Review for Final

- elementary mechanics
  - Newtonian mechanics
  - gravitation
  - dynamics of systems of particles
- Lagrangian and Hamiltonian Dynamics
  - Lagrangian mechanics
  - Variational dynamics
  - Hamiltonian dynamics
  - (theoretical mechanics)
- oscillations
  - SHO incl. damped and forced
  - coupled SHOs/normal modes
  - continuous limit
- central forces and scattering
- Emphasis will be on applications, not derivations
- Use Appendix B of lecture notes as review guide
Format of Final

- Similar to midterm in format
  - take-home
  - open notes, problem sets, solns, H&F
  - no collaboration, no other textbooks, no web searches, no symbolic manipulation programs, no calculators needed
- 4 hours
- Exam-taking tips
  - Don’t fixate on a problem -- they are not equally difficult.
  - “See” your way through the physics of the problem before digging into the algebra. Don’t “grind” -- keep the physics in mind to guide you.
  - if you are running out of time, try to indicate what you would do to complete the problem.
- Typo corrections out next week hopefully...
Elementary Mechanics

- Will not be emphasized -- had lots of it on the midterm -- likely will have no more than 1 or 2 (short) problems
- Newtonian mechanics (Section 1.1)
  - Solving problems -- finding the forces and determining the EOM
  - Using work and energy to solve problems
- Gravitation (Section 1.2)
  - gravity as the perfect example of a conservative force, where a potential energy can be defined -- defining and using the potential energy
  - Newton’s iron sphere theorem as an application of potential energy
- Dynamics of systems of particles (Section 1.3)
  - using separation between center of mass motion and internal dynamics to solve problems (e.g. rocket motion)
  - using conservation of momentum and energy for systems to solve problems (e.g., scattering)
Lagrangian Mechanics/Dynamics

• Foundations -- i.e., more than just the E-L eqn
  ¬ What is a holonomic constraint?
  ¬ Understand how to define problems in terms of coordinates that incorporate constraints

• Ph196-only topics
  ¬ Nonholonomic constraints
  ¬ Understand how calculate generalized forces (PS3 #5)
  ¬ Remember the Generalized Eqn of Motion

\[ \sum_i \vec{F}_i^{(nc)} \cdot \frac{\partial \vec{r}_i}{\partial q_k} = F_k = \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} \]

• Applications -- using the E-L eqn
  ¬ Calculating the kinetic and potential energies -- remember to begin in a coordinate system you understand, then rewrite in generalized coords
  ¬ You will get most of the points in such problems for writing down E-L eqns properly, make sure you do at least that much. Remember the signs in the E-L eqn -- it is a “physics” sign, not an “algebra” sign.
Variational Dynamics

• Calculus of variations
  • again, more than just the E-L eqn, remember how to use the calculus of variations for generic minimization

• Lagrange multipliers
  • be comfortable with using them and determining constraint forces from them: *This is a topic a lot of students have problems with! It is subtle!*

• Ph196: Incorporating nonholonomic constraints
Hamiltonian Dynamics

• Noether’s theorem
  ♦ know how to check whether a transformation is a symmetry transformation of the Lagrangian
  ♦ know how to calculate conserved quantities

• Hamilton’s equations of motion
  ♦ be sure to understand how to use them
  ♦ remember the distinction between \((q, dq/dt)\) and \((q, p)\) as independent variables
  ♦ Liouville’s theorem -- applications are pretty straightforward, unlikely you will get a question dealing with how it is proven.
  ♦ Virial theorem -- proof is not so important as being able to use it
Theoretical Mechanics (Mostly Ph196)

- **Canonical Transformations**
  - Be sure you understand which variables are independent in which cases:
    - generating function: pairs (q,Q), (q,P), (p,Q), (p,P) are independent
    - canonical transformation: (q,p) or (Q,P) are independent
  - How do you know a generating function is valid?
  - What is preserved under a canonical transformation? Why?

- **Symplectic Notation and Poisson Brackets**
  - Be able to use them -- it may simplify some problems

- **Hamilton-Jacobi**
  - Don’t need to know the derivation
  - Understand how to use it to solve problems

- **Ph106**: Action-angle variables and adiabatic invariants
  - Notes are not very good on this topic because too much time spent on derivation (aimed at Ph196). Know how to use for problems. See PS5 #7 and PS6 #1. What does adiabatic invariance say is preserved in phase space?
Simple Harmonic Oscillator

• Basics
  - Understand how to determine stability and frequency of small oscillations
  - See, e.g., PS6 #3 and PS7 #5 for examples on how to expand about an equilibrium point.

• Damped SHO
  - Know the three cases
  - Understand how to use Green’s functions to determine response to transient drive

• Driven SHO
  - Understand the distinction between transient and steady-state response
  - Understand the complex amplitude response plane -- magnitude and phase of response, physical meaning
Coupled SHO and Waves

• Know how to use the technology!
  - determining normal mode frequencies
  - finding normal mode vectors (including normalization)
  - applying initial conditions to find mode coefficients to get full time evolution (esp. understand intuitively how initial conditions excite different modes)

• Understand why degeneracy occurs, how to deal with it
• Understand the limiting process to go from loaded string to continuous string, and how it relates to normal modes
Central Forces

- **Formalism**
  - Know how to go from the 3D problem to the 1D problem and how to write the effective potential and effective 1D Lagrangian.
  - Understand the $1/r \rightarrow u$ transformation and how to *use* the $u(\theta)$ orbit and energy equations for more general cases -- e.g. PS7 #5 and the example precessing orbit solution given in PS7

- **Kepler applications**
  - Understand the Keplerian orbits, how geometry is related to energy and angular momentum
  - Using conservation of energy and angular momentum to understand how to go from one orbit to another -- lots of examples in previous years’ problem sets, PS7 #3

- **Scattering**
  - how to go from $(\ell, E)$ to $(b, v_\infty)$
  - getting $b(\theta \star)$ and differential cross section
  - CM $\leftrightarrow$ lab frame transformations (PS7 #6)
Study Tips

- Use Appendix B of the notes as a review guide -- you should feel comfortable with everything there.

- *Do problems!*
  - Certainly, you should understand the derivations as best you can...
  - but, ultimately, we can only really test your ability to apply what you’ve learned; it’s hard to think up variants on the derivations.
  - Try redoing the examples from the lecture notes without the notes in front of you
  - Use the sample problems from previous years. Email me for solutions.
  - Go to the library and xerox problems out of Thornton, Symon, Goldstein, Fowles and Cassidy, etc.
Next Term

• Rotations, rotations, rotations
  • How to deal with rotating coordinate systems
  • Dynamics of rigid bodies (torque, precession, nutations, etc.)
  • Special relativity
    • Can be viewed as a generalization of rotation formalism

• A taste of classical field theory
  • How to generalize the action and E-L formalism to fields
  • Noether’s theorem for fields
  • Scalar field, EM field, hopefully the relativistic electron
  • Local symmetry transformations and gauge invariance
  • (Spontaneous symmetry breaking?)