Absorption Distance

The number of intercepted absorbers per unit redshift is:

$$\frac{dN}{dz} \equiv N(z) = \rho A c H_0 \frac{dX}{dz}$$

where ρ is the comoving number density, A is the cross section, and $X(z)$ is the so-called absorption distance (originally introduced by Bahcall & Peebles 1969, ApJ, 156, L7):

$$X(z') = \int_0^{z'} (1+z)^2 E(z; \Omega_M, \Omega_\Lambda)^{-1/2} dz$$

and

$$E(z; \Omega_M, \Omega_\Lambda) = (1+z)^2(1+z\Omega_M) - z(2+z)\Omega_\Lambda$$

Objects with a constant mean comoving density should be distributed uniformly in $X(z)$ (in effect, this distance corrects for the expansion along the line of sight).

Special cases for $\Omega_\Lambda = 0$:

For $\Omega_M = 0$: $X(z) = \frac{1}{2}[(1+z)^2 - 1]$, $\frac{dX}{dz} = (1+z)$

For $\Omega_M = 1$: $X(z) = \frac{2}{3}[(1+z)^{3/2} - 1]$, $\frac{dX}{dz} = (1+z)^{1/2}$

It is traditional to express $N(z) \sim (1+z)^\gamma$. This is an adequate empirical approximation for most part, but it need not be a correct mathematical formulation in general; it only works for special cases of $(\Omega_M, \Omega_\Lambda)$.

Note also that ρ and A can be functions of z, for evolving populations of absorbers.