Physics 106b – Midterm Exam – Due Feb 4, 2005

Instructions

Material: All lectures through Jan 25, Sections 4 and 5 of lecture notes, Hand and Finch Chapters 4, 7, and 8 except those sections not covered in lecture notes. Review the material ahead of time, consult myself, the TAs, your fellow students, or other texts if there is material you are having trouble with.

Logistics: Do not look at the exam until you are ready to start it. Please use a blue book if possible (makes grading easier), but there will be no penalty if you don't have one.

Time: 2 hrs at 100% credit + 2 hrs at 50% credit. Fixed time. You may take as many breaks as you like, but they may add up to no more than 30 minutes (2 x 15 minutes, 3 x 10 minutes, etc.). If you know the material well, you shouldn't need any of the extra 2 hrs. Clearly indicate the 100% credit and 50% credit parts of the exam.

Reference policy: Hand and Finch, official class lecture notes, problem sets and solutions, your own lecture notes or other notes you have taken to help yourself understand the material. No other textbooks, no web searches, no interaction with your fellow students after you have looked at the exam. Calculators will not be needed. The algebra is negligible, symbolic manipulation programs will not be needed and are not allowed.

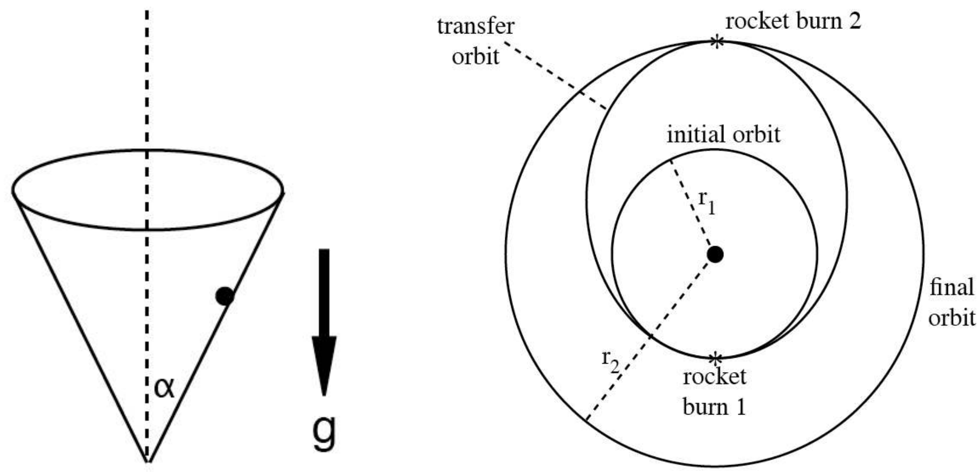
Question policy: Obviously, it will be difficult to ask questions if you are on a fixed-time policy. I have solved all the problems fully, so I have done my best to ensure there are no ambiguities. You are in the same boat as your fellow students, so if everyone has trouble with a particular problem, the grades will be curved accordingly.

Due date: Friday, Feb 4, 5 pm, my office (311 Downs). 5 pm means 5 pm. **No late exams will be accepted!**

Grading weight: The exam is one-third of the class grade.

Suggestions on taking the exam:

- Go through and figure out roughly how to do each problem first; make sure you've got the physical concept straight before you start writing down formulae.
- Don't fixate on a particular problem. They are not all of equal difficulty. Come back to ones you are having difficulty with.
- Don't get buried in algebra (this really should not be an issue on this exam). Get each problem to the point where you think you will get most of the points, then come back and worry about the algebra. Save the algebra for the 50% period.



Left: Problem 1. Right: Problem 2.

1. (25 points) Consider a particle of mass m moving frictionlessly on the inside surface of an inverted cone of opening half-angle α. The particle is subject to a uniform gravitational field g pointing vertically downward (parallel to the axis of the cone). Do the following:

   (a) Write down the Lagrangian in plane polar coordinates.

   (b) Find the equation of motion for r and show that the motion in r is equivalent to one-dimensional motion in an effective potential

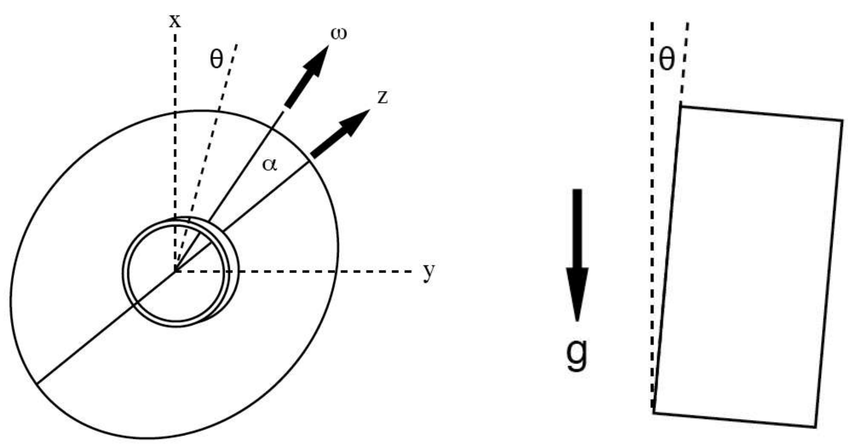
   $$V_{eff}(r) = \frac{l^2}{2\,m\,r^2} + m\,g\,r\,\cot\alpha$$

   where l is the angular momentum.

   (c) If the initial conditions are $r(t=0) = r_0$, $\dot{r}(t=0) = 0$, $\phi(t=0) = \phi_0$, and $\dot{\phi}(t=0) = v_0/r_0$ where r_0, ϕ_0, and v_0 are constants – that is, if the particle is given some initial horizontal velocity – show that the condition for a circular orbit (*i.e.*, $\dot{r} = 0$ for all time) is $v_0^2 = g\,r_0\,\cot\alpha$.

   (d) Show that, more generally, the turning points of the motion can be obtained from the solution of a cubic equation in r. What can you conclude about the existence of turning points based on our analysis of a similar relation in the symmetric top with torque problem?

2. (25 points) A Hohmann transfer is a technique to change from one Kepler orbit to another using a short rocket burn along the direction of motion at the perigee or apogee (smallest r or largest r points) of an orbit. Suppose a satellite is in a circular orbit of radius r_1 and it is desired to move to a circular orbit of radius $r_2 > r_1$. This can be accomplished using two rocket burns, one that puts the satellite in a transfer orbit with turning points (perigee and apogee) r_1 and r_2 and another burn that puts the satellite into the r_2 circular orbit from the transfer orbit. For each orbit transfer, calculate the amount by which the satellite's kinetic energy and velocity need to be changed to do the transfer.



Left: Problem 3. Note that θ is an angle in the xy plane and α is an angle in the xz plane.
Right: Problem 4.

3. (25 points) A gyroscope consists of a wheel of mass M and radius r, all of whose mass is located on the rim. The gyroscope is rotating with angular velocity $\dot{\theta}$ about its axis, which is horizontal and is at rest in a frame moving with the earth's surface. We choose a coordinate system in this same frame whose z-axis coincides with the gyroscope axis and whose origin lies at the center of the wheel. The angular velocity $\vec{\omega}$ of the earth lies in the xz plane, making an angle α with the gyroscope axis (α can be seen to be equal to the latitude).

   - Consider a mass element dm on the rim of the gyroscope at polar angle θ (measured from the x-axis through the y-axis, as usual). Find the x, y, and z components of the torque $\vec{N}$ relative to the origin that arises due to the Coriolis force acting on the mass element dm.
   - Use the above result to show that the total Coriolis torque on the gyroscope is

$$\vec{N} = \hat{y}\, M\, r^2\, \omega\, \dot{\theta}\, \sin\alpha$$

4. (25 points) A door is constructed of a thin homogeneous slab of material; it has total mass M, height h, and width w. You may neglect the thickness of the door. Its hinges make an angle θ with the vertical, causing the door to swing to an equilibrium position under the influence of gravity. Obtain an equation of motion for the azimuthal angle the door makes with its equilibrium position in terms of the fixed tilt angle θ and the mass and dimensions of the door (and g). Hint: the natural coordinate system identifies the tilt angle θ with the Euler angle θ and the azimuthal angle with the Euler angle ψ.