Ay123 – Fall 2007

STELLAR STRUCTURE AND EVOLUTION

Problem Set 8

Due Wednesday, December 5, 2007

1. (10 pts) Assume the thickness of the Solar corona is 1 $R_\odot$, and it has a uniform temperature and number density $T = 10^6$ K, and $n = 10^8$ /cm^3. For the chromosphere, assume a thickness of 2000 km, $T = 15,000$ K, and $n = 10^{12}$ /cm^3. Assume both the corona and the chromosphere consist of pure hydrogen. Calculate the optical depth at 1000 Å and at 5000 Å for each of these two layers. List the opacity sources you have included and the source of the expression you use for each opacity source. Note that Gray, "The Observation and Analysis of Stellar Photospheres", 3rd edition, gives suitable formulae, as does the 2nd edition of this book, or you can use formulae from the radiative processes class, or any other suitable source. The formulae from Gray are summarized (correctly, I hope) in a sheet on the class web site.

2. (20 pts) Limb Darkening in the Sun

Assume the source function is linear in depth, $S_\nu = C_\nu + D_\nu \tau$, where C_ν and D_ν are constants for each ν.

a) Use the Solar limb darkening curves given in table 14.16 of the 4th edition of Allen's Astrophysical Quantities to find C_ν / D_ν for $\lambda = 0.2$, 0.5, 1.0, 2.0, and 5.0 microns. How well do the observed limb-darkening curves resemble those of a grey body ? What is the approximate wavelength of the Rosseland mean opacity for the Sun as judged by the limb darkening curves ?

b) Assume LTE. The emergent specific intensity at the center of the Sun's disk in the continuum is (units are ergs/cm^2/sec/steradian/micron) is 0.014×10^{10} at 0.2 μm, 3.63×10^{10}

at 0.5 μm, 1.21×10^{10} at 1.0 μm, 0.18×10^{10} at 2.0 μm, and 0.0057×10^{10} at 5.0 μm. Use the emergent intensity as a function of wavelength at the center of the solar disc to find tempertatures at $\tau_\nu = 1$ for each of the five wavelengths listed above.

c) Find C_ν and D_ν for the 5 wavelengths.

d) Assume $\tau_{\nu,i} =$ constant $\times\ \tau_{\nu,j}$ for each pair (i,j) of wavelengths from the five above. At the physical depth z such that $\tau_{1\mu} = 1$, what is τ at each of the other 4 wavelengths ?

f) Comment on the variation of the opacity as a function of wavelength in the solar atmosphere for $0.2 \leq \lambda \leq 5.0$ μm based on these values for limb-darkening and intensity at the center of the Solar disk.

3.(20 pts) Comparing Simple Approximations with Detailed Model Atmospheres

In the first part of this problem we compare a simple approximation for $T(\tau)$ in the Sun with the result of a detailed model atmosphere calculation for the Sun.

In the directory for the class Web page you will find a model atmosphere for the Sun from Bob Kurucz. (His web site is http://kurucz.harvard.edu). This file is called kurucz_solar_model. The model is caculated for 64 optical depths uniformly spaced on a log scale. The deepest (last) point for this model is at optical depth 100, and there are 8 points per factor of 10, so the spaceing in the log is 0.125, and the next to last point is at $10^{-0.125} \times 100 \equiv \tau = 75$. The Rosseland mean opacity is used to define this optical depth scale.

At each depth, the parameters given are column mass (the mass per unit area integrated from the surface to the desired depth) (units: gm/cm^2), T, P, number of electrons/cm^3, and ABROSS, which is the Rosseland mean absorption coefficient per gram of material. The information in the last two columns is not relevant here.

a) How close is this model to the grey case solution,

$$T^4 \;=\; \frac{3}{4}\, T_{eff}^4\, (\tau + 2/3) \;\; ?$$

What is the largest percent deviation between $0.1 < \tau < 2$ and at what τ does it occur ?

b) Calculate the limb darkening predicted by this Solar model. Compare this to the limb darkening from the Eddington approximation, $S(\tau) = S_0 + S_1\tau$.

c) Compare this with the limb darkening curve observed for the Sun referenced in problem 2.

Now we consider the scaling of model atmospheres with identical chemical compositions and surface gravities but slightly different T_{eff}. Let us denote the reference model with superscript 0. We expect

$$T(\tau) \;\approx\; \frac{T_{eff}}{T_{eff}^0}\, T^0(\tau).$$

Download the grid of Solar metallicity models from Bob Kurucz's web site (kurucz.harvard.edu/grids.html). Use the file "GRIDP00NOVER". Consider the surface gravity $\log(g) = 4.0$ dex. Look at the models between 4500 and 5500 K (they are on a grid with 250 K steps). Check how well this rule is obeyed using the $T_{eff} = 5000$ K model as the reference model atmosphere. What is the largest percent deviation between the scaled $T(\tau)$ and the actual calculated model value for τ from 0.1 to 2.0 for each of these specific models and at what optical depth does this occur ?