HW 3
AY 123, Spring 2003
Due Tues April 22, beginning of class

Problem 1.

In the class Web page you will find a model atmosphere for the Sun from Bob Kurucz. (His web site is http://kurucz.harvard.edu). The model is calculated for 64 optical depths uniformly spaced on a log scale. The deepest (last) point for this model is at optical depth 100, and there are 8 points per factor of 10, so the spacing in the log is 0.125, and the next to last point is at $10^{-0.125} \times 100 \equiv \tau = 75$. The Rosseland mean opacity is used to define this optical depth scale.

At each depth, the parameters given are mass per unit area integrated from the surface to the desired depth (gm/cm^2), T, P, number of electrons/cm^3, and ABROSS, which is the Rosseland mean absorption coefficient per gram of material. The information in the last two columns is not relevant here.

(If you do not wish to use this model, you can use the semi-empirical Solar model by Eugene Avrett in Allen, Astrophysical Quantities, edition 4, section 14.4.)

a) How close is this model to the grey case solution, $T^4 = (3/4)[Teff^4(\tau + 2/3)]$? What is the largest percent deviation between $0.1 < \tau < 2$?

b) Calculate the limb darkening predicted by this Solar model. Compare this to the limb darkening from the Eddington approximation, $S(\tau) = S_0 + S_1\tau$.

c) Compare this with the limb darkening curve observed for the Sun given in Allen, Astrophysical Quantities, edition 4, section 14.7.

d) What happens to the limb darkening curve at wavelengths where the opacity is larger overall than the Rosseland mean ? smaller ?

Problem 2.

a) Integrate the equation of radiative equilibrium, assuming a Rosseland mean opacity law of Kramers form, (i.e. $\kappa_0 \rho T^{-s}$, $s = 3.5$, but keep s general until part (c)) to find the relation between temperature and pressure as a function of depth in a star in radiative equilibrium (energy carried by radiation). Assume the radiation pressure is sufficiently small that it can be ignored.

b) Show that the surface temperature and pressure boundary conditions become unimportant relative to the pressure-temperature relation in the deep interior.

c) Show that if $s = 3$ that the pressure-temperature (and hence density-pressure) relation is exactly that of a $\gamma = 4/3$ polytrope, and if $s = 3.5$, it is nearly the same.

Problem 3.

Use the Solar interior model and graphs of opacity given in Iglesias & Rogers, 1992, ApJS, 79, 507 and 1996, ApJ, 464, 943 (available either in the Astro Library or the ADS scanned article service, http://adsabs.harvard.edu/article_service.html) or those given in Allen's Astrophysical Quantities to estimate:

a) the ratio of radiation energy density to gas energy density near the center of the Sun and at $R = 0.5R_\odot$.

b) The Rosseland mean opacity at the center of the Sun and at $R = 0.5R_\odot$. In each case, how much larger is it than just the electron scattering contribution ?

c) The mean free path of a typical photon at the center of the Sun and at $R = 0.5R_\odot$.

d) The typical time it takes a photon to diffuse out of the Sun.