Neutrino Oscillations

Gary Cheng
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Introduction: Theory

- Neutrinos have mass eigenstates $\nu_1$, $\nu_2$, $\nu_3$ that are superpositions of the flavor eigenstates $\nu_e$, $\nu_\mu$, $\nu_\tau$, the quantum states in which neutrinos are produced.

- The difference between the mass eigenstates and the flavor eigenstates of neutrinos is what causes neutrino oscillations.
The mass eigenstates $\nu_1, \nu_2, \nu_3$ are the neutrino eigenstates of vacuum space.

The phase change acquired during the time evolution of the mass eigenstates $\nu_1, \nu_2, \nu_3$, whose components add up to make the detectable flavor eigenstates $\nu_\text{e}, \nu_\mu, \nu_\tau$, gives the finite probability that a neutrino emitted as one flavor eigenstate is later detected as another flavor eigenstate after traveling through space.
Introduction: Theory 3

- Two neutrino example:

$$U = \begin{pmatrix} \nu_\alpha & \nu_1 \\ \nu_\beta & \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$
Introduction: Theory 4

\[ |\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle \]

\[ |\nu_\alpha(L)\rangle \approx \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L} |\nu_i\rangle \]

\[ |\nu_\alpha(L)\rangle \approx \sum_\beta \left[ \sum_i U_{\alpha i} e^{-i(m_i^2/2E)L} U_{\beta i} \right] |\nu_\beta\rangle \]

\[ P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2[1.27 \Delta m^2(L/E)] \]

\[ P(\nu_\alpha \to \bar{\nu}_\alpha) = 1 - \sin^2 2\theta \sin^2[1.27 \Delta m^2(L/E)] \]
Neutrino Mixing Matrix (MNS Matrix)

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{i\delta} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
e^{i\alpha_1/2} & 0 & 0 \\
0 & e^{i\alpha_2/2} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}, \quad \{\delta, \alpha_1, \alpha_2\} \equiv \text{CP - Violating Phases}\]
Neutrino Mixing Matrix (MNS Matrix)

\[
U = \begin{pmatrix}
\nu_e & \nu_1 & \nu_2 & \nu_3 \\
\nu_{\mu} & c_{12}c_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{13}e^{-i\delta} \\
\nu_{\tau} & -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{12}c_{23}s_{13}e^{i\delta} & s_{13}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\times \text{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)
\]
Origin of Neutrino Oscillations

- Ray Davis’s Homestake Experiment observed a deficit in the number of solar $\nu_e$ neutrinos reaching the Earth as predicted by the standard solar model. Roughly 1/3 of the expected number of neutrinos is detected.

- The experiment consisted of a large tank of liquid $\text{C}_2\text{Cl}_4$ placed underground with the reaction:

$$\nu_e + ^{37}\text{Cl} \rightarrow \text{e}^- + ^{37}\text{Ar}$$
Description of Super-Kamiokande

- Consists of a 50,000 ton cylindrical tank of ultra-pure water surrounded by 11,242 inward facing photomultiplier tubes (PMTs).

- Placed 1000 meters within the Kamioka Mine to reduce background.
Confirmation of Neutrino Deficit

- Super-Kamiokande confirms the deficit in solar $\nu_e$ neutrinos reaching the Earth. Only about 35% of the expected number of neutrinos is detected.

- $\nu_e$ either scatters an electron or interacts with H to produce a positron. The energetic electron or positron emits Cherenkov radiation as it travels through the water and the radiation is detected by the photomultipliers.
Solar Neutrinos

- The electron density in the sun as the $\nu_e$ travels outward from the center of the sun affects the mixing angle. This is known as the matter effect.

\[
\mathcal{H} = \mathcal{H}_V + \mathcal{H}_M(r) \\
= \frac{\Delta m^2}{4E} \begin{bmatrix}
- \cos 2\theta & \sin 2\theta \\
\sin 2\theta & \cos 2\theta
\end{bmatrix} + \begin{bmatrix}
V(r) & 0 \\
0 & 0
\end{bmatrix}
\]
Result of Solar Neutrino Matter Effect

- From neutrino oscillations in vacuum, it is impossible to distinguish the mixing angle $\theta$ from $\theta' = \pi/2 - \theta$.

- But because of the $V(r)$ term in the Hamiltonian as the solar $\nu_e$ travels through the electron density in the sun, it is possible to distinguish $\theta$ from $\theta'$. 
Mass Hierarchy of Neutrinos

“Normal” Hierarchy

“Inverted” Hierarchy

\[ \Delta m^2_{\text{sol}} \]

\[ \Delta m^2_{\text{atm}} \]
Current Accepted Values of Mass Differences and Mixing Angles

- **Atmospheric (and Accelerator):**
  \[ 1.9 \times 10^{-3} \text{ eV}^2 < \Delta m^2_{\text{atm}} < 3.0 \times 10^{-3} \text{ eV}^2 \]
  \[ \sin^2 2\theta_{\text{atm}} > 0.90 \]

- **Solar (and Reactor):**
  \[ \Delta m^2_{\odot} = (8.0^{+0.6}_{-0.4}) \times 10^{-5} \text{ eV}^2 \]
  \[ \theta_{\odot} = (33.9^{+2.4}_{-2.2})^\circ \]
Determining the Parameters of Neutrino Oscillations

- From the neutrino oscillation probability formula, the oscillations depend on three critical parameters: $\Delta m^2$ (difference between the masses of the neutrinos squared), $L$ (distance from emitted neutrino), $E$ (energy of neutrino).
Separation of the Mixing Angles

- The separation of the three mixing angles is due to the following factors:
  - $\theta_{13}$ is small compared to $\theta_{23}$ and, to a first approximation, can be neglected.
  - The two order of magnitude difference between $\Delta m_{12}$ and $\Delta m_{23}$ ($\Delta m_{23} \approx \Delta m_{13}$) allows the neutrino oscillations to “separate” because the wavelength of oscillations are on entirely different length scales for a given energy of the neutrino.
Solar Neutrino Approximation

- The baseline for solar neutrinos is the distance from the sun to the earth (about 150 million km).

- The solar $\nu_e$ energy is in the few MeV range.

- The oscillations detected are of the mixing angle $\theta_{12}$ and $\theta_{\text{solar}} \approx \theta_{12}$ (the length scale of $\theta_{13}$ and $\theta_{23}$ are too short to make a significant contribution).
Atmospheric Neutrino Approximation

- The baseline for atmospheric neutrinos is the diameter of earth (about 12,500 km).

- The atmospheric $\nu_\tau$ energy is in the hundreds of MeV to few TeV range.

- The oscillations detected are of the mixing angle $\theta_{23}$ and $\theta_{\text{atm}} \approx \theta_{23}$ (the length scale of $\theta_{12}$ is too long to make a significant contribution and $\theta_{13}$ is small and neglected).
Criteria for Neutrino Oscillation Experiments

- All current neutrino oscillation experiments are designed with $\Delta m^2(L/E)$ in mind.

- For the given $\theta$ to be measured, the distance of the detector from neutrino production source (L) and the energy of the neutrino (E) are chosen so that $\Delta m^2(L/E)$ is on the order of magnitude of $\pi$. 
SNO (Solar Neutrinos)

- 1000 tons of heavy water ($\text{D}_2\text{O}$) in a 850 cm spherical vessel surrounded by approximate inward facing 9600 PMTs.

- Located 2 km underground in a mine in Ontario, Canada.
SNO (Sudbury Neutrino Observatory)

- The SNO Experiment could detect all three flavors of neutrinos $\nu_e$, $\nu_\mu$, $\nu_\tau$ coming from the sun through three possible interactions:
  - Charged current reaction
  - Neutral current reaction
  - Electron scattering
Charged Current Reaction

- $\nu_e + d \rightarrow p + p + e^-$

- Only sensitive to $\nu_e$. 
Neutral Current Reaction

- \( \nu_i + d \rightarrow n + p + e^- + \nu_i \)
- Sensitive to all three neutrino flavors with equal cross-sections.
Electron Scattering

- $\nu_i + e^- \rightarrow \nu_i + e^-$

- Sensitive to all three neutrino flavors, but $\nu_e$ sensitivity dominates by a factor of 6.
SNO Results

\[ \frac{\phi(\nu_e)}{\phi(\nu_e) + \phi(\nu_{\mu,\tau})} = 0.340 \pm 0.023 \text{ (stat)} +0.029 -0.031 \text{ (syst)} \]
KamLAND (Reactor Anti-neutrinos)

- KamLAND (Kamioka Liquid scintillator Anti-Neutrino Detector) detects antineutrinos from dozens of Japanese nuclear reactors, mostly located 150-200 km away.

- 1000 tons of 80% dodecane and 20% pseudocumene in a roughly 13 m diameter nylon/EVOH balloon surrounded by 1,879 PMTs.
KamLAND Results

2.6 MeV prompt analysis threshold

- KamLAND data
- best-fit oscillation
- best-fit decay
- best-fit decoherence

Ratio

$L_0/E_{\nu_e}$ (km/MeV)
Solar Exp. and KamLAND Data
LSND (Liquid Scintillator Neutrino Detector) detects the excess of electron antineutrinos above background oscillating from muon antineutrinos.

Muon antineutrinos are created from the decay of at rest $\mu^+$.

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$
LSND Experiment

- The detector consists of a 167-ton tank of mineral oil surrounded by 1220 PMTs.

- The baseline for the muon antineutrino oscillation is roughly 30 m.

- LSND results indicated a fourth sterile neutrino which was refuted by MiniBoone in 2007.
$\Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}} \neq \Delta m^2_{\text{LSND}}$
MiniBoone

- Detector is 40 m diameter sphere containing 800 tons of mineral oil surrounded by 1520 detectors.
- MiniBoone detects excess $\nu_e$ in a $\nu_\mu$ beam with a neutrino oscillation baseline of 450m.
MiniBoone Result 1

The Track-based $\nu_\mu \rightarrow \nu_e$ Appearance-only Result:

Counting Experiment: $475 < E_{\nu}^{QE} < 1250$ MeV

data: 380 events
expectation: $358 \pm 19$ (stat) $\pm 35$ (sys) events

significance: $0.55 \sigma$
MiniBoone Result 2
Future Experiments in Neutrino Oscillations

- Neutrino oscillation experiments (Double Chooz, Daya Bay, etc.) are underway to measure the small $\theta_{13}$ angle.

- Both Double Chooz and Daya Bay will watch for the disappearance anti-$\nu_e$ from nuclear power reactors by comparing flux and energy spectrum between detectors located close and far from the reactors.