Galaxies and Cosmology

AY21 Winter 2008

Solutions to Final Exam

written by Yacine Ali-Haïmoud, based on solutions by Thiago S. Gonçalves

March 21, 2008

1 Definitions

(a) Dwarf and regular ellipticals

The main point here is that dwarf ellipticals are not only small elliptical galaxies. Their formation history is different - regular ellipticals are probably the result of mergers - and a large part of the material has been stripped by supernova winds into the intergalactic medium. It is also important to notice that dwarf galaxies are dominated by dark matter.

(b) Schechter luminosity function

The Schechter LF is a distribution function that indicates the number of detected galaxies per luminosity. On the faint end, there is a power law (with more faint galaxies), with a sharp exponential cut-off at higher luminosities. The function is given by

$$\phi(L)dL = n_* \left(\frac{L}{L_*}\right)^{\alpha} \exp\left(-\frac{L}{L_*}\right) \frac{dL}{L_*}$$

Typical values from the notes:

$$
\begin{aligned}
n_* &= 0.01\ \text{Mpc}^{-3} \\
L_* &= 10^{10} L_\odot \\
\alpha &= -0.75,
\end{aligned}
$$

where n_* represents number density of galaxies in the universe, L_* is the typical luminosity and α is the power law exponent.

(c) Reionization era

The reionization era, which takes place for $6 < z < 15$ approximately, is the process of ionization of matter once the first stars were formed (before that, the Universe went through 'dark ages', and was mostly neutral). It gives an insight about the epoch of formation the first objects, and it affects the CMB spectrum.

(d) Blazars

Blazars were objects that were first thought to be stars, due to their high luminosity. Today we know they are actually active galactic nuclei, in which we look straight down the emission jet. They are very blue, and remarkably do not present any emission or absorption lines.

(e) Synchrotron emission

Synchrotron emission happens when relativistic electrons move in a magnetic field, being accelerated by it. The spectrum has three distinct parts: the middle one is the optically thin emission, as one would expect. At low frequencies, however, we have self-absorption (the emission cannot be stronger than a black body). At high frequencies, on the other hand, electrons are too energetic and rapidly emit radiate their anergy away; thus there is a break on the power law. The figure in Notes 16 shows those three regimes clearly.

(f) Black holes

The main difference is that Kerr black holes have angular momentum, while Schwarzschild do not. This allows infalling matter to reach closer to the center of the black hole, which means more efficient accretion.

(g) Unified model of AGN

The figure in Notes 17 is very clear. One should mention that the unified model states that all AGN are the same object, only viewed through different angles. Notice the difference between radio active and radio quiet AGN.

(h) Types of absorbers

There are three types of absorbers, classified according to their column densities: Ly-α absorbers (10^{14} cm^{-2} $\leq N_{HI} \leq 10^{16}$ cm^{-2}), with unsaturated lines and metallicity smaller than solar; Lyman limit absorbers ($N_{HI} \geq 10^{17}$ cm^{-2}), with saturated lines and density big enough to absorb all ionizing photons shortward of 912 Å; and finally the Damped Lyman-α systems ($N_{HI} \geq 10^{20}$ cm^{-2}), probably a proto-disk of forming galaxy.

(i) The cosmic web

The cosmic web is the large scale structure of the Universe : galaxies are distributed in clusters, walls, and filaments, surrounded by big voids. This can be studied through redshift surveys.

2 Fundamental plane

(a) Elliptical galaxies can be seen as a two-dimensional "surface" in the three dimensional "space" represented by the coordinates L (luminosity), σ (velocity dispersion), and R (effective radius) :

$$L \propto \sigma^{2.65} \; R^{0.65} \tag{1}$$

Note that the **canonical form** of the fundamental plane relation is equivalent to this one and reads :

$$R \propto \sigma^{1.4}\ I^{-0.8} \tag{2}$$

where R is the radius, I the mean surface brightness, and σ the velocity dispersion.

(b) Using the virial theorem, one can get a very crude derivation of the FP :

$$\sigma^2 = \frac{GM}{R}$$

Assuming a constant mass-to-light ratio, we thus have :

$$L \propto M \propto \sigma^2 R \tag{3}$$

To get the canonical form of the FP relation, we set $M = M/LL$, and $L \propto I\ R^2$, which gives then

$$R \propto \sigma^2 I^{-1} (M/L)^{-1} \tag{4}$$

(c) One crucial assumption is that the galaxies are virialized. This explains roughly the slopes. The small scatter is still not well understood. It requires, at least, similar environments for the galaxies.

Note, also, that one assumption that comes in the derivation of the FP relation is **homology**, which basically states that all the constants hidden in the $\propto$ are the same for all galaxies, and do not depend on M. See lecture notes 12 for more details. Of course, the real galaxies are not a homologous family, which gives a tilt in the relation.

(b) This can be used as a distance indicator (e.g. getting the luminosity distance once one has the velocity dispersion and the radius).

3 Central MBH of the Andromeda Galaxy

(a) First, we note that 700 kpc is at a small enough distance that we can neglect the non-Euclidean component of the angular size distance. We compute $\theta \leq 0.5$ arcsec $\times \pi/(180 \times 60 \times 60) = 2.4 \times 10^{-6}$ radians. So that $r = \theta\, d$

= 1.69 pc. Near the MBH, the orbits will be nearly Keplerian and circular. Thus,

$$\frac{GM}{r^2} = v^2/r$$

It is appropriate to take the velocity v as approximated by the dipersion velocity. We see that velocity increases montonically with decreasing radius. So a lower limit on the velocity at the large radii will set a lower limit on the implied mass internal to the test particle. Thus

$$M > v^2 r/G = 6.2 \times 10^7 M_\odot$$

(b) We now do part a) in reverse. The BEL region will be of size, $r = GM/v^2 = 5.3 \times 10^{16}$ cm $= 1.7 \times 10^{-8}$ Mpc. HST can resolve lengths larger than $l = 20$ Mpc $\times 4.8 \times 10^{-7} = 9.7 \times 10^{-6}$ Mpc. VLBI can resolve lengths larger than $l = 200$ Mpc $\times 4.8 \times 10^{-7} = 9.7 \times 10^{-10}$ Mpc. Thus HST cannot resolve the BEL but VLBI surely can.

4 Eddington Luminosity

The Eddington luminosity is the limit when radiation pressure equals the gravitational pull on a particle. If $L > L_{edd}$, the body is not gravitionally stable.

The radiation force on a particle is equal to the flux divided by the speed of light (pressure) times the effective area, given by the Thomson cross-section, $\sigma_T \approx (e^2/m_e c^2)^2$. Equating that to the gravitational force on a proton (considering that the gas is composed mostly of hydrogen),

$$\frac{L\sigma_T}{4\pi r^2 c} = \frac{GM_{BH} m_p}{r^2} \Rightarrow L_{edd} = \frac{4\pi G M_{BH} m_p c}{\sigma_T}.$$

Now, in the case of accretion, the radiated energy is a fraction of the energy being accreted as mass, meaning $L = \eta \dot{M} c^2$. If the luminosity is the Eddington value, then

$$\eta \dot{M} c^2 = \frac{4\pi G M m_p c}{\sigma_T}$$

$$\text{therefore} \quad M(t) = M_0 e^{t/\tau}$$

where $\tau = (\eta c \sigma_T)/(4\pi G m_p) = 47$ Myr is the characteristic timescale.

We took an efficiency $\eta = 0.1$ for this calculation, as was stated in the problem.

5 Cosmic Far-Inrared Background

(a) Number of starbursts

The total energy emitted by a starburst, of luminosity L in its restframe, lasting for a (retsframe) time Δt is equal to $< L > \Delta t = 3.6 \times 10^{60}$ erg.

This energy is emitted as radiation, i.e. photons. Let us suppose to simplify that all those photons have the same frequency. When we observe them today, the number of photons is still the same, but their frequency has redshifted by a factor $(1 + z)$, so the observed energy of a starburst today is $(1 + z)^{-1}$ of that, i.e. $E_{starburst} \approx 1 \times 10^{60}$ erg.

To account for the energy density of the cosmic far-infrared background observed today, we thus need a comoving density of starbursts of

$$n_{starburst} = u_{CIRB}/E_{starburst} \approx 7 \times 10^{-75} \text{ starbursts.cm}^{-3} \quad (5)$$

$$= 3 \times 10^{7} \text{ starbursts.Gpc}^{-3} \quad (6)$$

We can estimate the volume of the observable universe in an Einstein- de Sitter model : $V = \frac{4\pi}{3}(2c/H_0)^3 = 7 \times 10^3$ Gpc^3 (taking $H_0 = 50 km/s/Mpc$). We thus get the total number of starbursts as

$$N \approx 2 \times 10^{11} \text{ starbursts.}$$

(b) Comoving number density

We found the comoving density in the previous question :

$$n_{starburst} = 3 \times 10^{7} \text{ starbursts.Gpc}^{-3} \quad (7)$$

Estimating about 10^{11} galaxies in the universe, the corresponding number density is $n_{gal} = 1.4 \times 10^7$ Gpc^{-3}, which is of the same order of magnitude of the number density of starbursts needed to account for the CIRB.

(c) He and metals

Each nucleon (proton or neutron) corresponds to 7 MeV. The number of nucleons, then, is the total energy emitted divided by that value. That means 3.2×10^{65} nucleons, or $M = 5.4 \times 10^{41}$ g, per starburst. The metal yield is a fifth of that, so $M_{met} = 1.1 \times 10^{41}$ g, per starburst.

The final metallicity is the metal mass divided by the total mass of the galaxy, $10^{11} M_{\odot}$:

$$Z = 0.005.$$

6 Cosmic X-Ray Background

(a) Energy density

To determine the energy density from the intensity,

$$u = \frac{4\pi}{c}\nu I_\nu = 1.3 \times 10^{-16} \text{ erg cm}^{-3}.$$

Comparing to the CMBR energy density, $u_{CMBR} = aT^4 = 3.2 \times 10^{-13}$ erg cm^{-3}, we notice that the CXBR is about 3 orders of magnitude fainter.

(b) Supermassive black holes

We can use the same number for the comoving number density of galaxies as we found in the previous problem, $n_{gal} = 1.4 \times 10^7$ Gpc^{-3}. If the efficiency was 10%, then the observed energy density today will be

$$u = n_{gal} \cdot \frac{1}{1+z}\eta M_{bh} c^2 = 2 \times 10^{-16} \text{ erg cm}^{-3},$$

which is of the same order of magnitude than the observed CXBR and smaller than the CMBR. Keep in mind, however, that not all energy is emitted in x-rays, and our number are only an approximation.

7 Quasar at $z = 2$

(a) Luminosity distance

To compute the luminosity distance,

$$D_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}.$$

For an Einstein-de Sitter universe, $\Omega = \Omega_m = 1$, and $E(z) = (1+z)^{3/2}$. So

$$D_L(z) = (1+z)\frac{2c}{H_0}\left[1 - \frac{1}{\sqrt{1+z}}\right].$$

For the values given, $D_L(z = 2) = 15.2$ Gpc.

(b) Magnitudes

If we use the Sun as reference for the magnitude system,

$$M - M_\odot = -2.5 \log \left[\frac{L}{L_\odot} \right]$$
$$M_\odot = 4.8,\ L = 5 \times 10^{12} L_\odot \Rightarrow M = -26.9.$$

For the apparent magnitude,

$$m - M = 5 \log \left(\frac{D_L}{10 \text{ pc}} \right)$$
$$D_L = 15.2 \text{ Gpc} \Rightarrow m = 19.0.$$

(c) X-ray flux

Computing the observed flux,

$$F = 0.3 \cdot \frac{L}{4\pi D_L^2} = 2.0 \times 10^{-13} \text{ erg s}^{-1} \text{ cm}^{-2}.$$

(d) CXRB

The total number will be the total CXRB flux divided by the flux from an individual quasar:

$$N = \frac{4\pi \nu I_\nu}{F} = 1.8 \times 10^7 \text{ quasars}.$$

We see that this is a much smaller number than what we found for the supermassive black holes in the previous problem. To compute the projected surface density, divide this number by the solid angle of a whole sphere (in degrees, that is $(360^o)^2/\pi = 41253$ deg^2. That yields approximately 440 quasars per square degree in the sky.