Introduction, Overview, Basic Physics (Section 1)

Definition of "star"

self gravitating gas sphere

radiation emitted from its surface ("photosphere")

energy generated (or was in the past) by

thermonuclear reactions deep inside at high T

($4\,{}^{1}H \rightarrow {}^{4}He^{*}$ via several reactions + energy)

$$\Delta E = \Delta mc^2 = m({}^{4}He) - 4m({}^{1}H)$$

energy transported through star via

radiative diffusion

convection

conduction (rare)

Stellar Evolution in a nutshell (vastly simplified)

mass is key parameter (unit $M_\odot$)

– age – other key parameter

usually higher M, hotter star

Burning stages H → He → C → Ne → O → Si → Fe

(Fe end of line for nuclear fusion)

core composition H He C/O O/Ne Si Fe

: higher core T to reach more advanced burning

reached by star mass	$\gtrsim 0.1\ M_\odot$	0.5	$6\ M_\odot$	$10\ M_\odot$

Star Death

$M < 8\text{–}10\ M_\odot$ white dwarf (+ thermonuclear SN)

$M > 10\ M_\odot$ core collapse → CC SN + neutron star or black hole

Elemental Abundances

Big bang ^1H 76 (± 1)% mass fraction

^4He 24 (± 1)

^3He 10^{-5} ^7Li 10^{-10}

no heavier elements (primordial composition)

⇒ everything else made in stars, expelled in SN + stellar winds

(exception - a few light element isotopes can be produced by ~~CR~~ spallation / collisions of cosmic ray nuclei, but these nuclei must themselves be made in stars)

Solar abundances (mass fractions) (of total mass)

^1H 0.738 "X"

^4He 0.249 "Y"

$> He$ 0.013 "Z" (called "metals")

Number fractions are usually relative to H "abundance of an element"

define $\log_{10}(\varepsilon_H) = 12$

$$\log_{10} \varepsilon_X = \log_{10}\left(\frac{n_X}{n_H}\right) + 12$$

mass fraction: $\left(\frac{N_X}{N_H}\right)\left(\frac{m_X}{m_H}\right) = \frac{M_X}{M_H}$ $\quad (m_X \approx m_u A_X)$

atomic mass unit; atomic # of isotope X

Most abundant metals

	$\log \varepsilon(X)$ (by number) ($H=12$)		mass fraction	rank
1	O	~8.69	5.78×10^{-3}	1
2	C	~8.43	2.38×10^{-3}	2
3	Ne	~7.93	6.28×10^{-4}	7
4	N	~7.83	6.99×10^{-4}	5
5	Mg	~7.6	7.05×10^{-4}	4
6	Si	~7.3	6.69×10^{-4}	6
7	Fe	~7.5	1.31×10^{-3}	3
8	S	~7.12	3.11×10^{-4}	8

Notation
$$[Fe/H] = \log_{10}\left(\frac{N_{Fe}}{N_H}\right)_{star} - \log_{10}\left(\frac{N_{Fe}}{N_H}\right)_{\odot}$$
sun

$[Fe/H] = 0 \Rightarrow$ solar abundance ratio

Also common: $[X/Fe]$ Similar to above, Fe as "typical" heavy metal

First stars $Z \approx 0$ no heavy metals, only Big Bang, none known, lowest $[Fe/H] \approx -7$

$Z \approx 10^{-7} Z_\odot$, must be very old formed in first generation of stars

<u>Stellar populations</u>

metals are all made in stars during late phases of stellar evolution, so stellar spectroscopy provides info on <u>birth</u> environment of a star

In MW 2 distinct populations
Pop I: disk, spiral arms, young (age $< 10^9$ yr)
metal rich $Z \sim 0.5$ to $1 Z_\odot$

Pop II galactic halo + globular cluster stars,
age $\gtrsim 10^{10}$ yr, metal poor, $Z \sim (0.01 - 0.1) Z_\odot$

Galactic chemical evolution: when stars die in SN explosions, ejected gas is metal rich + it enriches the ISM with metals.

<u>Review: Thermodynamics, State of gas etc</u>

phase space 3D space + 3D momentum $dx\,dy\,dz\,dp_x\,dp_y\,dp_z$

number density of particles in volume

$$n(p)\,dp = \frac{f \cdot g}{h^3}\, d^3 p \qquad (d^3 p = 4\pi p^2 dp)$$

(number/cm³)

h = Planck's constant 6.62×10^{-27} ergs

quantum mechanics — minimum elementary cell in phase space has volume h^3

g = particle degeneracy (statistical weight) (same / very similar particles, different spins...)

f = phase space density, also called distribution function

$$= \frac{1}{\exp[(E-\mu)/kT \pm 1]}$$

T = temperature (K)

k Boltzman constant 1.38×10^{-16} ergs/K

fermions (+) spin 1/2

bosons (−) spin 0, 1, 2

E particle energy

μ chemical potential

Total number density $n = \int_0^\infty n(p)\,dp$

mass density = $n \cdot m$ (m = mass of particle)

If know g+T ~~cos~~ and E(p) can calculate

$$E = \text{total energy} = \underbrace{E_0}_{mc^2 \text{ rest mass}} + E_{kinetic}$$

non. rel limit ($p \ll mc$) $E_K = \frac{p^2}{2m}$ $\quad v = p/m$

relativistic limit ($p \gg mc$) $E_K = pc$ $\quad v \simeq c$

total kinetic energy / unit volume (internal energy of an ideal gas)

$$U = \int_0^\infty E_K(p)\, n(p)\, dp = (\text{non-rel}) \int_0^\infty \frac{p^2}{2m}\, n(p)\, dp$$

Pressure $P = \int_0^\infty \langle v_x p_x \rangle\, n(p)\, dp$

"Momentum transferred across a wall / unit time integrated over all particles"

$$P = \frac{1}{3}\int_0^\infty v(p)\, p\, n(p)\, dp$$

Since pressure is isotropic, $\langle v_x p_x \rangle = \frac{1}{3} vp$

$$P\ (\text{non-rel}) = \frac{1}{3}\int_0^\infty \frac{p^2}{m}\, n(p)\, dp = \frac{2}{3} U,$$

non-rel $\boxed{U = \frac{3}{2} P}$

$\boxed{P_{non\ rel} = \frac{2}{3} U}$ $\quad$ $\boxed{P_{rel} = \frac{1}{3} U}$, $\boxed{U_{rel} = 3P}$

Radiation / Planck distribution

Observe from stars: solar wind, radiation, solar (and SN1987A) neutrinos
but most info for stars is from photons (γ ray to radio)

Photons in local thermodynamic equilibrium (LTE) obey a Planck distribution

Substitute in our general distribution function:
$s = 1$ (bosons) $g = 2$ (2 polarization states), rest mass 0
so $\mu = 0$

$$n(p)\,dp = \frac{8\pi}{h^3} \frac{p^2}{(e^{pc/kT} - 1)} dp$$

$$\boxed{E = pc = h\nu = \frac{hc}{\lambda}}$$

A body whose photons are emitted with a Planck distribution is called a blackbody.

A blackbody absorbs all photons that fall on it

Radiation from a BB depends only on its thermal energy T & is described by the Planck function.

Planck function in frequency units

$$\boxed{n_\nu\, d\nu = \frac{8\pi}{c^3} \frac{\nu^2}{(e^{h\nu/kT} - 1)} d\nu}$$

$$U_{\text{radiation}} = \int_0^\infty E_\nu\, n_\nu\, d\nu = \int_0^\infty h\nu\, n_\nu\, d\nu$$

set $x = h\nu/kT$

$$U_r = \frac{8\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3\, dx}{e^x - 1}$$

definite integral $= \pi^4/15$

$\boxed{U_r = aT^4}$ where $\boxed{a = \frac{8\pi^5}{15} \frac{k^4}{h^3 c^3} = 7.57 \times 10^{-15} \text{ ergs/cm}^3/\text{K}^4}$

$\left(P_{rad} = \frac{1}{3} U_{rad} = \frac{a}{3} T^4\right)$

Total emitted radiation from a BB

$$B(T) = \int_0^{\infty} B_\nu(T)\, d\nu$$

$$U_r(T) = \frac{4\pi}{c} BT = \frac{1}{c}\int B(T)\, d\Omega = \frac{4\pi}{c} B(T)$$

solid angle

units of U are ergs/cm^3

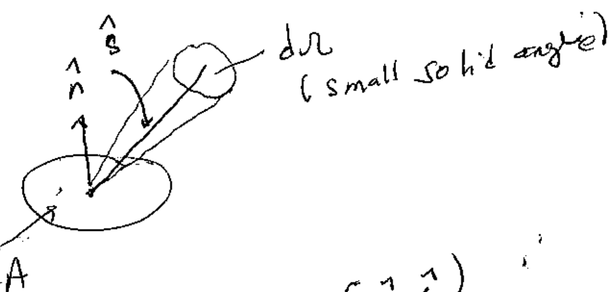
B ergs/cm^2/steradian/sec

$d\Omega$

$$\boxed{B_\nu(T)\, d\nu = \frac{c}{4\pi} U_\nu\, d\nu = \frac{2h}{c^2} \frac{\nu^3}{(e^{h\nu/kT} - 1)}\, d\nu}$$

Radiative Transfer Basics

specific intensity I



dE flowing through $d\Omega = I_\nu\, d\nu\, dA\, dt\, d\Omega\ (\hat{n} \cdot \hat{s})$
(ergs/sec/cm^2/hz/steradian)

unless there is emission, abs. or scattering, I_ν is constant along a ray from the source ($\frac{dI_\nu}{ds} = 0$)
length along ray $\hat{s}$)

Mean Intensity

$$J_\nu = \frac{1}{4\pi}\int I_\nu\, d\Omega \qquad d\Omega = \sin\theta\, d\theta\, d\phi$$

$I_\nu = J_\nu$ if I_ν isotropic (same in all directions)

by definition, U_ν = energy density $= \frac{4\pi}{c} J_\nu$

Flux (1st moment of I_ν)

units ergs/sec / ~~cm~~ cm^2

$$dF_\nu = I_\nu(\hat{s}\cdot\hat{n})\, d\Omega = I_\nu \cos\theta\, d\Omega$$

$$F_\nu = \int I_\nu \cos\theta\, d\Omega$$ = net flux going through dA

If radiation field is isotropic, $F_\nu = 0$

Flux emitted from surface of a BB

no incoming radiation

isotropic emission from surface

symmetric in φ about normal

(~~a~~ surface of a star ≈ surface of a BB, very approximately)

$$F_{BB} = \int_0^{2\pi} d\varphi \int_0^{\pi/2} B(T) \cos\theta \underbrace{\sin\theta\, d\theta}_{-d\cos\theta}$$

(already integrated over frequency

$$F_{BB} = \pi B(T) = \pi \left[\frac{ac}{4\pi}\right] T^4 = \frac{ac}{4} T^4 = \sigma T^4$$

$\sigma = 5.67 \times 10^{-5}$ ergs/cm²/sec/K⁴ — Stefan-Boltzmann constant

Luminosity of a star

Total emitted energy/sec (ergs/sec)

$$L = \underbrace{4\pi R_*^2}_{\text{surface area}} F_* = 4\pi R_*^2 \sigma T_*^4$$

L is constant

Flux depends on distance $\propto 1/R^2$

Stellar Effective Temperature

(1-9)

T_{eff} = temperature of a BB that has same total luminosity ($\int_0^\infty d\nu$) as the star

$$L_* = 4\pi R_*^2 \sigma T_{eff,*}^4$$

Sun T_{eff} 5780 K

$R_\odot$ 6.96×10^{10} cm $\Rightarrow$ $L_\odot = 3.85 \times 10^{33}$ ergs/sec

($M_\odot \approx 1.99 \times 10^{33}$ gm)

Wien's Law

λ or ν at which $B_\lambda(T)$ or $B_\nu(T)$ is maximum

$$\lambda_{max} = \frac{0.29 \text{ K cm}}{T}$$

As $T_{eff} \uparrow$, $B(T) \propto T^4$, total output $\uparrow\uparrow$ $\forall$
λ_{max} shifts to shorter λ (blued

Statistical Physics of Atoms / Ions etc

NR classical particles

recall $n(p)\,dp = \frac{4\pi}{h^3} g f p^2 dp$

$E = mc^2 + \frac{p^2}{2m} + E_{int} + M$

$\approx mc^2$

$$f = \frac{1}{\exp[(E-\mu)/kT \pm 1]}$$

$$f \approx \exp[-E/kT]$$

$$n(p)\,dp = \frac{4\pi}{h^3} \exp[-E/kT]\,dp$$

where E can be either kinetic or ~~excitation~~ excitation energy (ie internal excitation of an atom or ion)

So for 2 levels in an atom/ion

$$\frac{n_2}{n_1} = \frac{g_2}{g_1}\frac{e^{-E_2/kT}}{e^{-E_1/kT}} = \boxed{\frac{g_2}{g_1} e^{-(E_2-E_1)/kT} = \frac{n_2}{n_1}}$$

Boltzmann Law

This law is very important for stellar atmospheres as we need it to compute the level populations of various ions and atoms in stellar atmospheres.

Also need to be able to compute populations of various ionization stages of a given atom in stellar atmospheres.

Need fraction of total population of ion in a given level,
Set ground state to have energy 0, so

$$\frac{n_i}{n_1} = \frac{g_i}{g_0} \exp[-E_i/kT],$$ total population of atom is

$$n = \sum_{m=1}^{\infty} n_m = \frac{n_1}{g_1} \sum_{m=1}^{\infty} g_m \exp[-E_m/kT) = \frac{n_1}{g_1} Z$$

partition function
(Le Blanc calls this Z U_{ion})

$$\frac{n_i}{n} = \frac{g_i}{Z} e^{(-E_i/kT)}$$

Population of levels of hydrogen (actually H like atoms)
for H, $Z=1$

quantized levels $E_n = -\frac{Z^2 e^4 m_e}{2\hbar^2 n^2} = -13.6 \text{ eV} \cdot \frac{Z^2}{n^2}$

energy of state n

1 Rydberg

Why is $Z = \sum_{n=1}^{\infty} g_n e^{-E_n/kT}$ not ∞ in real life?

Because $r_n \propto a_0 n^2$ where for Hydrogen a_0 = Bohr radius = 0.5 Å

as $n\uparrow\uparrow$, $r_n (\uparrow\uparrow)^2$, electron eventually closer to nuc 2 than to nuc, no longer bound to its original nucleus, & sum must be cut off

transition between 2 levels of H-like ion

$$\Delta E_{n,m} = 13.6 \left[\frac{1}{n^2} - \frac{1}{m^2}\right]$$

hydrogen transitions — from ground state

Lyman series	Balmer	Paschen
Lyα 2→1	3→2 6563 Å	4→3
β 3→1	4→2	5→3
all Lyman lines in UV	visible λ	near IR λ
Lyman limit ∞→1 (λ = 912 Å)		(8210 Å)
Balmer limit ∞→2 (3646 Å)		

Since lifetime in an excited state ($n>1$) is very short ($\tau \sim 10^{-9}$ sec) unless continuous re-excitation (ie hot gas, lots of collisions etc), most atoms will be in the ground state.

hotter T, more atoms in excited states & more ionized atoms

Ionization Balance

H atoms, neutral & singly ionized

$$\frac{n_{II}}{n_I} = \frac{g_{II}}{g_I} \exp\left[-\underbrace{(E_{II} - E_I)}_{\Delta E}\right] / kT$$

n_{II} – singly ionized, n_I – neutral

$$\Delta E = I_H + \tfrac{1}{2} m_e v^2$$

$\frac{1}{2} m_e v^2$ ← KE of free e^-

I_H – ionization potential, 13.6 eV

$$\frac{dn_{II}(v)}{n_I} = \frac{g_{II}}{g_I} \exp\left[-\Delta E / kT\right] g_e$$

free volume / electron

$$d^3x = 1/n_e$$

$$g_e = 2 \frac{1}{h^3} d^3x \, d^3p$$

2 – spin

$$\frac{dn_{II}}{dn_I} = \frac{8\pi m_e^3}{h^3 n_e} \frac{g_{II}}{g_I} e^{(-\Delta E/kT)} v^2 dv$$

Integrate this

$$\frac{n_{II}}{n_I} = \frac{1}{n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} 2 \frac{g_{II}}{g_I} \exp\left[-IP/kT\right]$$

IP ← 13.6 eV energy

for H ground state $g_I = 2$ (2 spin orientations)

ionized $g_{II} = 1$ (just proton)

IPs:

HI → HII	13.6 eV
HeI → HeII	24.6 eV
HeII → HeIII	54.4 eV
CaI → CaII	6.1 eV
OI → OII	13.6 eV

Equation of State for a Multi Component Gas

mixture of 2 gases, H + He as example

Assume electrically neutral, all e^- are from ionizations

Cases: (ions) H II ①, He II ②, He III ③ (only if very hot

(neutrals) H I ④, He I ⑤

electrons from ionizations (very low mass $m_e \ll m_p$)

$$P_{ions} = P_1 + P_2 + P_3 + P_4 + P_5$$

$$P_{total} = P_{ions} + P_e$$

$$n_{ions} = \sum_{1-5} \frac{\rho}{m_0} \frac{X_i}{A_i}$$

ρ — density gm/cm^3; m_0 — mass of 1 atomic unit (1p + 1e) HI atom

X_i = mass fraction of i^{th} ion

A_i = mass of i in atomic units (H = 1, He = 4)

Gets very complicated very quickly. End up with a set of coupled linear equations.

Problem is P_e has contributions from all ions (except neutrals), but need P_e to calculate ion. equilibrium of each species

Becomes even more complicated if gas is cooled have to include molecules. Then, for example,

$n(H) = n(HI) + n(HII) + n(CH) + n(OH) + 2n(H_2O) + ...$