eaa.iop.org
DOI: 10.1888/0333750888/2134

Sunyaev–Zeldovich Effect

Anthony Lasenby

From

Encyclopedia of Astronomy & Astrophysics

P. Murdin

ISBN: 0333750888

Institute of Physics Publishing
Bristol and Philadelphia

Sunyaev–Zeldovich Effect

The Sunyaev–Zeldovich (SZ) effect is a spectral effect on the COSMIC MICROWAVE BACKGROUND (CMB) caused by interaction with the electrons in the hot gas inside a CLUSTER OF GALAXIES. In the Rayleigh–Jeans portion of the spectrum it manifests itself as a small ($\sim$0.5 μK) dip in the temperature of the CMB in the direction of the cluster. First proposed in 1970, about 20 yr of observational effort were required before it was reliably detected. Many tens of clusters have now detected in the SZ effect, and early predictions of its usefulness for cosmology are now starting to bear fruit.

The basic mechanism of the SZ effect (see figure 1) involves scattering of CMB photons by electrons in the distributed intracluster gas that exists in clusters of galaxies and in fact makes up about a quarter of their total mass. A rich cluster may have a total mass $>3 \times 10^{14} M_\odot$ and gas in hydrostatic equilibrium in the potential well of the cluster heats to temperatures of order 10^7–10^8 K. The hot electrons in this plasma have two effects: firstly they produce thermal bremsstrahlung radiation, which provides the cluster x-ray luminosity. Secondly, they interact with CMB photons passing through the cluster, via Thomson scattering. This preferentially increases the energy of the photons involved in the scattering, giving an effective shift of the CMB BLACK-BODY RADIATION curve to higher frequencies and hence a dip in the Rayleigh–Jeans region of the spectrum, and an increment in the Wien region. (The precise form of the temperature shift as a function of frequency is shown in figure 2.) The magnitude of the effect is proportional to the line integral of electron pressure through the cluster, and hence directly proportional to electron number density, n_e, and temperature. This is in contrast to the thermal bremsstrahlung responsible for the x-ray emission, which is proportional to n_e^2. This difference makes the SZ effect more sensitive to the broadly distributed component of gas, rather than the dense central regions favored in x-ray emission. A further, very useful, aspect of the SZ effect is that for the same physical gas parameters it is independent of REDSHIFT. This makes it as easy to detect at high redshift, where optical or x-ray detection of a cluster may be very difficult, as at low redshift, thus making it a useful cosmological probe. In addition, the combination of x-ray and SZ data on a cluster makes possible a determination of the HUBBLE CONSTANT independent of the usual distance ladder techniques. The results for H_0 from this method are already of interest and are expected to be of increasing importance in the future, as both the systematic and the random errors in the technique start to decrease.

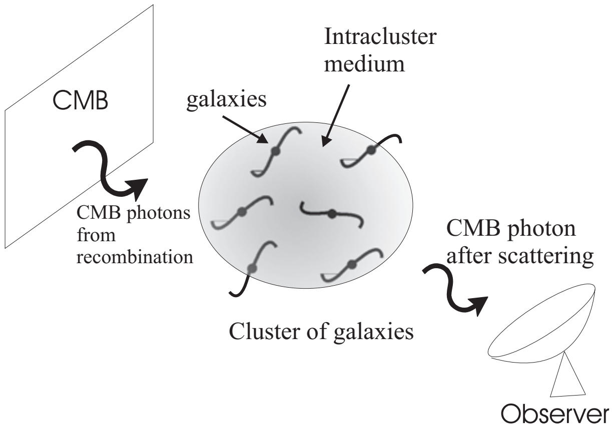

Figure 1. Schematic showing the origin of the SZ effect. Hot intracluster gas in a cluster of galaxies gives a small increase in the net energy of CMB photons arriving from the early universe.

The physics of the Sunyaev–Zeldovich effect

Probably the best way to derive the SZ effect, including relativistic corrections, is in the context of an extended version of the *Kompaneets equation*—this is discussed further below. We first begin with a heuristic derivation: consider the injection of energy, via Compton scattering, from an electron gas into an isotropic photon gas. We shall suppose that the electron gas is non-relativistic and with no bulk motions and that the radiation temperature $T_{\rm rad}$ is $\ll$ the electron temperature T_e. (This is well satisfied in practice.) With these conditions it is not immediately obvious that there should be any effect—if the scattering is elastic, then one cannot change the basically isotropic distribution of photons or electrons. It is true there is a 'Doppler scattering' effect

$$\frac{\Delta\nu}{\nu} = \frac{v_e}{c} \times \text{angular factor}$$

where v_e is the velocity of the colliding electron, but this will average to zero, since the electron velocities are isotropic. (Note that this formula will tell us what happens when there *is* a bulk motion.) The SZ effect arises as follows. The scattering mechanism is Thomson scattering, for which we know

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sigma_T \frac{3}{16\pi}(1 + \cos^2\theta)$$

where θ is the angle between the photon and electron directions, i.e. there will be maximum scattering in the forward and backward directions. We therefore approximate the problem as one-dimensional and consider in the cosmological (photon gas) frame the head-on collision of a photon with energy $h\nu$ with an electron with a velocity $v = \beta c$ directed away from the incoming photon. In the electron rest frame the photon energy is transformed to $h\nu[(1-\beta)/(1+\beta)]^{1/2}$ and there is an equal scattering probability in the forward and back directions. In these directions there is no frequency shift, from the familiar Compton formula

$$\lambda' - \lambda = 2\lambda_e \sin^2(\theta/2) \qquad \lambda_e = \frac{h}{m_e c}.$$

Transforming back to the cosmological frame we find that the photon has energy $h\nu(1-\beta)/(1+\beta)$ (i.e. *red*shifted) if its direction has been reversed in the encounter, or $h\nu$ if not. Now suppose that the photon came from the other direction, so that initially in the photon's frame the electron velocity was directed towards it. In this case the same analysis yields a *blue*shift to an energy of $h\nu(1+\beta)/(1-\beta)$ if the photon direction is reversed and a constant energy of $h\nu$ if not. The mean frequency after collision for photons with momentum parallel to electron velocity is thus

$$\nu' = \nu\frac{1}{4}\left(2+\frac{1-\beta}{1+\beta}+\frac{1+\beta}{1-\beta}\right) = \frac{\nu}{1-\beta^2} \approx \nu(1+\beta^2).$$

Thus there is a net second-order effect.

What happens in the interval of time Δt? Restoring the three-dimensional dependence on scattering angle, the mean shift is

$$\langle \text{probability of shift in} \Delta t \times (\beta\cos\theta)^2\nu \rangle$$

averaged over the isotropic photon momentum distribution, i.e.

$$\mathrm{d}\nu = \frac{1}{3} n_e \sigma_T c \Delta t \nu \frac{3kT_e}{m_e c^2}.$$

(Here $(n_e\sigma_T)^{-1}$ is the photon mean free path.) Thus

$$\frac{1}{\nu}\frac{\mathrm{d}\nu}{\mathrm{d}t} = \left(\sigma_T c n_e \frac{kT_e}{m_e c^2}\right).$$

The right-hand side has dimensions of $(\text{time})^{-1}$ and we write it as $\mathrm{d}y/\mathrm{d}t$. Integrating, we obtain

$$y = \int_{t_i}^{t_f} \sigma_T c n_e \frac{kT_e}{m_e c^2}\,\mathrm{d}t.$$

The quantity y is called the Comptonization parameter and here is worked out for the case of injection of energy from an initial time t_i to a final time t_f. Equating this to the time taken for a photon to traverse a cluster yields the final formula

$$y = \int \sigma_T n_e \frac{kT_e}{m_e c^2}\,\mathrm{d}l$$

or $y = (\sigma_T k/m_e c^2) \times$ line integral of pressure through cluster. The equation

$$\frac{1}{\nu}\frac{\mathrm{d}\nu}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}t} \qquad \text{implies} \qquad \nu(t) = \nu(0)\mathrm{e}^y.$$

In the cluster case we have $\nu_{\text{out}} = \nu_{\text{in}}\mathrm{e}^y$. This shift in frequency yields an apparent *deficit* in the Rayleigh–Jeans region of the CMB spectrum and an increase in the Wien region (see figure 2). We have

$$\begin{aligned}\Delta I(\nu) &= I\left(\frac{\nu}{\mathrm{e}^y}\right) - I(\nu) \qquad \text{generally}\\ &= 2\left(\frac{\nu}{\mathrm{e}^y}\right)^2\frac{kT_{\text{rad}}}{c^2} - \frac{2\nu^2 kT_{\text{rad}}}{c^2}\\ &\qquad \text{in the Rayleigh–Jeans region}\\ &\approx I(\nu)\times(-2y) \qquad \text{for small } y.\end{aligned}$$

For a rich cluster, taking the parameters of the COMA CLUSTER as an example, $l \sim 3$ Mpc, $n_e \sim 10^3$ m^{-3} and $T_e \sim 7\times10^7$ K. Thus a typical y is 7×10^{-5}, making the assumption of its smallness in general well justified.

The result just quoted for the Rayleigh–Jeans region is the correct one (neglecting relativistic corrections), as is the general idea of a decrement in the Rayleigh–Jeans region and an excess in the Wien region. However, our derivation has ignored several important physical factors, in particular that the photon energy $h\nu$ varies after collision in the electron frame, that the photons are not isotropic in the electron frame and that photons are bosons, the latter implying that there is stimulated emission also to consider. Putting all this together to find the correct equation yields the Kompaneets equation. The general solution as a function of frequency, expressed in terms of Rayleigh–Jeans brightness temperature ($T_{\text{RJ}} = (\lambda^2/2k)I$), is

$$\Delta T_{\text{RJ}} = \left(\frac{x}{\mathrm{e}^x-1}\right)^2 e^x\left(x\coth\frac{x}{2}-4\right) y T_{\text{rad}}$$

where y is as above and $x = h\nu/kT_{\text{rad}}$. This is what a typical heterodyne system would measure. If we were to measure *flux* rather than temperature, i.e. use a bolometer-type system, then

$$\Delta I \propto \nu^2(\Delta T_{\text{RJ}})_{\text{equiv}} \propto x^2 \times \text{above}.$$

The importance of this is that the effect is thus much increased in the sub-mm region, where we are likely to be using bolometer techniques anyway. The relevant curves are shown in figure 2, which also shows the same curves for a simple change in the black-body temperature of the CMB, such as would be relevant to primordial perturbations. One can see from figure 2 that observing the flux effect in three regions, centered at approximately 2 mm, 1.2 mm and 0.8 mm say, would give a unique signature for the SZ effect, separating it quite distinctly from any anisotropies due to primordial perturbations. Pursuing this point of different frequency dependences further, we note that if a cluster has a systemic *radial* velocity then the bulk electron flow induces Doppler scattering with magnitude given by the bulk motion formula above:

$$\frac{\Delta T}{T} \sim \frac{v_{\text{pec}}}{c}\tau$$

where τ is the optical depth to Thomson scattering through the cluster, $\tau \sim \int n_e\sigma_T\,\mathrm{d}l$. The frequency dependence of this effect will be that of the black-body curves of figure 2, and so by observing at the peak of the black-body curve one can separate the direct 'thermal' SZ effect, which nulls here, from the kinetic or 'Doppler' one, which has a peak here (when measured in terms of flux). To see what the hopes are of being able to map the peculiar velocity field of clusters in this way, we take the τ appropriate to the Coma cluster, ~1%, and assume a cluster velocity of 600 km s^{-1}, which may not be untypical of the higher velocities which could be expected. This yields a black-body temperature change of 0.05 mK. For comparison,

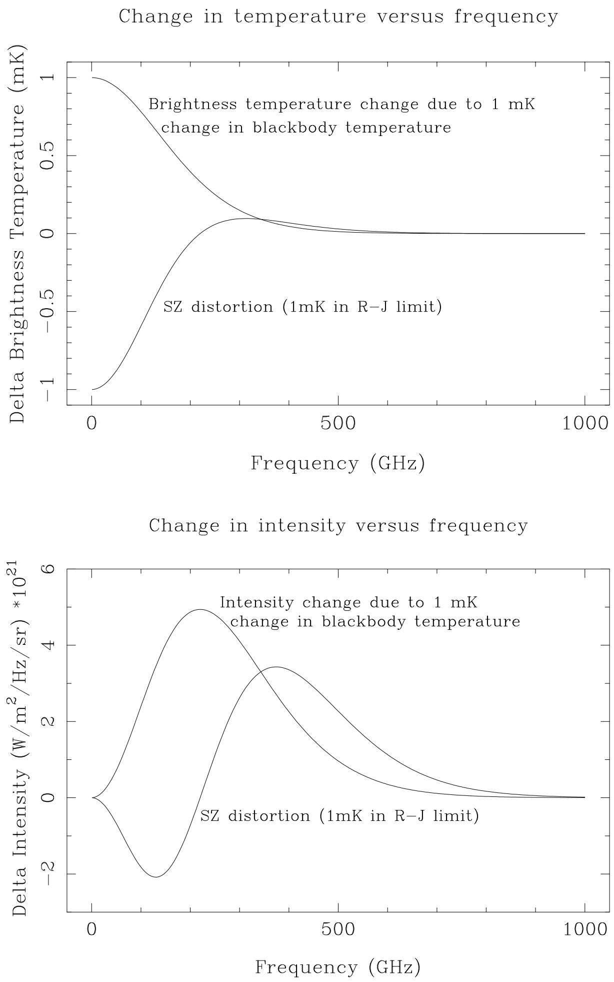

Figure 2. Top: Plots of ΔT (brightness temperature) for a 1 mK change in black-body temperature (top curve) and due to the SZ effect with a Rayleigh–Jeans limit of 1 mK. Bottom: Same but for intensity distortion.

taking the Coma cluster as an example again, the direct SZ effect expected is 0.5 mK, and indeed the measured values in several clusters appear to be of this order. Thus the mapping of the peculiar velocity field of clusters will be a difficult task, but the numbers do not look hopeless. Transverse motions of clusters also give a perturbation to the CMB via the 'moving lens effect', with a distinctive polarization signature, but for similar velocities the $\Delta T/T$ effect tends to be a few $\times 10^{-6}$ rather than a few $\times 10^{-5}$ as for the thermal and radial kinetic effects. The angular scale of the SZ effect on the sky is set by the angular diameter subtended by the cluster, and is typically a few arcminutes.

Relativistic effects

For clusters with electron temperature above about 10 keV, relativistic effects neglected in the usual Kompaneets equation must be introduced, if one wishes to match theory and experiment together properly, even at the accuracy level of the current observations, and certainly for future observations. First calculations of the corrections were made numerically by Wright and Raphaeli and recently Challinor and Lasenby have found an analytical extension of the Kompaneets approach, which allows results to be derived as power series in the electron temperature parameter $\theta_e = k_B T_e / m_e c^2$. As an example, in the Rayleigh–Jeans region, one finds

$$\frac{\Delta T_{\rm RJ}}{T_{\rm rad}} \approx -2y\left(1 - \frac{17}{10}\theta_e\right)$$

to first order in θ_e. To give an example of the magnitude of this effect, for typical cluster parameters ignoring this correction would give an overestimation by about 5% in the value of the Hubble constant derived via the SZ route (described below). For clusters with temperature greater than about 10 keV there are substantial modifications to the frequency dependence of the SZ effect in the Wien region of the spectrum (by tens of per cent even at 15 keV) and the correct relativistic formulae (including θ_e^2 corrections) must be used.

Observations

A variety of approaches have been used to try to detect and measure the SZ effect. Here we highlight the techniques that have been used, roughly in order of their historical appearance.

Single-dish observations

The effect was first claimed to have been discovered by Parijskij, in 1972, in the Coma cluster. This was unlikely given the frequency of observation and the fact that Coma has a strong cluster halo source, in addition to other radio sources near the center. There were further claims in 1978 and 1981 for a detection in several clusters by Birkinshaw, Gull and Northover, using a dish at Chilbolton, UK, at 10 GHz, but again radio source confusion and possible systematic effects in the observations make it unlikely the the decrements seen (at about 1 mK amplitude) were real. The first reports of a detection that are widely believed, and seem to have withstood the test of further accumulation of data, came from Birkinshaw, Gull and Hardebeck in 1984, using the 40 m Owens Valley dish of Caltech, at 20 GHz. It might be wondered why there is any difficulty or question about these measurements, given that the effects are about 10 times bigger than a 95% confidence blank sky limit which had been set with the same telescope in a search by Readhead *et al* for primordial CMB anisotropy. The answer lies in the (necessarily) different observing strategies adopted in the two cases. For the intrinsic anisotropy work, Readhead *et al* were able to minimize telescope movement by working with fields very close to the north celestial pole and observing only near transit. For observations of specific clusters, there is no such option, since the cluster must be followed around an appreciable portion of the sky in order to build up sufficient sensitivity. This leads to two problems. First, the combination of telescope on–off motions and beamswitching used for both experiments in order to reduce atmospheric effects can still leave residual systematics due to incomplete subtraction of sidelobes of the two beams. This is known as 'differential spillover'.

The second problem is that, as the source sweeps around the sky, the 'off' beams themselves sweep over circular arcs, displaced from the cluster center by the beamthrow. If these 'reference arcs' contain radio sources, these will produce negative dips in the central signal as the beams pass over them, leading to a false impression of a decrement having been detected. The only solution to this is an independent survey for the confusing radio sources. To achieve sufficient sensitivity in such a survey requires the power of the Very Large Array, but the area that needs to be covered for each cluster means that several VLA fields need to be mosaiced together, requiring quite large amounts of observing time.

Bolometer measurements
A bolometric system is able to achieve great sensitivity by having a very large fractional bandwidth and working at high frequencies. The disadvantage is that they are strongly affected by the atmosphere at these high frequencies and so have to be run from mountain tops, balloons (which have only limited observation time) or space.

The first really successful ground-based bolometer measurements have come from the Caltech SuZie group, working on the Caltech Submillimeter Observatory in Hawaii. They use a differencing bolometer scheme, coupled with drift scans across the source, which provides good stability and rejection of atmospheric fluctuations. The wavelengths observed are 1.2, 1.4 and 2.2 mm, which are properly spaced to make an attempt at separating the kinetic and thermal SZ effects. So far only upper limits have been found for the cluster peculiar velocities, but it is quite possible this instrument will be the first to detect non-zero velocities in the future. Scans at high signal to noise have been obtained on several clusters including A2163, which is the hottest known ($T_e \sim 14$ keV), making it necessary to use the correct *relativistic* formulae for the frequency dependence and amplitude of the SZ effect. Lamarre *et al*, using a balloon-borne bolometric system called PRONAOS, have made a determination of the spectrum of A2163 extending all the way to far-IR wavelengths (see figure 3), and in conjunction with the SuZie results this shows convincing evidence for the full expected frequency dependence of the SZ effect, albeit mixed in with dust emission at higher frequencies.

Interferometers
Interferometers provide currently provide the cleanest technique for making *images* of the SZ effect. An interferometer consists of a set of baselines formed by taking the elements of the interferometer in pairs. Each baseline is sensitive to a particular Fourier component on the sky and is relatively *in*sensitive to atmospheric

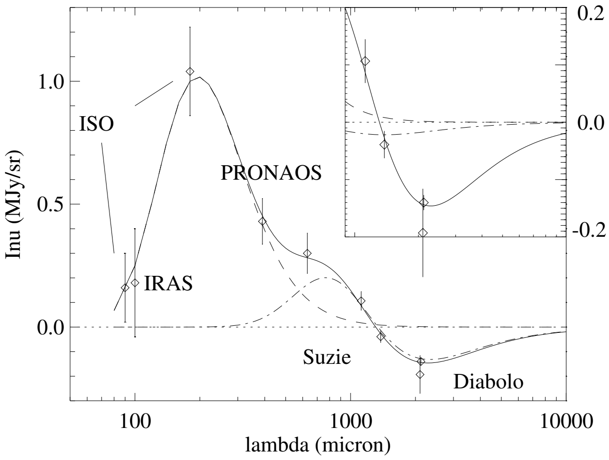

Figure 3. Spectrum of A2163, from Lamarre J M *et al* 1998 *Astrophys. J.* **507** L5. The solid curve shows a best-fit model including both the SZ effect (dot–dash curve) and predicted dust emission (dashed). The inset shows the expected contribution from the the kinetic SZ effect while the names are of the experiments used to trace different regions of the spectrum.

fluctuations and ground spillover and pickup (the problems which bedevilled earlier attempts to measure the SZ effect). Furthermore, close-in baseline pairs can be used to give a map of the SZ effects, while longer baselines can be used to subtract the effects of discrete radio sources, which are often variable, making it very desirable to measure them simultaneously with the SZ measurements. Two groups in particular have been pursuing this approach. The first image of the SZ effect came from the Ryle Telescope at Cambridge, UK, in 1993, and subsequently about 10 other clusters have been detected and mapped. In figure 4 a map is shown of the cluster A1413 made with the Ryle Telescope (Cambridge). The more centrally condensed nature of the x-ray emission versus the SZ effect is evident. The other group actively pursuing this approach have been using the OVRO and BIMA millimeter arrays in California, but with centimeter wave rather than millimeter wave receivers fitted. The lower observing frequency gives them sensitivity on angular scales appropriate to the SZ effect and they are able to use otherwise fallow summer observing periods to conduct the measurements. This campaign has resulted in the detection of about 25 clusters in SZ, and the highest signal-to-noise maps thus far made.

Determinations of H_0

The SZ effect can be used to determine H_0 in conjunction with x-ray information. We start with the equation for the x-ray flux:

$$S_x \propto \int \frac{n_e^2 T_e^{0.5}}{D_L^2}\,\mathrm{d}V$$

where n_e and T_e are the electron number density and temperature and D_L is the luminosity distance to the cluster. Via the x-ray and optical information we can

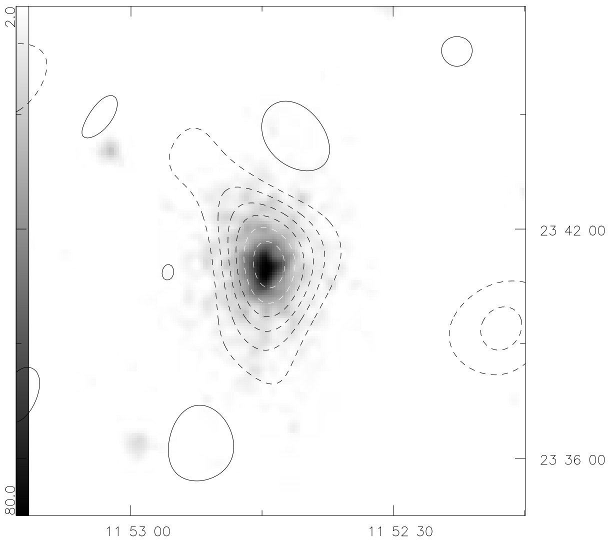

Figure 4. Ryle Telescope map of the cluster Abell 1413 (contours) overlaid on ROSAT PSPC x-ray image (greyscale). Contour levels for the radio map are −450 to 75 μJy/beam in steps of 75 μJy/beam. The numbers at the right are declination in degrees, minutes and seconds, indicating the overall angular scale. (Taken from Grainge K *et al* 1996 *Mon. Not. R. Astron. Soc.* **278** L17.)

determine T_e, θ and z, where θ is a characteristic angular size of the cluster and z is the cluster redshift. The SZ effect is determined by

$$\Delta T_{SZ} \propto \int n_e T_e \, \mathrm{d}l$$

i.e. a line-of-sight integral of pressure through the cluster, and we can relate dl to the other quantities via

$$\Delta l = \frac{f \theta D_L}{(1+z)^2}.$$

Here f is a prolateness factor, which, assuming the cluster has an ellipsoidal shape, is the ratio of the length of the major axis of the ellipse to the minor axis. The last equation uses the angular diameter distance formula to get the physical width of the cluster given its angular diameter and then converts this to a line-of-sight distance through the cluster using this assumed prolateness. This is done assuming what will turn out to be a ‘worst case’, where the cluster is indeed prolate and pointing straight at the observer. If we assume the cluster is at not too high a redshift, then the luminosity distance can be approximated by $D_L = cz/H_0$. (Obviously more accurate formulae can be used for this and other quantities at higher redshift.) Eliminating n_e via the known S_x, and D_L in favor of H_0, one finds

$$H_0 \propto \frac{f}{\Delta T_{SZ}^2}$$

with the constant of proportionality known. We note straightaway that if f is assumed to be 1 when it is really >1, i.e. where the cluster is indeed oriented towards the observer, then the H_0 derived by this method will come out too small. Since the H_0 values found so far using this approach have tended to come out *lower* than optical determinations, it is obviously this case that we need to worry about most in the first instance. Furthermore, for clusters which are selected on the basis of x-ray surface brightness, there would be a tendency for just this effect to happen, since the greater line-of-sight distance through the cluster would emphasize their x-ray brightness.

Putting back further factors we obtain

$$H_0 \propto \frac{f}{\Delta T_{SZ}^2} S_x T_e^2. \quad (1)$$

T_e can be found from the x-ray spectrum obtained, e.g. with the ASCA satellite, which covers the range up to ~10 keV. A typical cluster temperature is $T_e \sim 6$ keV and ROSAT only covers the range up to ~2 keV. The x-ray flux, S_x, can be obtained from e.g. ASCA, with resolution ~2′, or the ROSAT PSPC, with resolution ~15″. The cluster profile is modelled generally via an assumed isothermal atmosphere, with

$$n_e(r) = \frac{n_e(0)}{[1 + (r/r_c)^2]^{3\beta/2}}.$$

Typical values for clusters are $\beta \sim 0.6$, and core radius $r_c \sim 0.25$ Mpc, but one has to *fit* for these in any given case. For elliptical clusters, one needs to introduce different core radii along orthogonal axes. These are fitted iteratively until they match the x-ray observations. Ideally, one needs to include the physics of the pressure and cooling of the cluster in this process, and also to use simultaneously the SZ observations.

The values found so far for H_0 using this technique have encompassed a rather large range, but estimates are now beginning to stabilize. Birkinshaw, using results from nine well-measured clusters, finds a mean H_0 of 60 ± 10 km s^{-1} Mpc^{-1} (although systematic contributions to the error could be larger). Further results from the much larger samples being built up by the Cambridge and OVRO/BIMA interferometer teams are expected shortly. Carlstrom *et al*, working with detailed modeling of a single cluster, find good agreement with a value of 60 km s^{-1} Mpc^{-1} and moreover have shown that although the results for the core radius r_c and β are highly correlated after fitting, the resulting degeneracy in H_0 is quite small, which is encouraging for further work.

As well as estimating H_0, one can also use the SZ results to estimate the total baryon content of a cluster. In conjunction with information from GRAVITATIONAL LENSING in the cluster, or alternatively by assuming virial equilibrium for the intracluster material, one can then estimate the total matter fraction (baryonic plus dark) in the cluster. With a final assumption that the proportion of baryons to dark matter is similar to that pertaining in the universe as a whole, the total matter density of the universe can be found. Recently Grego *et al*, using 18 clusters from the OVRO/BIMA sample, have obtained the estimate $\Omega_M \sim 0.23^{+0.06}_{-0.04}$ (for an H_0 of 65 ± 10 km s^{-1} Mpc^{-1}, scaling inversely with H_0 for other values). This value represents that matter fraction relative to the CRITICAL DENSITY needed to make the universe spatially flat and ties in well with other recent indications that, if the universe really is spatially flat (as primordial CMB results seem to indicate), then the deficit must be made up in some other way than via dark matter. This lends extra support to the hypothesis of a non-zero COSMOLOGICAL CONSTANT. With a sufficiently large sample of clusters out to high redshift, it may be possible to use the luminosity distance dependence of the SZ effect to measure the cosmological constant directly.

Future uses of the SZ effect

The redshift independence of the SZ effect makes it an excellent tool for investigating the evolution of structure with redshift. On the other hand, current instruments are severely limited in their ability to do the kinds of 'blank sky' searches for the SZ effect that would be necessary to fully exploit this potential, owing to their relative lack of sensitivity and the time it takes to detect just one cluster in an already-known position. Accordingly, proposals for new ground-based instruments (all interferometers), able to make maps of the SZ effect over large areas of the sky, are now being made and will, it is hoped, be funded. The 'SZ source counts' (the number of clusters per square degree with SZ effect above a certain level) are strong functions of the matter density and geometry of the universe and so should provide significant information on Ω_M. In addition, the large number of clusters detected this way could act as a 'finder' for clusters to be investigated by future x-ray missions, which will not have the same sensitivity at high redshift as dedicated SZ telescopes. As well as ground-based telescopes, some future space missions will also be able to detect large numbers of clusters via the SZ effect. The PLANCK SURVEYOR mission, due for launch by ESA in 2007, has the potential to detect tens of thousands of clusters via the SZ effect, and it is hoped that for a large number of these a detection of the 'kinetic effect' will be possible as well, thus making possible a totally new kind of mapping of peculiar velocity fields on large scales.

Bibliography

The first predictions of the SZ effect were made in two papers:

Sunyaev R and Zeldovich Y 1970 *Comments Astrophys. Space Phys.* **2** 66

Sunyaev R and Zeldovich Y 1972 *Comments Astrophys. Space Phys.* **4** 173

The first image of the SZ effect is presented in

Jones M, Saunders R, Alexander P, Birkinshaw M, Dillon N, Grainge K, Lasenby A, Lefebvre D, Pooley G, Scott P, Titterington D and Wilson D 1993 *Nature* **365** 320

An extensive review of the SZ effect, covering both theory and observation, is given in

Birkinshaw M 1999 *Phys. Rev.* **310** 97

while an up-to-date review of imaging of the SZ effect and its future uses is contained in

Carlstrom J E, Joy M K, Grego L, Holder G P, Holzapfel W L, Mohr J J, Patel S and Reese E D 1999 astro-ph/9905255

Relativistic corrections to the SZ effect are discussed in

Challinor A D and Lasenby A N 1998 *6th Course on Topics in Astrofundamental Physics (Erice 1997)* ed N Sanchez and A Zichichi (Dordrecht: Kluwer) pp 409–42

Anthony Lasenby