QUANTUM GRAVITY – AN UNFINISHED REVOLUTION¹

Claus Kiefer

University of Cologne, Faculty of Mathematics and Natural Sciences, Institute for Theoretical Physics, Cologne, Germany

Abstract

It is generally assumed that the search for a consistent and testable theory of quantum gravity is among the most important open problems of fundamental physics. I review the motivations for this search, the main problems on the way, and the status of present approaches and their physical relevance. I speculate on what the situation could be in 2050.

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1 Present understanding and applications

1.1 The mystery of gravity

Already one year after the completion of his theory of general relativity, Albert Einstein predicted the existence of gravitational waves from his new theory. At the end of his paper, he wrote:²

... the atoms would have to emit, because of the inner atomic electronic motion, not only electromagnetic, but also gravitational energy, although in tiny amounts. Since this hardly holds true in nature, it seems that quantum theory will have to modify not only Maxwell's electrodynamics, but also the new theory of gravitation.

Thus already in 1916 Einstein envisaged that quantum theory, which at that time was still in its infancy, will have to modify his newly developed theory of relativity. More than hundred years later, we do not have a complete quantum theory of gravity. Why is that and what are the prospects for the future?

Gravitation (or simply gravity) is the oldest of the known interactions, but still the most mysterious one. It was Isaac Newton's great insight to recognize that gravity is responsible for the fall of an apple as well as for the motion of the Moon and the planets. In this way, he could unify astronomy (hitherto relevant for the region of the Moon and beyond) and physics (hitherto relevant for the sublunar region) into one framework. In the Newtonian picture as presented in his *Principia* from 1687, gravity is understood as action at a distance: any two bodies in the Universe attract each other by a force which is inversely proportional to the square of their distance (see Appendix). For this, he had to introduce the so far unknown concepts of *absolute space* (which has three dimensions) and *absolute time* (which has one dimension). These entities exist independent of any matter, for which they act like a fixed arena that cannot be reacted upon by the dynamics of matter. Newton's discovery marked the beginning of modern celestial mechanics, which allowed the study of the motion of planets and other astronomical bodies with unprecedented accuracy.

The strength of the gravitational force between two bodies is proportional to their *masses*. Masses can only be positive, in constrast to electric charges, which can be both positive and negative. This difference is the reason why charges can attract each other (if they, unlike the masses in gravity, differ in sign) as well as repel each other (if they have the same sign). For elementary particles, mass, by which we mean rest mass, is an intrinsic property (the same holds for charge).

²This is my translation from the German. The original reference can be found in Kiefer (2012), p. 26.

There can also exist particles with zero mass, of which the only observed one is the photon; such particles must propagate with the speed of light *c*. Elementary particles are also distinguished by their intrinsic angular momentum (spin), by which they can be divided into bosons (having integer spin) and fermions (having half-integer spin).

Newton's theory of gravity was superseded only with the advent of general relativity in 1915. It was Einstein's great insight to recognize that gravity can be understood as representing the *geometry* of space and time as unified to a fourdimensional entity called spacetime. In this way, gravity acquires its own dynamical local degrees of freedom. Spacetime then no longer plays the role of a fixed background acting on matter, but takes itself part in the game and can be reacted upon – both by matter and by itself. Gravity itself creates a gravitational field as is reflected by the non-linear nature of Einstein's field equations (see Appendix). That gravity possesses its own degrees of freedom can best be seen by the existence of gravitational waves, which propagate with the speed of light and which were detected directly for the first time by the laser interferometers of the LIGO collaboration in 2015. That gravity (and thus spacetime) is fully dynamical is also called *background independence*.

Gravity is very weak. The gravitational attraction between, say, electron and proton in a hydrogen atom is about 10^{40} times smaller than their electric attraction. A metallic body can be prevented from falling to the massive Earth by holding it with a small magnet. Still, because masses are only positive, it is the dominating force for the Universe at large scales, because positive and negative electric charges, being present in roughly equal amounts, average to zero at those scales.

Newton had carefully distinguished between gravity (interaction between bodies) and inertia (resistance of bodies to changes in their momenta). These two concepts are unified in Einstein's theory as expressed by the equivalence principle. The geometry of spacetime thus leads in appropriate limits to the traditional gravitational interaction as well as to inertial forces such as centrifugal or Coriolis forces.

Gravity is of a universal nature. Everything in the world is in spacetime and is thus subject to its geometry, that is, to gravity. So far, Einstein's theory successfully explains all observed gravitational effects from everyday life (e.g. the working of the GPS) to the Universe as a whole. Figure 1 presents a famous photograph showing galaxies at distances that cover cosmic scales in space and in time – because light propagates at the finite speed c we see these galaxies in a very early state of their evolution, billions of years ago. Astronomers measure cosmic scales in Megaparsec (Mpc) and Gigaparsec (Gpc). In conventional units, 1 Mpc $\approx 3.09 \times 10^{22}$ metres (m), and 1 Gpc is thousand Mpc. The size of the observable

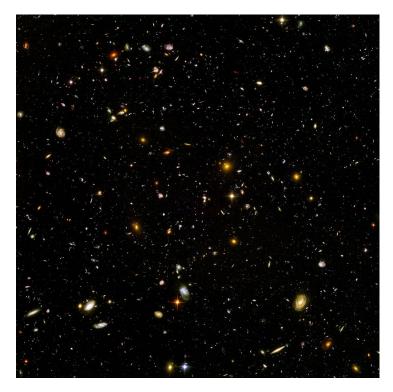


Figure 1: A glimpse into the macroscopic world: the *Hubble Ultra Deep Field*, a photograph taken from September 2003 to January 2004 in a small celestial region in the constellation Fornax. Figure credit: NASA and the European Space Agency.

Universe is estimated to be about 14 Gpc.³

Strictly speaking, there are two features for which it is presently open whether they can be fully accommodated into Einstein's theory or not: Dark Matter and Dark Energy. The two can only be observed by their gravitational influence; Dark Matter exhibits the same clumpiness as visible matter (and exhibits itself, for example, in the rotation curves of galaxies), but Dark Energy is of a homogeneous and repulsive nature and is responsible for the present accelerated expansion of our Universe (as measured by observing supernovae at increasing distances). Some scientists speculate that new physics is needed to account for Dark Matter and Dark Energy, but at present this is far from clear.

General relativity is what one calls a classical, that is non-quantum, theory. Our current theories for the other interactions are all *quantum* theories or, more

³This is the so-called particle horizon: the distance in today's Universe up to which we can see objects, that is, the distance over which information (basically in the form of electrodynamic or gravitational waves) had enough time since the Big Bang to reach us. The age of our Universe is estimated to be about 13.8 billion years.

precisely, these interactions are described within a quantum framework, which uses concepts drastically different from classical physics. For example, whereas classical mechanics makes essential use of *trajectories* for bodies, the equations of which are determined by their initial positions and momenta, quantum mechanics no longer contains such trajectories in its mathematical description.⁴ It instead features wave functions Ψ from which observable quantities such as energy values for spectra and interference patterns of particles can be obtained. The relation to positions, momenta (and other classical concepts) proceeds via the probability interpretation, and the limits can be expressed by the indeterminacy (or uncertainty) relations. The quantum-to-classical transition can be understood and experimentally studied using the concept of decoherence (Joos *et al.* 2003, Schlosshauer 2007). The quantum framework and formalism seems to be of universal nature.

Quantum theory is usually applied in the realm of microphysics. This is the world of molecules, atoms, nuclei, and elementary particles. Quantum theory thus lies at the basis not only of physics, but also of chemistry and biology. The smallest scales investigated experimentally so far are the scales explored by particle accelerators such as the Large Hadron Collider (LHC) at CERN. Figure 2 shows a glimpse into these smallest scales – the decay of the Higgs particle into other particles. Such "microscopic" pictures are far more abstract than photos of the kind shown in Fig. 1; a great amount of theoretical insight is involved to construct them.

These smallest explored scales are of the order of 10^{-18} m. Comparing it to the above cosmic scale of 14 Gpc, which is about 4×10^{26} m, we see that this corresponds to a difference of about 44 orders of magnitude.⁵

The non-gravitational degrees of freedom are described by the Standard Model of particle physics. It provides a partial unification (within the framework of gauge theories) of strong, weak, and electromagnetic interaction. The Standard Model is a quantum *field* theory, that is, a quantum theory with infinitely many degrees of freedom. So far, there are no clear hints for physics beyond the Standard Model. For theoretical reasons, one expects a unification of interactions at high energies. Some approaches to unification make use of supersymmetry (SUSY) in which fermions and bosons are fundamentally connected. Despite intensive search at the LHC, no evidence for SUSY was found.

Particle physicists measure energies in electron volts (eV). For high energies, one uses Megaelectronvolts (MeV), $1 \text{ MeV} = 10^6 \text{ eV}$, Gigaelectronvolts (GeV), $1 \text{ GeV} = 10^3 \text{ MeV}$, and Terraelectronvolts (TeV), $1 \text{ TeV} = 10^3 \text{ GeV}$. The

⁴The trajectories that appear in the so-called de Broglie–Bohm interpretation of quantum theory are of a non-classical nature.

⁵It is interesting to see that the geometric mean of the largest and the smallest explored distance corresponds to about 10 kilometres, which is an everyday scale.

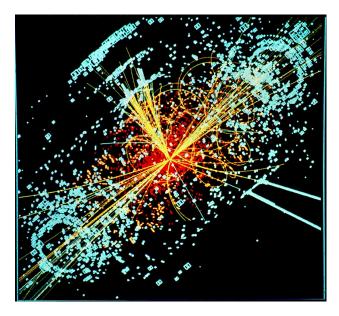


Figure 2: A glimpse into the microscopic world: simulation of the hypothetical decay of a Higgs particle into other particles at the detector CMS at CERN. Figure credit: Lucas Taylor / CERN - http://cdsweb.cern.ch/record/628469 (Creative Commons License).

LHC reaches a collision energy of 13 TeV. Because of Einstein's famous relation $E = mc^2$, masses can be measured in eV over c^2 . The proton mass is about 938 MeV/ c^2 , and the mass of the famous Higgs particle discovered at the LHC in 2012 is about 125 GeV/ c^2 .

The fields in the Standard model all carry energies and thus generate a gravitational field. Because they are quantum fields, they cannot be inserted directly into the classical Einstein field equations. Only a consistent unification of gravity with quantum theory can describe the interaction of all fields at the fundamental level.

1.2 What are the main problems?

What do we mean when we talk about quantum gravity? Unfortunately, this term is not used in a consistent way. Here, we call quantum gravity any theory (or approach) in which the superposition principle is applied to the gravitational field.

The superposition principle is at the heart of quantum theory: for any physical states of a system (described e.g. by wave functions Ψ and ϕ), any linear combination $\alpha \Psi + \beta \phi$, where α and β are complex numbers, is again a physical state. This principle is confirmed by an uncountable number of experiments. For more than one system it leads to entanglement between systems, which is relevant for

atoms (e.g. the qubits used in quantum information), for particles (e.g. neutrino oscillations), and many other cases.

Now, because gravity couples universally to all degrees of freedom, this should entail also a superposition of different gravitational fields, for which a quantum theory of gravity is needed. At a famous conference at Chapel Hill (US) in 1957, Richard Feynman explained this by a gedanken experiment, see Fig. 3.

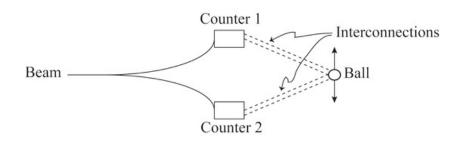


Figure 3: Stern–Gerlach type of gedanken experiment, in which the detectors for spin up respective spin down are coupled to a macroscopic ball. If the particle has spin right, which corresponds to a superposition of spin up and down, the coupling leads to a superposition of the ball being moved up and down, leading to a superposition of the corresponding gravitational fields. Figure adapted from DeWitt and Rickles, p. 251, see DeWitt (1957).

In this, the superposition of microscopic states (e.g electron spins) is transferred to the spatial superposition of a macroscopic ball, for which the gravitational field is measurable. But how do we describe the gravitational field of an object which is in a spatial superposition at different locations? Only a theory of quantum gravity can achieve this. There exist attempts to realize superpositions à la Feynman in the laboratory; see, for example, Bose *et al.* (2017) and Marletto and Vedral (2017). Whether this is possible and whether one can draw conclusions on quantum gravity from this, is currently subject of discussion.

There are other reasons in favour of the search for quantum gravity. As already mentioned above, if one aims at a unification of all interactions in Nature (a "theory of everything" or TOE), one has to accommodate gravity into the quantum framework, since the quantum fields of the non-gravitational interactions act as sources for gravitational field. One may, of course, envisage in principle a unified theory in which gravity stays classical. But there are at least two reasons that speak against this possibility. First, it is not very satisfactory to have such a fundamental hybrid theory. Second, there are various counter-arguments from the observational point of view against some hybrid theories, see e.g. Kiefer (2012) for details. But there exist no logical arguments that would force the quantization of gravity, and hybrid theories can indeed be constructed (Albers *et al.* 2008). Einstein's theory, by itself, is incomplete. One can prove singularity theorems which state that, given some assumptions, there are regions in spacetime where the theory breaks down (Hawking and Penrose 1996). Concrete examples include the regions inside black holes and the origin of our Universe ("time zero"). Only a more general theory, such as a quantum theory of gravity, may be able to resolve these singularities and thereby allow a full description of black holes and the Universe.

There is also another kind of singularities. Quantum field theories are plagued by divergences which arise from probing spacetime at arbitrarily small scales, leading (by the indeterminacy relations) to momenta and energies of arbitrarily high values. On paper, these "infinities" can be handled by regularization and renormalization. Regularization means that divergent expressions can be made finite by a mathematical procedure of "isolating" the divergences (infinities). Renormalization means that the isolated divergences can be absorbed in physical parameters of the theory. These parameters cannot, of course, be calculated from the theory, but can only be determined empirically. The paradigmatic example is quantum electrodynamics (QED) and the parameters swallowing the infinities are the electric charge and the mass of the electron. Once this is done for finitely many parameters (typically a small number), the theory becomes predictive. While this procedure is consistent and can be successfully applied to the Standard Model, the question arises whether a fundamental theory including gravity is finite by construction, that is, whether no divergences occur in the first place. Perhaps the root for both types of singularities (gravitational and quantum field theoretical) lies in the assumed continuum nature of spacetime.

Before we embark on a brief discussion of the main approaches, let us address the physical scales where we definitely expect quantum effects of gravity to become relevant (due to the universality of the superposition principle, such effects can, in principle, become relevant at any scale).

In the most recent version of the Système International d'unités (SI), which is valid since 2019, physical units are based as much as possible on fundamental constants.⁶ In this, Planck's constant h, the speed of light (c), and the electric charge (e) are attributed fixed values. The units metre (m) and kilogram (kg) can then be inferred from h and c, while the second (s) is determined from atomic spectra. For us, h and c are relevant:

$$c = 299\,792\,458\,\frac{\mathrm{m}}{\mathrm{s}},\tag{1}$$

$$h = 6.626070040 \times 10^{-34} \,\mathrm{J} \cdot \mathrm{s},$$
 (2)

The gravitational constant G is known with much lower accuracy. On the NIST Reference on Constants, Units, and Uncertainty, one finds the following 2018

⁶See, e.g., Hehl and Lämmerzahl (2019) for a thorough discussion.

value for G:

$$G = 6.67430(15) \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}.$$
 (3)

It thus cannot serve the same purpose as h and c (otherwise, we could base our time unit on G). Einstein's theory also contains the cosmological constant Λ , which has dimension of an inverse squared length. From current observations one finds the value

$$\Lambda \approx 1.2 \times 10^{-52} \,\mathrm{m}^{-2} \approx (0.35 \,\mathrm{Gpc})^{-2},$$
 (4)

which, however, is not precise enough for using Λ as a standard of units.

The three constants G, h (resp. $\hbar = h/2\pi$), and c provide the relevant scales for quantum gravity, because one can construct from them (apart from numerical factors) unique expressions for a fundamental length, time, and mass (or energy). Because Max Planck had formulated them already in 1899, they are called Planck units in his honour. The Planck length reads

$$l_{\rm P} := \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \,\mathrm{m},$$
 (5)

the Planck time is

$$t_{\rm P} := \frac{l_{\rm P}}{c} = \sqrt{\frac{\hbar G}{c^5}} \approx 5.391 \times 10^{-44} \,\mathrm{s},$$
 (6)

and the Planck mass is

$$m_{\rm P} := \frac{\hbar}{l_{\rm P}c} = \sqrt{\frac{\hbar c}{G}} \approx 2.176 \times 10^{-8} \,\mathrm{kg} \approx 1.22 \times 10^{19} \,\mathrm{GeV}/c^2,$$
 (7)

from which one can derive the Planck energy

$$E_{\rm P} := m_{\rm P} c^2 \approx 1.22 \times 10^{19} \,\,{\rm GeV} \approx 1.96 \times 10^9 \,\,{\rm J} \approx 545 \,\,{\rm kWh}.$$
 (8)

Whereas Planck length and Planck time are far remote from everyday (and experimentally accessible) scales, Planck mass (energy) seems to be of a more everyday nature. The point, however, is that the Planck mass is more than 10^{19} times the proton mass $m_{\rm pr}$ and more than 10^{15} times the maximal collision energy attainable at the LHC. This means that to generate particles with masses of order the Planck mass or higher, one needs to construct an accelerator with galactic dimensions. This is one of the most important problems in the search of quantum gravity: we cannot probe the Planck scale directly by experimental means.

The size of structures in the Universe is determined by the squared ratio of proton mass and Planck mass, sometimes called the "finestructure constant of gravity",

$$\alpha_{\rm g} := \frac{Gm_{\rm pr}^2}{\hbar c} = \left(\frac{m_{\rm pr}}{m_{\rm P}}\right)^2 \approx 5.91 \times 10^{-39}.\tag{9}$$

It is the smallness of this ratio that is responsible for the usual smallness of quantum-gravitational effects in astrophysics. It is an open question whether this number can be calculated from a fundamental theory or whether it remains un-explained as a phenomenological parameter that can only be determined from observations.

1.3 What are the main approaches and applications?

Before addressing the full quantization of gravity, it is appropriate to have a brief look at what is known about the relation between quantum theory and classical gravity.⁷

The relation between quantum *mechanics* (quantum theory with finitely many degrees of freedom) and gravity is studied by using the Schrödinger (or Dirac) equation in a Newtonian gravitational field. This is the regime where experiments are available, for example by observing interference fringes of neutrons or atoms. The combination of quantum *field* theory (QFT) with general relativity ("QFT in curved spacetime") is much more subtle. The perhaps most famous prediction there is that black holes are, in fact, not black but radiate with a thermal spectrum. This effect was derived from Stephen Hawking in 1974 and is called Hawking radiation. The temperature of a black hole is given by

$$T_{\rm BH} = \frac{\hbar\kappa}{2\pi k_{\rm B}c},\tag{10}$$

where κ is the surface gravity characterizing a stationary black hole. Within Einstein–Maxwell theory (coupled gravitational and electrodynamical fields), one can prove the *no-hair theorem* for stationary black holes: they are uniquely characterized by the three parameters mass (M), electric charge (Q), and angular momentum (J). Astrophysical black holes are described by the two parameters M and J (Kerr solution).

For a spherically-symmetric (Schwarzschild) black hole with mass M, the Hawking temperature is

$$T_{\rm BH} = \frac{\hbar c^3}{8\pi k_{\rm B} GM} \approx 6.17 \times 10^{-8} \left(\frac{M_{\odot}}{M}\right) \,\mathrm{K}.\tag{11}$$

The smallness of this value means that this effect cannot be observed for astrophysical black holes, which have a mass of at least three solar masses $(3M_{\odot})$.

Figure 4 shows an example of an observed black hole – a supermassive black hole with $M \approx 6.5 \times 10^9 M_{\odot}$ in the centre of the galaxy M87. For such black holes, the Hawking effect is utterly negligible.

⁷References on this and the following sections can be found e.g. in Kiefer (2012). See also Carlip (2001) and Woodard (2009) for general accounts of quantum gravity.

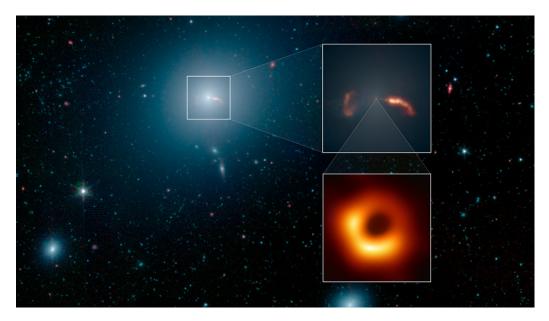


Figure 4: Shadow of the supermassive black hole in the centre of the bright elliptical galaxy M87. For this and all other black holes observed so far, only a consistent quantum theory can explain what happens in their inside regions. Image credit: NASA, JPL-Caltech, Event Horizon Telescope Collaboration.

There is, in fact, an analogue of the Hawking effect in flat (Minkowski) spacetime. An observer moving with constant acceleration *a* through the standard vacuum state of flat spacetime experiences a bath of thermally distributed particles with "Unruh temperature"

$$T_{\rm U} = \frac{\hbar a}{2\pi k_{\rm B}c} \approx 4.05 \times 10^{-25} \ a \left[\frac{\rm m}{\rm s^2}\right] \, {\rm K}.$$
 (12)

One immediately recognizes the similarity with (10), with *a* replaced by κ . The reason for the appearance of this temperature is the fact that there is no unique vacuum (and thus no unique particle concept) for non-inertial observers in flat spacetime.

If black holes have a temperature, they also have an entropy, which is given by the "Bekenstein–Hawking expression"

$$S_{\rm BH} = \frac{k_{\rm B}Ac^3}{4G\hbar} \equiv \frac{k_{\rm B}A}{(2l_{\rm P})^2},\tag{13}$$

where A denotes the area of the black hole's event horizon. In the Schwarzschild case, we can express the entropy as

$$S_{\rm BH} \approx 1.07 \times 10^{77} k_{\rm B} \left(\frac{M}{M_{\odot}}\right)^2.$$
 (14)

 $S_{\rm BH}$ is indeed much greater than the entropy of the star that collapsed to form the black hole. The entropy of the Sun, for example, is given approximately by $S_{\odot} \approx 10^{57} k_{\rm B}$, whereas the entropy of a solar-mass black hole is about $10^{77} k_{\rm B}$, which is twenty orders of magnitude larger. All the above expressions contain the fundamental units c, G, \hbar and thus point towards the need for constructing a quantum theory of gravity. Such a theory should be able to provide a microscopic interpretation of the entropy formula (13).

Besides black holes, quantum effects are also important in cosmology. Assuming that the Universe underwent an (almost) exponential expansion at a very early state (a phase called *inflation*), density perturbations of matter and gravity (gravitons, see below) are generated out of quantum vacuum fluctuations. All the structure in the Universe (galaxies and clusters of galaxies) is believed to arise from these perturbations. The power spectrum of these density perturbations (also called "scalar modes") can be derived to read

$$\mathcal{P}_{\rm S} = \frac{1}{\pi} \left(t_{\rm P} H \right)^2 \epsilon^{-1} \approx 2 \times 10^{-9},$$
 (15)

where ϵ is a 'slow-roll parameter' that is peculiar to the chosen model of inflation, and H is the Hubble parameter (expansion rate) of the Universe during inflation. One recognizes the explicit appearance of the Planck time $t_{\rm P}$, Eq. (6), in this formula. The power spectrum of these density fluctuations is recognized in the anisotropies of the cosmic microwave background (CMB) radiation, see Fig. 5. The number 2×10^{-9} on the right-hand side of (15) comes from observations.

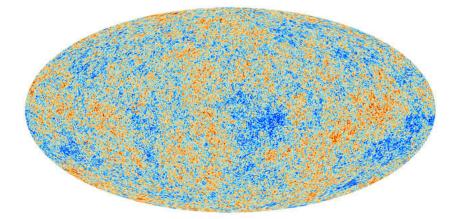


Figure 5: Anisotropy spectrum of the Cosmic Microwave Background (CMB). Image credit: ESA/PLANCK Collaboration

All what has been said so far points towards the need for a quantum theory of gravity. But how can such a theory be constructed? The first attempts date back

to work done in 1939 by Léon Rosenfeld, who was then an assistent to Wolfgang Pauli in Zürich. In two papers, he pioneered two approaches: the 'covariant approach' and the 'canonical approach'. Both approaches aim at the construction of a quantum version of general relativity. What is the status of these approaches?

The covariant approach has its name from the fact that a four-dimensional (covariant) formalism is employed throughout. In most cases, this formalism makes use of path integrals (in which, according to the superposition principle, four-dimensional spacetimes are summed over), see the Appendix. Similar to the photon in quantum electrodynamics, a particle is identified as the mediator of the quantum-gravitational field, the *graviton*. It is massless, but has spin 2 (whereas the photon has spin 1). That it is indeed massless is indirectly confirmed by the detection of gravitational waves - they move with speed of light *c*. From this, the LIGO and Virgo collaborations report a limit of the graviton mass $m_{\rm g} \leq 7.7 \times 10^{-23}$ eV. As remarked above, gravitons (also called "tensor modes") are generated from the vacuum during an inflationary phase of the early Universe. Similar to the density spectrum (15), one can derive for them the power spectrum

$$\mathcal{P}_{\mathrm{T}}(k) = \frac{16}{\pi} \left(t_{\mathrm{P}} H \right)^2.$$
(16)

A central quantity is the ratio between tensor and scalar modes,

$$r := \frac{\mathcal{P}_{\mathrm{T}}}{\mathcal{P}_{\mathrm{S}}} = 16\epsilon.$$
(17)

So far, no observations have indicated a non-vanishing value for r. Observing such a value would constitute a direct test of quantum gravity at the linearized level.

As all relevant quantum field theories, also the covariant quantization of general relativity exhibits divergences. But there is a major difference to the situation in the Standard Model. Whereas the perturbation theory for the Standard Model is renormalizable, this does not apply for gravity. It is thus *not* possible to absorb divergent terms into a finite number of observable parameters; at each order of the perturbation theory, new types of divergences appear, and one would need infinitely many parameters to absorb them, rendering the theory useless. But the question arises whether higher terms in the perturbation expansion are indeed relevant. They come in powers of the parameter

$$\frac{GE^2}{\hbar c^5} \equiv \left(\frac{E}{m_{\rm P}}\right)^2 \sim 10^{-32},\tag{18}$$

where E is the relevant observation energy, here taken to be 14 TeV, the energy of the planned LHC-upgrade. This is a very small parameter, so perturbation

theory should in principle be extremely accurate. One could thus adopt the point of view that quantum general relativity is an *effective field theory* only, that is, a theory that is anyway valid only below a certain energy and must be replaced by a more fundamental, potentially renormalizable or finite theory above that energy. An approach that makes use of standard quantum field theory up to the Planck scale is *asymptotic safety*. In this, G and Λ are not constants, but (as is typical for quantum field theory) variables that depend on energy. They may approach non-trivial fixed points in the limit of high energy and thus lead to a viable theory of quantum field theory at all scales. It is imaginable that the scale dependence of G could mimic Dark Matter; in this case, it would be hopeless to look for new particles as constitutes of Dark Matter.

To calculate quantum-gravitational path integrals is far from trivial and definitely not possible analytically. For this reason, computer methods are heavily used. One promising approach is *dynamical triangulation* which bears this name because the spacetimes to be summed over in the path integral are discretized into tetrahedra. This leads to interesting results about the possible microstructure of spacetime.

One candidate for a finite quantum field theory of gravity is supergravity, which combines SUSY with gravity more precisely; more precisely, a particular version called N = 8 supergravity. Heroic calculations over many years have shown that there are no divergences in the first orders of perturbation theory. Whether this continues to hold at higher orders and, moreover, whether this holds at all orders, is far from clear. Only a new, so far unknown, principle can be responsible for this theory to be finite.

A candidate for a finite theory of quantum gravity of a very different nature is superstring theory (or M-theory). In the limit of small energy, the above covariant perturbation theory is recovered, but at higher energies, string theory is of a very different nature. Actually, its fundamental entities are not only onedimensional entities as the name suggests, but higher-dimensional objects such as branes. Moreover, the theory makes essential use of a higher-dimensional spacetime (with 10 or 11 as the number of dimensions). The theory is not a direct quantization of gravity – quantum gravity appears only in certain limits as an emergent theory. In contrast to theories of quantum general relativity, string theory has the ambition to provide a unified quantum theory of all interactions (the TOE mentioned above). Such a theory should also allow to understand the origin of mass in Nature. One aspect of this is the hierarchy problem. We observe widely separated mass scales – neutrino masses ($\sim 0.01 \text{ eV}$), electron mass ($\sim 0.5 \text{ MeV}$), and topquark mass (≈ 173 GeV), all of which are much smaller than the Planck mass (7). So far, the origin of this hierarchy is not understood. It is not clear whether there is new physics between the Standard Model energy scale (as exemplified by the Higgs and the top-quark mass) and the Planck scale.

Out of string theory and the discussion of black holes grew insights about a possible relation between quantum-gravity theories and a class of field theories called conformal quantum field theories. The latter are defined on the boundary of the spacetime region in which the former are formulated. This is known as gauge/gravity duality, holographic principle, or AdS/CFT conjecture (see e.g. Maldacena 2011). Some claim that it will play a fundamental role in a full theory of quantum gravity.

The alternative to covariant quantization is the canonical (or Hamiltonian) approach. The procedure is here similar to the procedure in quantum mechanics where one construct quantum operators for positions, momenta, and other variables. This includes also the quantum version of the energy called Hamilton operator. In quantum mechanics, the Hamilton operator generates time evolution by the Schrödinger equation. In quantum gravity, the situation is different. Instead of the Schrödinger equation, one has constraints - the Hamiltonian (and other functions) are constrained to vanish. This is connected with the disappearance of spacetime at the fundamental level. Spacetime in general relativity is the analogue of a particle trajectory in mechanics; so after quantization spacetime disappears in the same way as the trajectory disappears (recall the indeterminacy relations) only space remains. This is sometimes referred to as the "problem of time", although it is a direct consequence of the quantum formalism as applied to gravity. It is connected with the fact that already the classical theory has no fixed background, so there is no such background available to serve for the quantization of fields - different from the situation with the non-gravitational quantum fields of the Standard Model. Background independence is one of the main obstacles on the route to quantum gravity.

If one uses the standard metric variables of Einstein's theory, one arrives at quantum geometrodynamics with the Wheeler-DeWitt equation as its central equation. Due to mathematical problems, the full equation remains poorly understood, but it can be applied to problems in cosmology and for black holes. An alternative formulation makes use of variables that show some resemblance with the gauge fields used in the Standard Model. It is known under the name loop quantum gravity. At the kinematic level (before the constraints are imposed), it is well understood, but the exact construction of the Hamiltonian constraints and the recovery of quantum field theory in curved spacetime present problems. Applications of loop quantum gravity also include cosmology and black holes.

An important feature of the Wheeler-DeWitt approach is the possibility of building a bridge (at least at a formal level) from quantum gravity to quantum field theory in curved spacetime. In this way, spacetime (and, in particular, time) emerges as an approximate concept. This procedure is similar to the recovery of geometric optics ("light rays") from fundamental wave optics. In this, the separation of scales (the separation of Planck mass from masses of the Standard Model)

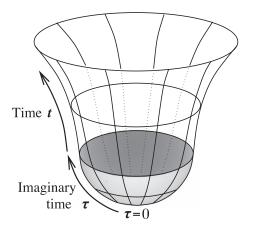


Figure 6: Hartle–Hawking instanton: the dominant contribution to the Euclidean path integral is assumed to be from half of a four-sphere attached to a part of de Sitter space. From Kiefer (2012).

is crucial.⁸ This emergence of time can also be described in the covariant approach. Using the method of path integrals, Hartle and Hawking have constructed a certain four-dimensional geometry that elucidates the emergence of time by attaching a Euclidean ('timeless') geometry to a Lorentzian one. This "Hartle–Hawking" instanton is shown in Fig. 6. It is frequently discussed in the application of quantum gravity to cosmology (quantum cosmology).

The Wheeler-DeWitt equation has a very peculiar structure. It is asymmetric with respect to the size of three-dimensional space and may thus allow to understand the origin of the arrow of time from fundamentally timeless quantum gravity (Zeh 2007): there is an increase of entropy with increasing size of the Universe.

Besides the approaches already mentioned, there are a variety of others, and only space prevents me from discussing them in more detail. Many of these other theories make use of a discrete structure, either fundamentally imposed or derived from other principles. The reader may wish to consult Oriti (2009) for more details.

⁸A review of this and other conceptual issues can be found in Kiefer (2012, 2013) and the references therein.

2 Challenges and opportunities in the Horizon 2050

2.1 Theoretical challenges and opportunities

What can or should we expect in the coming decades? Physics is an experimental science. There can only be progress if we have testable predictions that can falsify a given approach and discriminate between different approaches. To derive such predictions is one of the main theoretical challenges.

It makes sense to distinguish between predictions at the linearized level and at the full level. The linearized level of quantum general relativity also follows from unified theories such as superstring theory, so tests at that level are very general. Looking at atomic physics, one can calculate the transition rate from an excited state to the ground state by emission of a graviton. In one example (Kiefer 2012, p. 40) this gives a lifetime τ of the excited state as big as

$$\tau \approx 5.6 \times 10^{31}$$
 years. (19)

It thus seems forever impossible to observe such a transition. One should, however, not forget that the predicted lifetime of a proton in the simplest unified theory of particle physics (the minimal SU(5) theory) is about 10^{32} years, which one was able to falsify in the Super-Kamiokande experiment in Japan; it turned out that the proton has a lifetime of at least about 10^{34} years. The problem with (19) is that this decay is drowned in electromagnetic transitions, which are very fast. But if one could identify transitions in atomic or molecular physics that emit photons at no or low rate, there may be the option to observe gravitonic emissions in, for example, thin interstellar clouds. To the best of my knowledge, however, no one so far has attempted to identify and calculate such processes.

The power spectra (15) and (16) are, in a certain sense, already effects of linearized quantum gravity. The reason for this claim is that the calculation makes use of variables that combine gravitational (metric) and matter variables in a quantum sense. This is confirmed by the appearance of the Planck time t_P in these expressions. Calculations have also been performed to derive corrections to these expressions by going beyond the linear approximation. This has been achieved in particular for the canonical theory in both the geometrodynamic and the loop version. The corrections are proportional to the inverse Planck-mass squared and turn out to be too tiny to be observable at present. Similar correction terms should appear for the power spectra of galaxy distributions; so far, however, calculations of such terms do not seem to exist. Quite generally, one would expect that the first signatures of quantum gravity come from small effects. This was the case for quantum electrodynamics, where the theoretical understanding and the successful observation of the Lamb shift in atomic spectral lines led to the general acceptance of the theory. A second major challenge is the construction of a viable full quantum theory of gravity, preferable one that gives a unified description of all interactions. On the one hand, it is not clear whether one can construct a separate quantum theory of gravity alone, without unification. Asymptotic safety may provide an example of a stand-alone theory, but most likely, such a separate theory would be an effective theory, one that is valid only below a certain energy scale. This would be sufficient for calculating small effects, but would lack an understanding of quantum gravity at the fundamental level. On the other hand, it is far from obvious that the programme of reductionism will continue to work and that a "theory of everything" can be found. Superstring theory, the main candidate for such a theory so far, has not proven successful in the last fifty years.

The case of superstring theory also exhibits a deep general dilemma. One might expect that a really fundamental theory would enable one to predict most of the fundamental constants of Nature from a small number of parameters. One important example is (9), which sets the scale at which structures in the Universe appear, and which string theory cannot predict so far. But since one knows that only a very fine-tuned set of physical parameters (masses, coupling constants, etc.) allow the existence of a Universe such as ours and the formation of life, this would leave the open question why this is so. If, on the other hand, the fundamental theory does not lead to such a prediction and if, moreover, all possible parameter values are allowed in the world (which would then constitute a kind of 'multiverse'), it would leave us only with the anthropic principle as a way to understand the Universe (see e.g. Carr 2007). It may, of course, happen that we have a mixture of the two cases, so that most constants are determined by the fundamental theory and a few (such as the cosmological constant and the Higgs mass) can only be determined anthropically. A decision about this dilemma is one of the most important theoretical challenges, if not the most important one.

We have remarked above that general relativity is incomplete because it predicts the occurrence of spacetime singularities. The general expectation is that a quantum theory of gravity will avoid singularities. The present state of quantum gravity approaches is not mature enough to enable the proof of theorems, but preliminary investigations in various approaches indicate that singularity-free quantum solutions can indeed be constructed. It is one of the main theoretical challenges of the next decades to clarify the situation and get a clear and mathematical precise picture of the conditions under which singularity avoidance follows. This would also throw light on one important open question in the classical theory – *cosmic censorship* (see e.g. Penrose 2007). Black holes such as the one in Fig. 4 are characterized by the presence of a horizon from behind which no information can escape to external observers. The singularity predicted by general relativity is thus hidden. The hypothesis of cosmic censorship states that all singularities arising from a realistic gravitational collapse are hidden by a horizon, thus preventing the singularity from being "naked". Singularity avoidance from quantum gravity would immediately lead to the non-existence of hidden *and* naked singularities and would thus prove cosmic censorship to be true in a trivial sense.

2.2 Observational challenges and opportunities

Progress in quantum gravity can eventually only come from observations and experiments. As we have seen, quantum effects of gravity are usually small and become dominant only at the Planck scale. Laboratory experiments thus may look hopeless. One can try to generate superpositions of gravitational fields in the sense mentioned in connection with Fig. 3, but it is unlikely that this could enable one to discriminate between different approaches. One may also use laboratory experiments to decide whether the superposition principle is violated for gravitational fields as advocated, for example, in Penrose (2007). The main obstacle in this is to avoid standard decoherence effects from environmental degrees of freedom (Schlosshauer 2007). Laboratory experiments are also useful to test acoustic analogies to the Hawking and Unruhe effects, from which insight relevant for quantum gravity may be drawn.

The main observational input should thus come from astrophysics and cosmology, but also from particle physics. For this to be successful, large international collaborations are typically needed. We have already mentioned the anisotropy spectrum for the CMB, which was precisely measured by international projects such as PLANCK, WMAP, BOOMERANG, and others. Whether quantum gravity effects can be seen in future projects of this kind, remains open. A major step would be the identification of a non-vanishing value for the r-parameter (17), from which the existence of gravitons could be inferred.

Another important class of experiments are gravitational-wave experiments. They are not designed primarily for quantum-gravity effects, but they may be helpful also in this respect by detecting, for example, a stochastic background of gravitons from the early Universe. One project is the Laser Interferometer Space Antenna (LISA) scheduled for launch in 2034.⁹ A planned terrestrical project is the Einstein Telescope (ET) scheduled for starting observations in 2035.¹⁰

Aside from cosmology, black holes are perhaps the most important objects for exploring quantum gravity experimentally. Due to Hawking evaporation, black holes have a finite lifetime. Taking into account the emission of photons and gravitons only, the lifetime of a (Schwarzschild) black hole under Hawking radi-

⁹https://www.lisamission.org

¹⁰http://www.et-gw.eu

ation is (see e.g Page 2013)

$$\tau_{\rm BH} \approx 8895 \left(\frac{M_0}{m_{\rm P}}\right)^3 t_{\rm P} \approx 1.159 \times 10^{67} \left(\frac{M_0}{M_{\odot}}\right)^3 \text{ years.}$$
(20)

It is obvious that this lifetime is much too long to enable observations for astrophysical black holes. This would only be possible if small black holes exist, which most likely can only result from large density fluctuations in the early Universe – for this reason they are called *primordial black holes*. So far, observations gave only upper limits on their number and on the rate for their final evaporation. Since gamma rays are emitted in the final phase, gamma-ray telescopes are crucial for their detection, for example the Fermi Gamma-ray Space Telescope launched in 2008.¹¹ There are also speculations about the presence of a primordial black hole with the size of a grapefruit in the Solar System ("Planet X"); whether this is really the case must be checked by future observations, for example by the upcoming Vera C. Rubin Observatory in Chile.¹²

Hawking's calculations that led him to the black-hole temperature (10) break down when the mass of the black holes approaches the Planck mass (7). This means that the final phase can only be understood from a full theory of quantum gravity (beyond the approximation of small correction terms). Observations may then shed light on the "information-loss problem", that is, whether the radiation remains thermal up to the very end (and may thus lead to loss of information about the initial state) or not.

Quantum-gravity effects may also seen in particle accelerators. This may be due, for example, to the existence of higher dimensions or due to the presence of supersymmetry. So far, no hints for this or other quantum-gravity related effects were found at the LHC¹³ or other machines. The upgrade High Luminosity Large Hadron Collider (HL-LHC) is planned to start operation in 2027. Plans for various other big machines scheduled for operation before 2040 exist.

2.3 A brief outlook on the year 2050

There is a quote attributed to Mark Twain: "Prediction is difficult – particularly when it involves the future," which definitely also applies to predictions about the status of quantum gravity in 2050. Looking thirty years back (my postdoc years), most of the present quantum-gravity approaches did exist, some of them already for a while. Since then, there has been progress in both the mathematical formulation and the conceptual picture, but no final breakthrough was achieved. A

¹¹https://fermi.gsfc.nasa.gov

¹²https://www.lsst.org

¹³https://home.cern/science/accelerators/large-hadron-collider

hypothetical researcher time travelling from 1991 to 2021 would have no problems to follow the current literature on quantum gravity. But what about the next thirty years?

An optimistic picture would perhaps look as follows. We have a leading candidate for a quantum theory of gravity that provides an explanation of the cosmological constant (more generally, Dark Energy) and perhaps Dark Matter. It predicts testable effects for quantum-gravitational correction terms to power spectra of galaxies and the CMB and sheds light on the final phase of black-hole evaporation. Gravitons are observed as relics from the early Universe and in the form of tensor modes from the CMB. Primordial black holes are observed and their final phase can be studied in detail. Ideally, this theory should give a unified description of gravity and the other interactions.

A pessimistic version would look very differently. We still work on essentially the same approaches to quantum gravity as today and see no possibilities for testing them. The above mentioned projects for the 2030s and 2040s turn out to be very successful for astronomy and particle physics, but fail to shed light on quantum gravity. Already in 1964, Richard Feynman wrote, see Feynman (1990, p. 172): "The age in which we live is the age in which we are discovering the fundamental laws of nature, and that day will never come again. It is very exciting, it is marvellous, but this excitement will have to go." What he means is that there are limits to performing experiments for fundamental physics coming from their sheer size and financial needs, and that these limits may appear rather soon. Still, I think, at least the next thirty years should remain exciting, and perhaps major progress, theoretically and empirically, will emerge from a totally unexpected side. ...

Appendix

In this Appendix, I shall summarize some formulae which were omitted in the main text. For a clear and concise account of classical (Newtonian and Einsteinian) gravity I refer to Carlip (2019).

The famous inverse-square law of Newtonian gravity reads

$$\mathbf{F} = -\frac{GM_1M_2}{r^2}\hat{\mathbf{r}}.$$
(21)

This force can be derived from a potential Φ , which obeys Poisson's equation

$$\nabla^2 \Phi = 4\pi G\rho, \tag{22}$$

where ρ is the matter density.

In general relativity, Poisson's equation is replaced by the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu},$$
(23)

which are of a non-linear nature (a gravitational field generates again a gravitational field, and so on). A fundamental role is played by the metric $g_{\mu\nu}$, which instead of the one function Φ in the Newtonian case contains ten functions. The physical dimension of the energy-momentum tensor $T_{\mu\nu}$ is energy density (energy per volume), which is equal to force per area (stress). Einstein once spoke of the left-hand side as marble (because of its geometric nature) and the right-hand side as timber (because of the non-geometric nature of matter fields). In fact, $T_{\mu\nu}$ contains the fields of the Standard Model. Because these fields are quantum operators, the Einstein equations cannot hold exactly but must be modified by an appropriate quantum equation.

Covariant quantum gravity can be defined by a path integral P, which contains a sum over all permissible metrics $g_{\mu\nu}$ and over all non-gravitational fields ϕ ,

$$P = \int \mathcal{D}g_{\mu\nu}\mathcal{D}\phi \exp\left(\frac{\mathrm{i}}{\hbar}S\right),\tag{24}$$

where S denotes the total action of the system. In the canonical approach, one has constraints which are also fulfilled by the path integral, building in this way a bridge between the two approaches.

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References

- Albers, M., Kiefer, C., and Reginatto, M., 2008. Measurement Analysis and Quantum Gravity. *Physical Review D*, 78, 064051 [17 pp.]
- Bose, S., Mazumdar, A., Morley, G.W., Ulricht, H., and Toroš, M., 2017. Spin Entanglement Witness for Quantum Gravity. *Physical Review Letters*, 119, 240401 [6 pp.]
- [3] Carlip, S., 2001. Quantum gravity: a progress report. *Reports on Progress in Physics*, **64**, 885–942.
- [4] Carlip, S., 2019. *General Relativity. A Concise Introduction*. Oxford University Press, Oxford.
- [5] Carr, B. (ed.), 2007. *Universe or Multiverse?* Cambridge University Press, Cambridge.
- [6] DeWitt, C. (ed.), 1957. Proceedings of the conference on the role of gravitation in physics, University of North Carolina, Chapel Hill, January 18–23, 1957. WADC Technical Report 57-216 (unpublished). These Proceedings have recently been edited in: D. Rickles and C. M. DeWitt (eds), Edition Open Sources, http://www.edition-open-sources.org/sources/5/
- [7] Feynman. R., 1990. *The Character of Physical Law*. The M.I.T. Press, Cambridge, Massachusetts.
- [8] Hehl, F.W. and Lämmerzahl, C., 2019. Physical dimensions/units and universal constants: their invariance in special and general relativity. *Annalen der Physik*, 531, 1800407 [10 pp.].
- [9] Joos, E., Zeh, H. D., Kiefer, C., Giulini, D., Kupsch, J., Stamatescu, I.-O., 2003. *Decoherence and the Appearance of a Classical World in Quantum Theory*. 2nd ed. Berlin: Springer.
- [10] Kiefer, C., 2012. *Quantum Gravity*. 3rd ed. Oxford: Oxford University Press.
- [11] Kiefer, C., 2013. Conceptual Problems in Quantum Gravity and Quantum Cosmology. *ISRN Mathematical Physics*, Volume 2013, article ID 509316 (open access).
- [12] Maldacena, J., 2011. The gauge/gravity duality. arXiv:1106.6073 [23 pp.].
- [13] Marletto, C., Vedral, V., 2017. Witness gravity's quantum side in the lab. *Nature*, 547, pp. 156–158; see also the Correspondence in *Nature*, 549, p. 31.

- [14] Oriti, D. (ed.), 2009. *Approaches to Quantum Gravity*. Cambridge: Cambridge University Press.
- [15] Page, D.N., 2013. Time dependence of Hawking radiation entropy. *Journal* of Cosmology and Astroparticle Physics 09 (2013) 028 [28 pp.].
- [16] Penrose, R., 2007. *The Road to Reality: A Complete Guide to the Laws of the Universe*. New York City: Vintage.
- [17] Schlosshauer, M., 2007. *Decoherence and the quantum-to-classical transition*. Berlin: Springer.
- [18] Woodard, R. P., 2009. How far are we from the quantum theory of gravity? *Reports on Progress in Physics*, **72**, 126002 [42 pp.].
- [19] Zeh, H. D., 2007. *The Physical Basis of the Direction of Time*. 5th ed. Berlin: Springer.