),

\[
I_{BE}(\nu, T) = \frac{2k\nu^3}{c^2} \frac{1}{\exp(h\nu/kT + \mu) - 1}.
\]

Since a blackbody spectrum has zero chemical potential, this would produce observable distortions in the spectrum. Data from the FIRAS experiment on COBE show that \(|\mu| < 9 \times 10^{-7}\). Energy transferred from hot electrons to the CMB photons by electron scattering later than a few thousand years after the big bang will produce a Sunyaev–Zel’dovich or Kompaneets distortion of the spectrum:

\[
I_E(\nu, T) = B_\nu \left[ T_0 \left[ 1 + y \left( \frac{x(e^x + 1)}{e^x - 1} - 4 \right) \right] \right]
\]

with \(x = h\nu/kT_0\), characterized by the parameter

\[
y = \frac{\tau_{es}(T_e - T_r)}{m_e c^2}
\]

where \(\tau_{es}\) is the electron scattering optical depth. Data from the FIRAS experiment on COBE show that \(|y| < 1.5 \times 10^{-5}\). The FIRAS limits on \(y\) and \(\mu\) show that any energy transferred to the CMB later than a month or two after the big bang is less than 60 parts per million of the total CMB energy.

**Isotropy of the CMB**

The CMB is completely isotropic to about one part per thousand, and, except for the velocity of the solar system, would be isotropic to better than 40 parts per million. Figure 3 shows the temperature of the sky without any contrast enhancement, and no structure can be seen. This is quite a remarkable property and would normally require that many messages be passed back and forth between different regions seen in different directions on the sky. However, in the space–time geometry of the simplest standard big bang model, the distance that light can travel between the big bang and a time \(t\) after the big bang is given by \(3ct\). It is larger than \(ct\) because the distances traveled at early times have grown with the expansion of the universe. The universe appears to be about 13 Gyr old, so the ‘radius of the sky’ (actually the ‘circumference of the sky’ divided by \(2\pi\)) as seen now is about 39 billion light years. At the time the universe became transparent, about 300 000 years after the big bang, light would have traveled 900 000 light years since the big bang. However, this distance would grow by a factor of 1100 during the expansion of the universe from then until now, so the apparent angle subtended by the distance light could have traveled between the big bang and recombination is given by \(\tan \theta = 1/39\) or \(\theta = 1.5^{\circ}\). Thus while many messages back and forth across the whole sky would be needed to establish the observed isotropy, in the standard big bang model one message could only travel 1% of the way around the sky in the time available. This is the ‘horizon’ problem in the standard big bang model, and thus the observed isotropy of the CMB is evidence in favor of the inflationary scenario which can solve the horizon problem.

**Anisotropy of the CMB**

When the contrast of the temperature map is increased by a factor of 400 as shown in the middle portion of figure 3, a systematic variation across the sky can be seen. Note that this figure uses an equal-area Mollweide projection to show the entire sky in an ellipse with a 2:1 aspect ratio and that the equator of the map is the galactic plane, with the center of the Milky Way galaxy occupying the center of the map. The ‘dipole’ pattern with a hot pole in the...
upper right and a cold pole in the lower left can be explained by a Doppler shift between the frame of reference of the solar system and a frame of reference at rest relative to the observable universe. Radiation received in the solar system with frequency $\nu$ at angle $\theta$ to the dipole axis would have had frequency $\nu' = \gamma (1 - \beta \cos \theta)\nu$ in the ‘universal rest frame’, where $\beta = v/c$ and $\gamma = 1/(1 - \beta^2)^{1/2}$. Thus the observed temperature as a function of angle is given by $T(\theta) = T_0 \gamma (1 - \beta \cos \theta) \approx T_0 + T_0 \beta \cos \theta$. The observed coefficient of $\cos \theta$ is $3353\pm24$ µK, while $T_0$ is 2.725 K, so $\beta = 0.00123$ and the velocity of the solar system relative to the center of mass of the local group is approximately in the opposite direction and much less well determined. Thus the velocity of the local group relative to the observable universe is $600\pm45$ km s$^{-1}$.

Since the pattern produced on the sky by a Doppler shift can be predicted, the dipole anisotropy can be removed from the map giving a no-dipole map. After this is done, the largest signal that remains is due to the radio and millimeter wave emission from the Milky Way. This emission is much stronger at centimeter wavelengths than at millimeter wavelengths, so if a fraction of a long-wavelength map is subtracted from a short wavelength map, then almost all of the emission from the Milky Way can be canceled while still leaving a strong CMB anisotropy signal. The bottom portion of figure 3 shows such a no-dipole, no-Galaxy map based on all of the data from the COBE DMR.

The inflationary scenario makes a specific prediction for the statistical properties of the CMB anisotropy seen by the COBE DMR. The statistical variance of the temperature due to spots now seen with sizes projected on the sky in the ranges $80^\circ$–$160^\circ$, $40^\circ$–$80^\circ$, $20^\circ$–$40^\circ$, $10^\circ$–$20^\circ$, $5^\circ$–$10^\circ$, etc should all be equal at the end of the inflationary epoch at a time less than 1 ps after the big bang. This is known as ‘equal power on all scales’. The actual perturbations are produced as quantum fluctuations with a size perhaps as small as $10^{-33}$ m, but the exponential growth during inflation increases the size of these perturbations. During the inflationary epoch the universe grows by at least 30 powers of 10, or more than $70 e$-foldings. Because the physical conditions and hence the strength of the fluctuations change little from one $e$-folding to the next, but the final angular size changes by a factor $e$, inflation naturally produces the same fluctuation power on angular scales of $e$, $e^2$, $e^3$, etc. degrees, which is equal power on all scales. As discussed earlier, signals traveling at the speed of light have enough time before recombination to modify structures smaller than $1.5^\circ$, so we only expect equal power on all scales for spots larger than $1.5^\circ$. However, the COBE DMR was only able to resolve spots larger than $7^\circ$, so it should see a pattern consistent with equal power on all scales if the inflationary scenario is correct.

The statistical properties of a random process on the celestial sphere, with a particular realization being the anisotropy signal $T(l,b)$, where $(l,b)$ specify the longitude and latitude of the point with temperature $T$, can be fully specified by either the angular correlation function, $C(\theta)$, or its Legendre transform, the angular power spectrum, $C_l$, if the random process is Gaussian and has no preferred axis. Since in the standard model the CMB anisotropy at a given point is the sum of the effects from a very large number of minute influences, the central limit theorem will guarantee that the random process is Gaussian. The angular correlation function is given by

$$C(\theta) = \frac{1}{4\pi} \int \frac{d^2l}{l^2} \langle T(l) T(l') \rangle = \frac{1}{4\pi} \int \frac{d^2l}{l^2} \langle T(l) T(l') \rangle + \frac{1}{4\pi} \int \frac{d^2l}{l^2} \langle T(l) T(-l) \rangle$$
the expectation value of the product of temperatures at pairs of points separated by an angle \( \theta \):

\[
C(\theta) = \langle \Delta T(l, b) \Delta T(l', b') \rangle |_{l, b, l', b'} = 0.
\]

The angular power spectrum is defined in terms of the coefficients of an expansion of \( T(l, b) \) into spherical harmonics \( T(l, b) = \sum_{l,m} a_{lm} Y_{lm}(l, b) \) as

\[
C_\ell = \langle |a_{\ell m}|^2 \rangle
\]

and the two are related by

\[
C(\theta) = (4\pi)^{-1} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos \theta)
\]

where \( P_\ell(x) \) is the Legendre polynomial of order \( \ell \). Note that a given spherical harmonic of order \( \ell \) is associated with an angular scale of about \( 180^\circ / \ell \). To have equal power on all scales, the total variance from all of the spherical harmonics in a range of indices \( \ell \) equal to the central index \( \ell \) should always add up to a constant, independent of the central index \( \ell \). However, there are \( \ell (2\ell + 1) \) spherical harmonics that contribute to this range, so equal power on all scales requires

\[
C_\ell = \text{const} \frac{\ell (\ell + 1)}{\ell (2\ell + 1)} \approx \frac{2 \times \text{const}}{\ell (2\ell + 1)}.
\]

The COBE DMR data are compared to an equal power on all scales model in figure 4. The data points for large angular scale (low \( \ell \)) in figure 5 show \( \ell (\ell + 1) C_\ell \) for the data from the COBE DMR. If the sky shows equal power on all scales, these data points should be consistent with a horizontal line, and they are for large angular scales.

The dominant cause of the temperature fluctuations for large angular scales is the gravitational redshift. Dense regions have negative gravitational potential wells, and the photons of the CMB lose energy as they leave the potential wells. This effect is larger than the normal density effect: dense regions have higher photon number densities and thus higher temperatures. However, at small angular scales the situation is changes, in part because the density fluctuations have time to change between the big bang and recombination. The photons and the baryons plus electrons (ordinary matter) are tightly coupled before recombination, so the density fluctuations travel as acoustic waves at the sound speed in the photon–baryon fluid. Because there are billions of photons for every baryon, the radiation pressure from the photons is large compared with the energy density from the baryon rest mass, so the speed of sound is very high: \( c_s \approx c/(3)^{1/2} \). If the wavelength of a fluctuation is set so that it goes through one-half cycle of oscillation before recombination, then the effects of the photon density and the gravitational redshift are in phase at recombination instead of partially canceling. Note that the gravitational potential is dominated by the dark matter density, and the dark matter is not affected by the photon–baryon acoustic
waves, so the gravitational potential is almost unchanged by the acoustic oscillations of the photon–baryon fluid. Being in phase produces a big peak in the angular power spectrum, much larger than the equal power on all scales level. Since light would only travel the distance spanned by a \( \approx 1.5^\circ \) angle in the time between the big bang and recombination, the sound wave would travel \( 1.5^\circ (3)^{1/2} \), and the wavelength of the oscillation is twice this, or \( 1.7^\circ \). Approximately 200 such waves would fit around a full circle, so \( \ell \approx 200 \) for this peak in the standard model which has a density equal to the critical density, with most of the matter being some form of dark matter which does not interact with electromagnetic radiation. The critical density is derived by setting the Hubble flow \( v = Hr \) equal to the escape velocity, \( v_e = (2GM/r)^{1/2} = r(8\pi G\rho/3)^{1/2} \), so \( \rho_c = 3H^2/8\pi G \). When the density is equal to the critical density, then general relativity requires that the geometry of space be a Euclidean, uncurved space. Observations suggest, however, that the density of matter is less than the critical density. If the total density is less than the critical density, then space has an open, negatively curved or hyperbolic geometry. For this open geometry, the ‘circumference of the sky’ is much higher, so the angular scale corresponding to the peak is smaller, and thus the angular frequency of the peak is higher, with \( \ell \approx 400 \) for a density that is one-quarter of the critical density. On the other hand, if the density of matter is one-quarter of the critical density but the rest of the critical density is provided by a vacuum energy density (or COSMOLOGICAL CONSTANT), then the angular frequency of the peak remains close to \( \ell \approx 200 \). The height of the peak is primarily determined by the ratio of the baryon density to the dark matter density. The general trend of the scattered data points from the medium to small angular scale experiments shown in figure 5 is consistent with the predicted curve for a standard cold dark matter model with a fairly high baryon to dark matter ratio. This could be due to a high baryon density or caused by a low value of the Hubble constant \( (H_0 = 40 \text{ km s}^{-1} \text{ Mpc}^{-1}) \) which would lower the dark matter density. However, the data are also consistent with a vacuum energy density dominated model.

In addition to the first acoustic peak at \( \ell \approx 200 \), the standard models predict additional peaks in a harmonic sequence. The data available in 1998 do not allow one to determine the position and amplitude of these peaks. However, better measurements of the angular power spectrum are planned from two ongoing space projects. The Microwave Anisotropy Probe (MAP), to be launched by NASA in late 2000, will produce data points having the noise level shown by the shaded band in figure 5. The PLANCK satellite, to be launched by ESA in 2007, will provide even better data extending up to \( \ell = 2000 \). The data from MAP will allow a determination of cosmological parameters such as the Hubble constant, \( H_0 \), the ratio of the matter density to the critical density, \( \Omega \), and the ratio of the vacuum energy density to the critical density, \( \lambda \), to an accuracy of 10%. The improved data from PLANCK will determine these parameters to 1% accuracy.

In 2001, new results from DASI (Degree Angular Scale Interferometer) and further detailed analysis of results from BOOMERANG (Balloon Observations of Millimetric Extragalactic Radiation and Geophysics) revealed hundreds of complex regions visible as tiny variations—typically only 100 millionths of a degree—in the temperature of the CMB. The new results show the first evidence for a harmonic series of angular scales on which structure is most pronounced. The presence of these harmonic peaks bolsters the theory that the Universe grew from a tiny subatomic region during a period of violent expansion a split second after the Big Bang.

Unconventional models
There are non-standard models for the formation of large-scale structure and CMB anisotropy in the universe that do not invoke fluctuations produced in the first picosecond after the big bang. Some models produce the CMB anisotropy through the action of topological defects acting after recombination (see TOPOLOGICAL DEFECTS IN COSMOLOGY). The best known type of topological defect is the cosmic string, a very narrow and very long object that is quite massive because its core is filled with a large vacuum energy density. If cosmic strings exist, then the CMB anisotropy on the sky will have extremely sharp edges like the essentially vertical cliffs that border the Yosemite valley in California. If such an edge is discovered, followup observations could easily establish that the edge was abrupt and prove the existence of cosmic strings.

The CMB and large-scale structure
CMB anisotropy measurement determine the gravitational potential at the time of recombination, and thus the gravitational forces that act on the initial smooth distribution of matter are known. In order to produce the clustered pattern seen today, these forces have to act freely on most of the matter in the universe over the whole time interval from 10 000 years after the big bang until the present. Since ordinary baryonic matter cannot move freely until recombination, the CMB anisotropy data already show that most of the matter of the universe is dark.

The accurate determination of the CMB angular power spectrum by MAP and PLANCK will establish the initial conditions that existed at recombination. Before recombination, ordinary matter was strongly tied to the photons, and large-scale structures such as clusters of galaxies were not able to grow. After recombination, these structures could start to grow, and the CMB data
will specify the properties of both the gravitational potential and the density fluctuations at recombination. With these initial conditions, detailed calculations of the behavior of matter between the time of recombination and now can be performed. These calculations often involve following the motions of tens of millions of particles using $N$-body computer codes. With the CMB data providing the initial conditions, and large scale surveys of galaxies like the Sloan Digital Sky Survey providing the final result, the significant physical effects that produce the observed large-scale structure can be identified.

**Conclusion**

Observations of the CMB have already provided a wealth of information about the universe. The CMB spectrum gives data from a month or two after the big bang. The anisotropy of the CMB shows the state of the universe at recombination, but that state was established during the inflation epoch less than 1 ps after the big bang. Thus study of the CMB provides data from the earliest evolution of the universe.

More detailed information from two satellite projects, MAP and PLANCK, will allow the accurate determination of hundreds of points on the angular power spectrum of the CMB. From these hundreds of data points, the values of several cosmological parameters will be estimated to an accuracy of 1–10%. With many data points being used to determine a small number of parameters, a very strong consistency check will be possible. If a consistent set of parameter values can be found, and the implied density and gravitational potential perturbations at recombination evolve into the observed large-scale structure under the influence of understood physical processes, then we will have learned much about the universe. If no consistent set of parameters can be found, or if the CMB data and the large-scale structure data disagree, then we will have new physical processes to find and understand. In any case, the study of the CMB will lead to a much better understanding of the universe.

**Bibliography**


*Edward Wright*