

## 1. Historical Notes for Ay 153

### 1.1. Nuclear Physics

First nuclear reaction observed by Rutherford in 1919.  $\alpha$ -particles emitted by radioactive decay of naturally radioactive Po reacting with nitrogen gas. *Po*, discovered to be radioactive by the Curies in 1898, has no stable isotopes.  $^{209}\text{Po}$  has a half life of 102 years, the longest of any of the *Po* isotopes It decays into  $^4\text{He} + ^{205}\text{Pb}$ . The  $\alpha$  particle is detected through  $^{14}\text{N} + ^4\text{He} \rightarrow ^{17}\text{O} + \text{proton}$ .

Understanding of nuclear fission reactions - Hahn and Strassman 1939

First nuclear reactions using accelerated particles, not naturally energetic particles (from radioactive decays), 1932, Cockcroft and Walton

p–p chain worked out by Hans Bethe in 1939 (Nobel prize, 1969)

triple  $\alpha$  process ( $3\text{He} \rightarrow ^{12}\text{C}$ ) Salpeter and Hoyle (1952 through 1954)

Prediction of behavior of nuclear reaction rates – Willy Fowler, Nobel prize 1983

Measurement of many nuclear reaction rates – Kellogg Radioation Lab at Caltech, through early 1990s.

### 1.2. Stellar Spectroscopy and Modelling

Saha – theory of ionization of atoms – 1921

Payne (Cecelia Payne Gaposchkin) applied this to stars and analyzed spectra for stellar abundances, first one to realize that stars are mostly H, Harvard, 1925. First woman tenured faculty member at Harvard.

Unsold - early model stellar atmospheres, 1938

Wildt – (1937) poor agreement of early model stellar atmospheres with real life, suggested missing  $H^-$  opacity, which in fact dominates opacity in stars like the Sun

## 2. Early Famous Women Astronomers

Annie Jump Cannon – worked with Pickering at Harvard College Observatory, did most of the classifications for the Henry Draper Catalog, her job title was “computer”.

Henrietta Leavitt – worked as an assistant to Baade at the Mount Wilson Observatory. In 1912 she discovered the period – luminosity law for Cepheids from measuring plates of the Magellanic Clouds

Cecelia Payne Gaposchkin – see above, she worked out the theory of stellar spectra based on the Saha equation, including the dependence of Balmer and other lines on spectral type.

Beatrice Tinsley – (1970s) worked on spectra and evolution of galaxies. She was a research associate at University of Texas, Austin, then a professor at Yale.

## 3. What is a star ?

Self gravitating sphere (or almost sphere) of gas with a finite definable radius, not easily deformed, not like a cloud in the Earth’s atmosphere

Nuclear reactions occur at least to the point where  ${}^3\text{He}$  is produced.

radiates energy into the surrounding medium. Jupiter also does this, some internal heat is generated due to radioactive decays, but that is much less than the energy it receives from the Sun. Jupiter is not a star.

#### 4. Stars as Physics Laboratories

Here we demonstrate that conditions are routinely achieved in stars which cannot be achieved in physics laboratories on the Earth, and hence by studying the properties of stars we probe regimes of physical parameters that cannot be reached any other way. The opposite extreme is often reached in the interstellar medium, same idea.

**Temperature**  $T(\text{ISM}) \sim 3 - 10\text{K}$ ,  $T(\text{Sun, center}) \sim 20 \times 10^6 \text{ K}$ ,  $T(\text{Sun, surface}) \sim 6000 \text{ K}$ ,  $T(\text{Earth, surface}) \sim 300\text{K}$

$T(\text{lab})$  from 0.1 K (for very small volumes only,  $< 10 \text{ cm}^3$ ) up to 100,000 K, but the latter, only instantaneously, as in an explosion

**Pressure** Earth's atmosphere at sea level = 760 mm of Hg =  $1 \times 10^6 \text{ dyne cm}^{-2}$  (1 mm of Hg = 1 torr, a good vacuum on Earth  $\sim 10^{-7}$  torr  $\sim 10^{-4} \text{ dyne cm}^{-2}$ )

Sun -  $P(\text{surface}) \sim 10^4 \text{ dyne cm}^{-2}$ ,  $P(\text{center}) \sim 10^{17} \text{ dyne cm}^{-2}$ ,  $P(\text{ISM}) \sim 10^{-14} \text{ dyne cm}^{-2}$

**Number Density** ( $N$  = mean number density, units atoms (or molecules, or other particles)/ $\text{cm}^3$ ).

$N(\text{ISM}) \sim 1 \text{ atom/cm}^3$ ,  $N(\text{Earth atmosphere, near sea level}) \sim 3 \times 10^{19} \text{ molecules/cm}^3$ ,  $N(\text{Sun, outer atmosphere}) \sim 5 \times 10^{16} \text{ atoms/cm}^3$ ,  $N(\text{neutron star, center}) \sim 10^{26} \text{ neutrons/cm}^3$ .

**Density** ( $\rho$ ) Air(Earth's atmosphere, sea level)  $\sim 10^{-3} \text{ gm/cm}^3$ . Water  $1 \text{ gm/cm}^3$ , lead  $2.2 \text{ gm/cm}^3$ , ISM  $2 \times 10^{-24} \text{ gm/cm}^3$ , Sun(center)  $100 \text{ gm/cm}^3$ , Neutron star (mean density)  $\gtrsim 2 \times 10^{12} \text{ gm/cm}^3$ .

**Timescales** Age of Earth (radioactive dating)  $\sim 3 \times 10^9 \text{ yr}$ , Life on Earth (fossils)  $\sim 6 \times 10^8 \text{ yr}$ , Supernova explosion  $< 1 \text{ minute}$ , Nuclear reaction timescale, radiation interaction  $< 1$

sec to  $10^9$  yr, Lab time scale  $< 1$  sec to 20 yr.

**New physics !** nuclear reactions, energy production, degeneracy, equations of state at high  $P$  and  $\rho$ , neutrino physics...

### Some numbers relevant to planets

$M(\text{Earth}) \sim 3 \times 10^{-6} M(\text{Sun})$ ,  $M(\text{Jupiter}) \sim 1/1000 M(\text{sun})$ , total mass of 9 planets  $\sim 450 M(\text{Earth}) \sim 1/1000 M(\text{Sun})$

Total angular momentum of planets  $\sim 3 \times 10^{50}$  gm cm<sup>2</sup>/sec, Solar angular momentum (almost all from its rotation)  $\sim 1.6 \times 10^{48}$  gm cm<sup>2</sup>/sec, 1/200 that of the 9 planets.

### The Vogt-Russell Conjecture

The properties of a star depend only on  $M, X, Y, Z, t$ , which are mass, fractional abundance of H, of He, of elements heavier than He (so  $X + Y + Z = 1$ ), and time (i.e. age).

These variables determine the observed parameters  $L, R, T_{eff}, g$ , which are luminosity, radius, effective temperature and surface gravity.

Other potential parameters which have a minor effect (at least in most circumstances) include  $\Omega, B$ , which are angular velocity (rotation) and magnetic field strength.

## Stellar Distances

Direct measurement via parallax - angle subtended by the orbit of the Earth around the Sun. The mean distance to the Sun (the mean radius of the Earth's orbit) is called the astronomical unit (AU).  $1 \text{ AU} = 1.5 \times 10^{13} \text{ cm}$ . At a distance of 1 parsec (1 pc) 1 AU subtends 1 arcsec, so, using the small angle approximation,  $1 \text{ pc} = 1 \text{ AU}/1 \text{ arcsec}$  (in radians)  $= 1.5 \times 10^{13} \times (57.2 \times 3600) = 3.08 \times 10^{18} \text{ cm}$ .

Movie illustrating parallax:

[www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/parallax.html](http://www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/parallax.html) (from Richard Pogge)

We now introduce the concept of proper motion, as stellar positions change through both parallax and proper motion, and astrometric programs seek to measure both of these.. Since stars move around with random motions within our Galaxy as well as rotate around its center, there are two components to a star's motion relative to the Sun. The first is the velocity along the line of sight, the radial velocity,  $v_r$ , which can be measured through the Doppler effect. The second is the motion perpendicular to the line of sight, i.e. in the plane of the sky. This motion, called "proper motion", can be detected as the position of the star on the sky will change with time, moving linearly with time in a fixed direction. Thus the parallax is a small annual oscillation over the larger and constant with time proper motion.

The proper motion( $\mu$ ) as seen on the sky in units of radians/sec is  $\mu = v_T/d = v \sin(\phi)/d = v_r \tan(\phi)$ , where  $v_T$  is the tangential projection of the velocity,  $d$  is the distance to the object, and  $\phi$  is the angle between the velocity vector and the line of sight. Putting in the appropriate constants, we find that  $\mu = v_t/4.74d$ , where  $v_t$  is given in km/sec,  $d$  in pc, and  $\mu$  in arcsec/yr.

The angular resolution of a telescope on the surface of the Earth is limited to about 1 arcsec by turbulence and thermal variations in the Earth's atmosphere. Only with specialized

techniques can this limit be overcome.

Best centroiding of image of a bright point source: about 1/100 of the image size. So from the Earth's surface, we can measure a parallax of 0.01 arcsec or larger, corresponding to distances of 100 pc or smaller. At larger distances, we must rely on indirect estimates based on calibrated properties, such as the period luminosity relationship of Cepheid variables, the peak brightness of supernovae, etc.

Telescopes in space are not affected by this, and the size of the image of distant point source that they produce is limited by diffraction; their performance is not degraded by the Earth's atmosphere. In such a case,  $\theta_d \sim 1.2\lambda/D(\text{tel})$ , where  $\lambda$  is the wavelength of observation,  $D(\text{tel})$  is the diameter of the telescope primary mirror, and  $\theta_d$  is the diffraction image size (in radians). If can again centroid to 1/100 of the image size, then for  $D(\text{tel}) \sim 4$  m, distances out to  $10^4$  pc (10 kpc) can be determined.

Astrometric space missions: Hipparcos (ESA satellite) - small telescope, 29 cm diameter primary mirror, 1 mas accuracy positions for 118,000 stars (mostly  $V < 7.9$  mag) launched 1989, took data for about 4 years, years of data analysis produced the Hipparcos catalog, released in 1997. Many arguments about how accurate the parallaxes in this catalog actually are. It is now clear that there are some problems specific to certain situations in the catalog.

GAIA – another ESA project, to be launched in 2011, collecting area  $30 \times$  that of Hipparcos, position measurements  $200 \times$  more accurate than Hipparcos.

SIM (space interferometry mission) – JPL project, very ambitious project, very technically challenging, launch maybe 2011, very high accuracy astrometry, search for planets, goal is to measure positions for stars with  $V < 15$  mag to an accuracy of  $4 \times 10^{-6}$  arcsec, with many measurements of the each star. This will yield parallaxes for distances

of up to  $10^5$  pc (100 kpc), which includes the entire Milky Way Galaxy. SIM can measure proper motions and parallaxes for the entire galaxy !

### **Parallax Projects**

Either on the ground or in space, a parallax project must observe a star repeatedly over a year (probably over several years) to measure a parallax.

The project must have a list of stars to be observed BEFORE the project can begin (an input catalog).

There must be a scheduling algorithm (minimize telescope moves, dead time, maximum coverage at the right time of the year for an object, etc).

There must be ongoing monitoring of the whole survey for quality.

After enough measurements have accumulated, data analysis for  $\pi$  values for each star in the input catalog.

Instrument and telescope must be DEDICATED, and cannot be shared. The precision required is so difficult to achieve that one can't afford any changes to setups, spatial scales, etc over the entire observational phase of the survey.

Problem: the reference frame of non-moving (fixed) objects. To do the analysis one needs a set of objects at infinite distance that are not moving against which motions of nearby objects can be measured.

Galaxies are too big.

QSOs are OK (most of the time), but they are rare, and they are much fainter than the survey objects.

Faint stars are numerous, but they are in fact all moving too.

At the precision of SIM, even QSOs are not ideal. If the QSOs are ejecting blobs that move away from the central engine, and if the spatial resolution is high enough, even a QSO will not be a point source, and since blobs expand away, the position of the centroid of the image of a QSO may vary slightly with time.

The solution for parallaxes turns into a big minimization problem.

### **Distances to nearby star clusters**

Assume a group of stars are moving together through space, and one can measure  $v_r$  and  $\mu$  for each object, which are the the group velocity vector projected appropriately. If the group is close enough to the Earth, then the proper motions of the member stars will appear to converge to a point (the convergence point), rather than be parallel to each other, as we would naively expect. With this information, the mean velocity of the group and its distance can be determined. Sketches below from Mihalas and Binney, *Galactic Astronomy*, fig. 3.1 and 3.2 illustrate the general idea.

The moving cluster method of determining distances is less direct than parallax measurements, and it requires special conditions (i.e. a group of stars moving together in space) to be applicable.

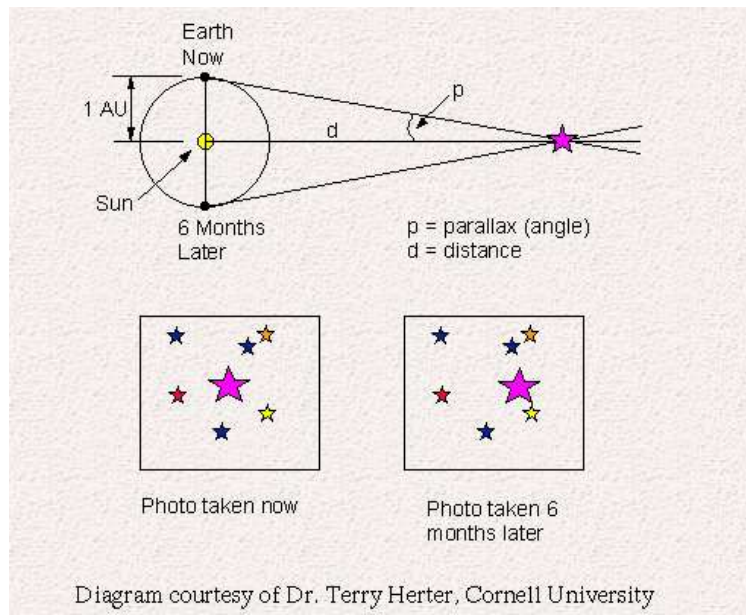


Fig. 1.— Top: the definition of parallax. The bottom panel illustrates the movement of a nearby star as compared to the background of faint distant stars.

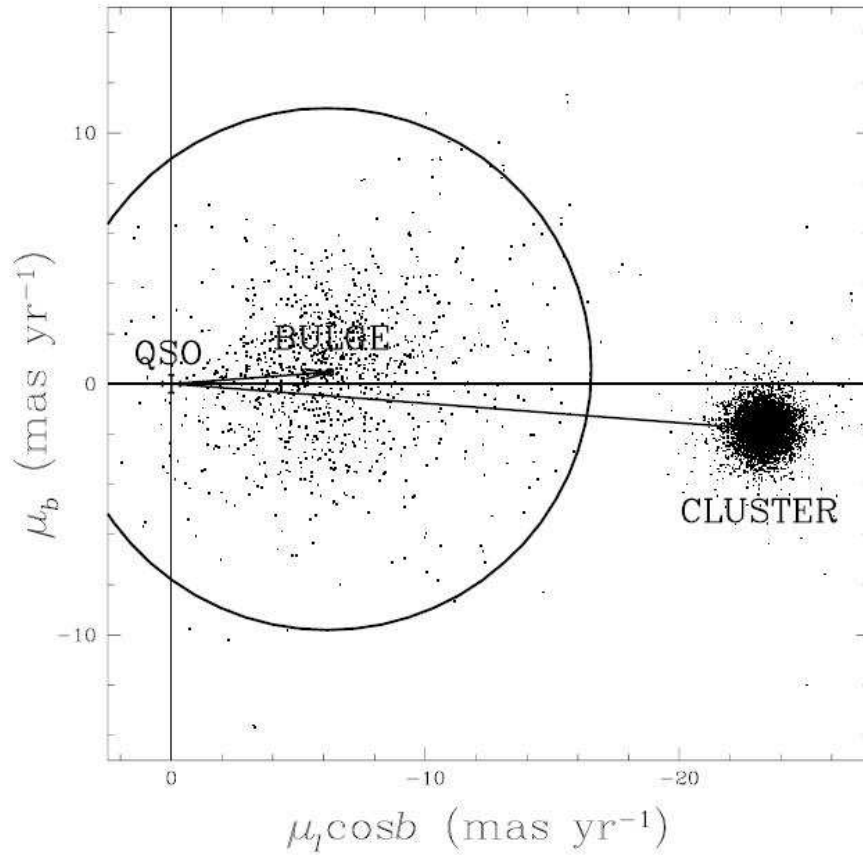
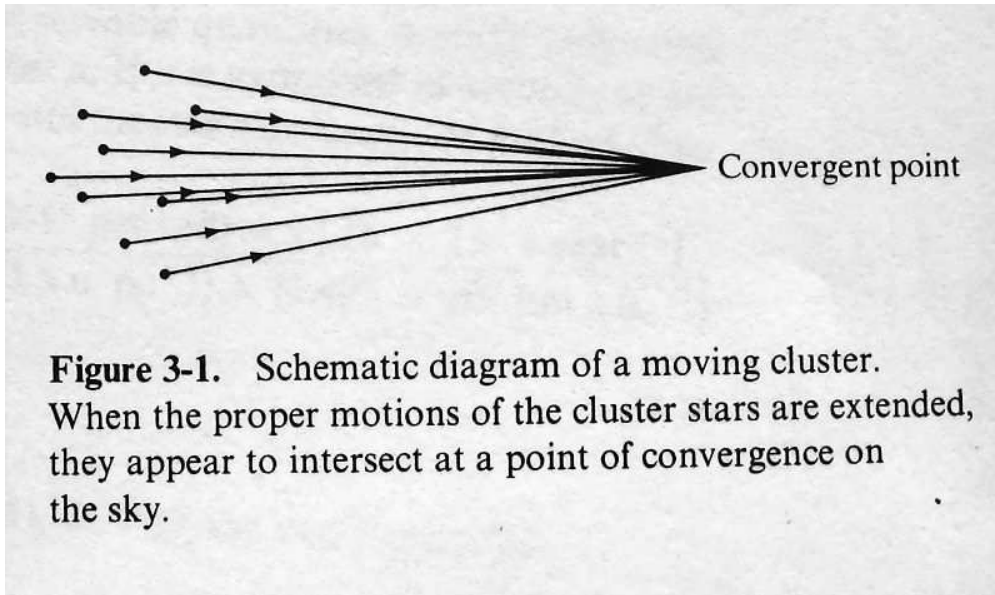
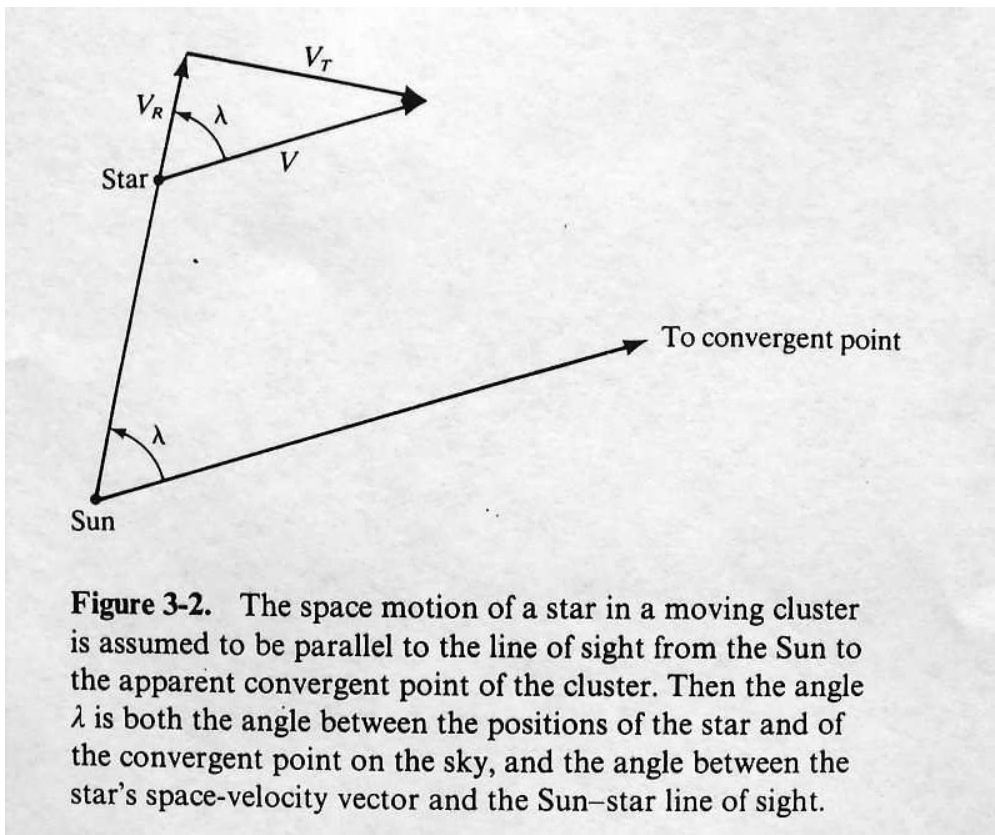


Fig. 6.— In this figure we show the proper motions of the bulge stars, and M4 stars, in Galactic coordinates. The extragalactic point source has been chosen as the origin. The meaning of the circle is given in the text.

Fig. 2.— The proper motions of the bulge stars and of the stars in the globular cluster M4 are shown in Galactic coordinates. These were deduced from multi-epoch HST images. A single extragalactic point source was chosen as the origin. This is Fig.6 of Bedin, Piotto, King & Anderson, 2003, AJ, 126, 247



**Figure 3-1.** Schematic diagram of a moving cluster. When the proper motions of the cluster stars are extended, they appear to intersect at a point of convergence on the sky.



**Figure 3-2.** The space motion of a star in a moving cluster is assumed to be parallel to the line of sight from the Sun to the apparent convergent point of the cluster. Then the angle  $\lambda$  is both the angle between the positions of the star and of the convergent point on the sky, and the angle between the star's space-velocity vector and the Sun-star line of sight.

## Mass and Radius Measurements for Stars

### Masses

The mass of the Sun can be determined if one knows the radius of the Earth's orbit around the Sun and assumes  $M(\text{Earth}) \ll M_{\odot}$ , and the orbit of the Earth around the Sun is approximately circular.

Newton's law:  $GM(\text{Earth})M_{\odot}/r^2 = M(\text{Earth})v^2/r$ . If  $r$  is known, then  $v$  (the orbital velocity) is known ( $v = 2\pi r/P$ ,  $P$  is the period, 1 year). This gives  $M_{\odot} = 1.99 \times 10^{33}$  gm.

All other stellar masses are from binaries except for a few gravitational redshifts for white dwarfs. The problems of binaries are: we only see the orbit projected onto the sky, not the full 3D orbit, and we do not know the inclination angle, the angle that the orbital plane makes with the plane of the sky.

Also for binary stars, the assumption that  $M_2 \ll M_1$  is rarely valid. (The convention is that the most massive component of a binary is star 1.) This means that the stars in the system orbit around the center of mass which may not be in or even close to the  $M_1$ , unlike the planet case, where the center of mass is within, and close to the center, of the Sun.

Visual Binaries, the orbit of each star is an ellipse about the center of mass of the system, a circle when the ellipticity is 0. Let  $a_1$  and axis of each orbit (the radius for circular orbits). Then we define  $a$  as  $a_1 + a_2$ . Assume the parallax  $\pi$  of the system is known. Then for an orbital separation between star 1 and 2 on the sky,  $\theta$ ,  $a/1 \text{ AU} = \theta/\pi$  and Newton's law becomes

$$M_1 + M_2 = \frac{4\pi^2 a^3}{P^2 G}$$

or  $(M_1 + M_2)/M_{\odot} = (\theta^3/\pi^3)(1\text{yr}/P)^2$ .

Consider the typical accuracy of the data:  $\pi = 0.050 \pm 0.05''$ , so 10% for  $\pi$ , same for  $a$ . This leads to a large ( $\approx 30\%$ ) error in  $M_1 + M_2$ .

How can we separate  $M_1$  and  $M_2$  to complete the solution ? We need  $v_r$  amplitudes or proper motion undulations of the two stars. The former requires a double lined spectroscopic binary, period  $P$ , radial velocity amplitudes of  $k_1$  and  $k_2$ , in km/sec. The latter requires a binary close enough to detect the proper motion undulations of each of the component stars.

Then set  $q$  to be the ratio of the masses  $q = M_1/M_2 = k_2/k_1$ .  $q$  is a measured quantity for a double lined spec. binary.  $1 + 1/q = (M_2 + M_1)/M_1$ , and  $1 + q = (M_2 + M_1)/M_2$ . For proper motion undulations,  $q = M_1/M_2 = \mu_2/\mu_1$ .

Let  $i$  be the angle of inclination of the orbit plane to the line of sight. Then  $P = 2\pi r/v = 2\pi r_2/(k_2/\sin(i)) = 2\pi r_1/(k_1/\sin(i))$

This provides the necessary separation, i.e.

$$r_1 = \frac{M_2}{(M_1 + M_2)}a = \frac{a}{(1 + q)}, \quad r_2 = \frac{a}{(1 + 1/q)}$$

Combining the above we get

$$G(M_1 + M_2) = \frac{4\pi^2 a^3}{P^2} = \frac{4\pi^2}{P^2} [(1 + 1/q)r_2]^3 = \left(\frac{P}{2\pi}\right) \left(\frac{k_2^3}{\sin^3 i}\right) (1 + 1/q)^3$$

So we finally have the separate equations:

$$(\sin^3 i)M_1(1 + 1/q)G = \frac{P}{2\pi}k_2^3(1 + 1/q)^3$$

and

$$(\sin^3 i)M_2(1 + q)G = \frac{P}{2\pi}k_1^3(1 + q)^3$$

With these two equations and the one for the sum of the masses of the two components, we can now solve for  $M_1$  and  $M_2$  separately.

The inclination of a binary system is unknown. Lets look at some mean values, assuming a random orientation of the orbital plane with respect to the plane of the sky. Let  $\theta = 90 - i$ . Then

$$\langle \sin(i) \rangle = \frac{\int_0^{2\pi} \int_0^{\pi/2} \sin(90 - \theta) \cos\theta d\theta d\phi}{\int \int \cos\theta d\theta d\phi} = \frac{2\pi \int_0^{\pi/2} \cos^2\theta d\theta}{2\pi}$$

Since  $\langle \cos^2\theta \rangle = 1/2$ ,  $\langle \sin(i) \rangle = \pi/4$ . But we need  $\langle \sin^3(i) \rangle$ , which is  $\int_0^{\pi/2} \cos^4\theta d\theta$ . Using a table of definite integrals, this is evaluated as  $3\pi/16$ . So now

$$G \langle (M_1 + M_2) \rangle = \left(\frac{P}{2\pi}\right) \frac{k_2^3 (1 + 1/q)^3}{(3\pi/16)}$$

## Eclipsing Binaries

In this case, the inclination of the orbit is known, i.e. the plane of the orbit is, to within a few degrees, perpendicular to the plane of the sky ( $i \approx 90^\circ$ ). Of course, this means that only a small fraction of spectroscopic binaries are eclipsing binaries.

For a system with  $i = 90^\circ$ , one of the two eclipses is a total eclipse, and the light curve has a flat bottom during that eclipse.

If  $i$  is not close enough to  $90^\circ$ , the eclipses are partial, as shown in the figure.

Given  $i$ , we can solve for the two masses individually,  $M_1$  and  $M_2$ .

The range of masses found for stars is from  $\sim 0.1M_\odot$  (Ross 614B) to  $60 M_\odot$  (Plaskett's star), with a maximum near  $120 M_\odot$  (HD 93129A). We can understand the lower mass

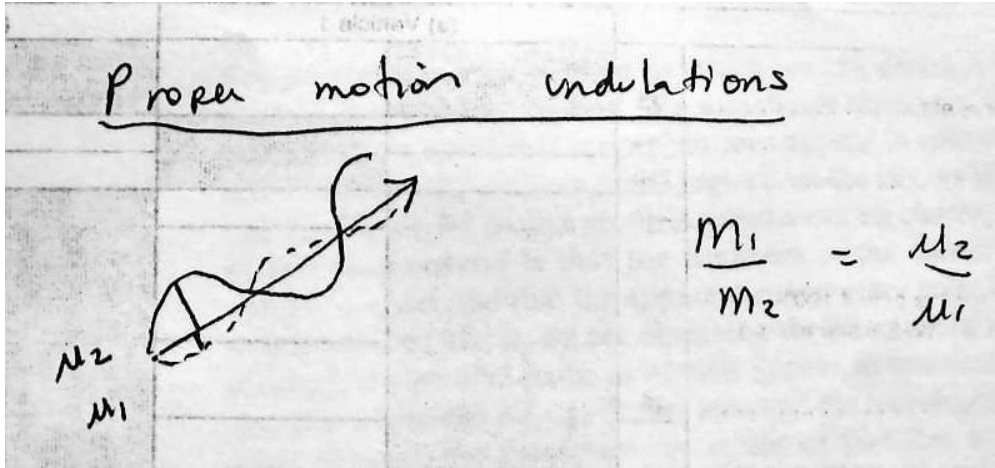


Fig. 3.— Sketch of proper motion undulations due to orbital motion in a binary star system.

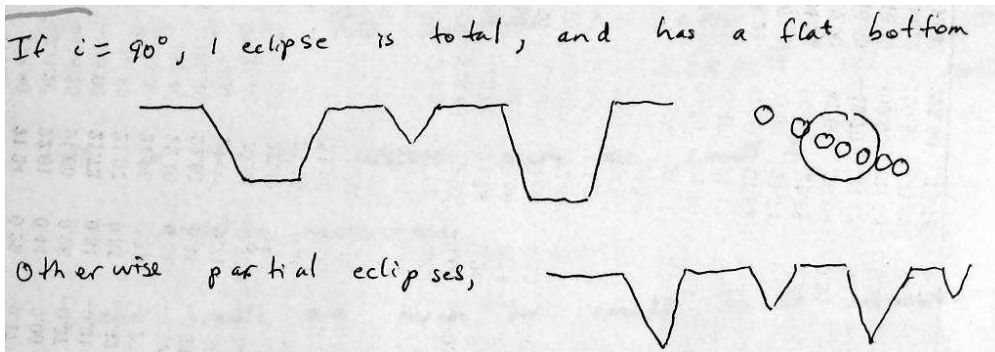


Fig. 4.— The light curve of an eclipsing binary with total eclipses ( $i = 90^\circ$ ) and with partial eclipses ( $i$  not exactly  $90^\circ$ ).

limit, the center of the star is too cool for thermonuclear reactions to ever ignite efficiently, at lower masses than this limit, stars are called “brown dwarfs”. We will see that the upper mass limit is related to pulsational instabilities brought on by the very high radiation pressure in such luminous stars.

Movie: simulation of orbiting binary stars by Terry Herter of Cornell,  
<http://instruct1.cit.cornell.edu/courses/astro101/java/binary/binary.htm>.

Of course, there are triple systems as well !

### Searching for Planets

A Jupiter around a Sun at a distance of 10 pc would produce an astrometric wobble with an amplitude of 0.5 milliarcsec (mas), while an Earth-like planet would have a wobble of only 0.3 mas. This is very small and impossible to measure without use of interferometry in space. The figure below, from [planetquest.jpl.nasa.gov/science/finding\\_planets.cfm](http://planetquest.jpl.nasa.gov/science/finding_planets.cfm), shows the result from a model for the entire Solar system, including perturbations from Saturn and other planets of the astrometric displacement of the Sun due to Jupiter.

Photometric search - look for transits, light of star + planet becomes light of star only, small drop in flux (about 1%) is hard to measure from the ground, easier with HST, see Brown et al, 2001, ApJ, 552, 699. With transits the radius of the planet can be derived, and since the mass is known from the orbit,  $\rho$  can be obtained, which constrains the composition of the planet. Maximum drop in flux for planet of HD 209458 is 1.5% of continuum,  $R_p = 1.35 \pm 0.06 R_J$ ,  $i = 86.68 \pm 0.14$  deg,  $R(star) = 1.146 \pm 0.050 R_\odot$ .

Radial velocity searches, which have been the most productive thus far, depend on the  $v_r$  accuracy that can be achieved. This is now down in the 1 m/sec range, a phenomenal technical achievement. See the latest result from HARPS group at ESO, Udry, Mayor, Benz et al, 2006, A&A

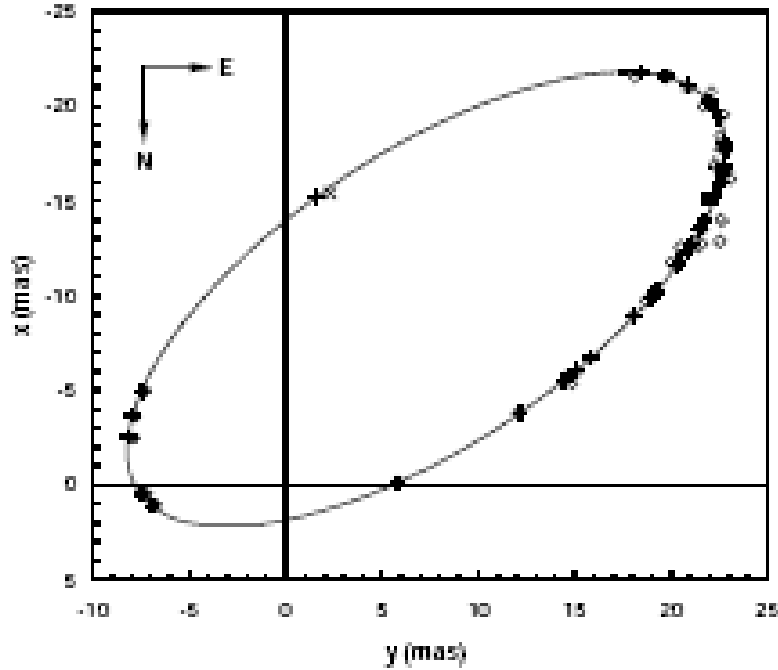
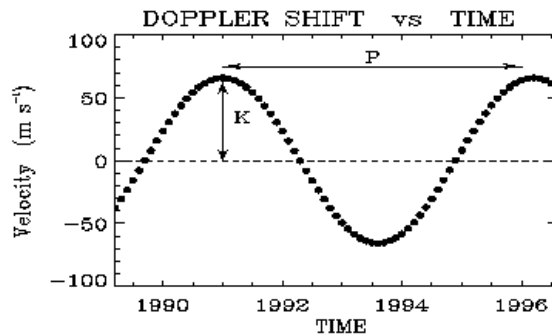


Figure 3. The orbit of  $\beta$  Centauri. The data points are the SUSI results from Table I except for the MAPPIT point which is from Robertson et al. (1998). SUSI observations: 1997  $\diamond$ ; 1998  $\triangle$ ; 1999  $\circ$ ; 2000  $\bullet$ ; 2002  $\square$ . MAPPIT observation: 1995  $\times$ . The points on the fitted orbit that correspond to the observational points are marked with  $+$ . Further details are given in the text.

Fig. 5.— The orbit of a binary star measured with an optical interferometer. Note that the scale is in milliarcsec (mas). (Figure 3, from Davis, Mendez, Seneta, Tango et al, 2004, MNRAS, orbit of  $\beta$  Cen)

## DOPPLER - WOBBLE TUTORIAL



Kepler:

$$r^3 = \frac{GM_*}{4\pi^2} P^2$$

Observe Period.

$$V_{PL} = \sqrt{GM_*/r}$$

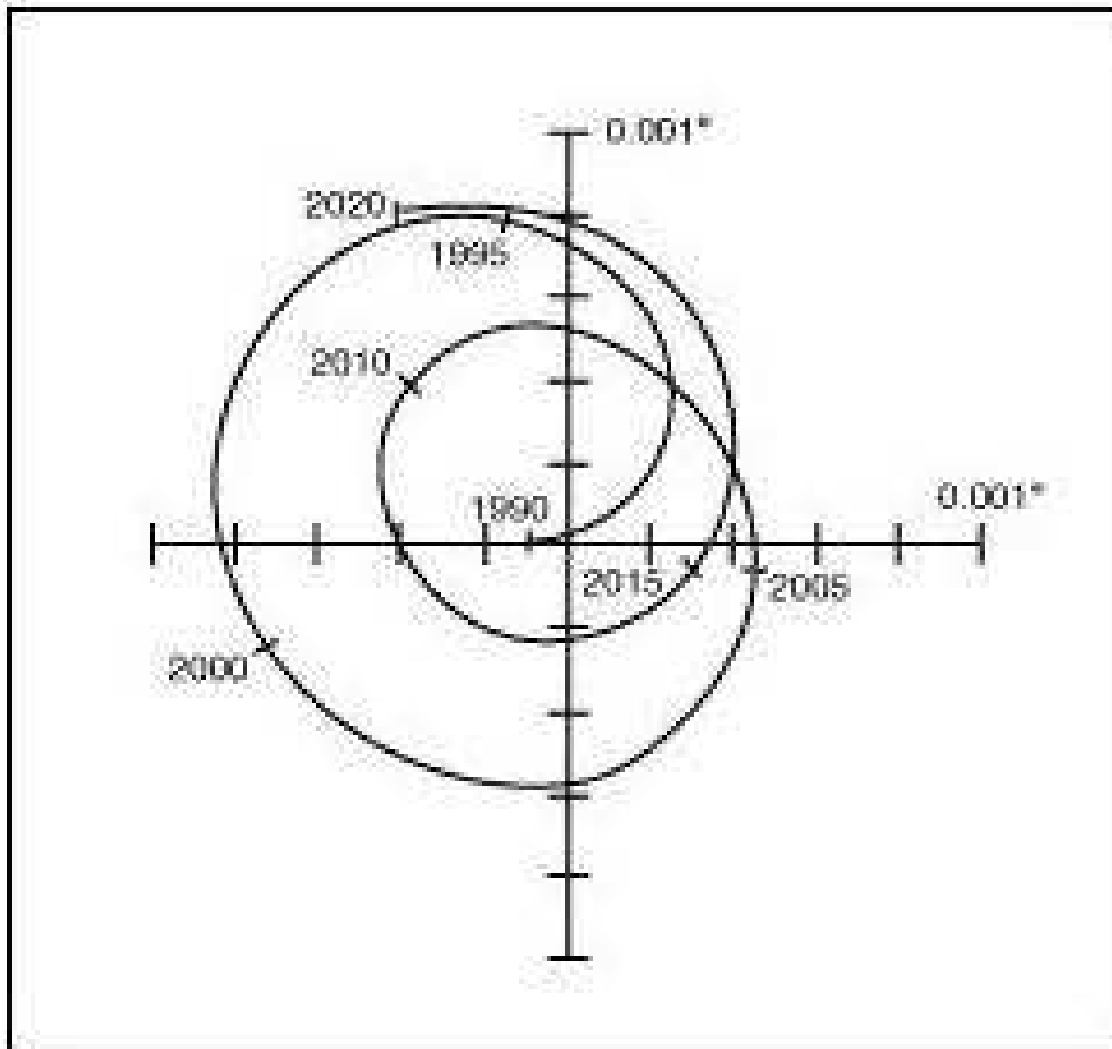
-> Vel. of Planet .

Momentum Conservation:

$$M_{PL} = M_* V_* / V_{PL}$$

Observe  $K = V_* \sin i$

$$\implies M_{PL} \sin i$$



*Astrometric displacement of the Sun due to Jupiter as it would be observed from 10 parsecs, or about 33 light-years.*

Phased light curve for all four transits, assuming a planetary orbital period of 3.52474 days. The time series for each transit has been scaled to have the same average intensity over the second and fifth (out-of-transit) orbits.

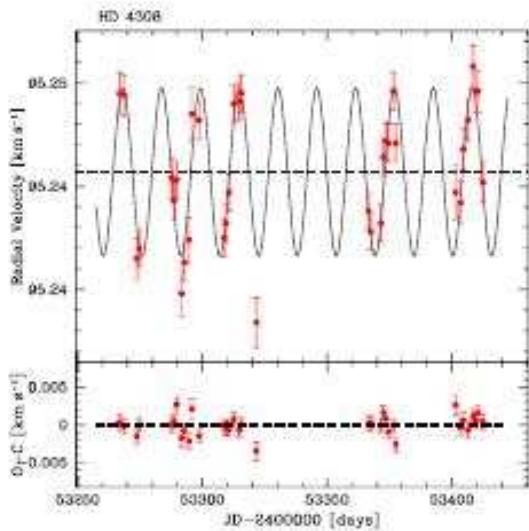
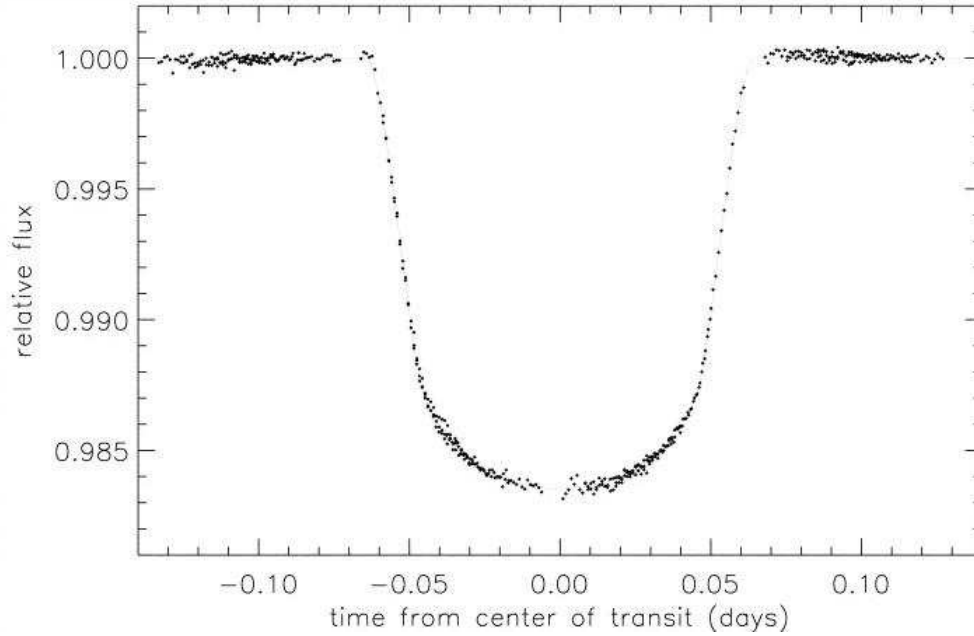


Fig. 5. Intermediate season of HARPS radial velocities for HD 4308. The best Keplerian fit to the data (solid curve) gives a minimum mass of  $14.1 M_{\oplus}$  and an orbital period of 15.56 days for the planet.

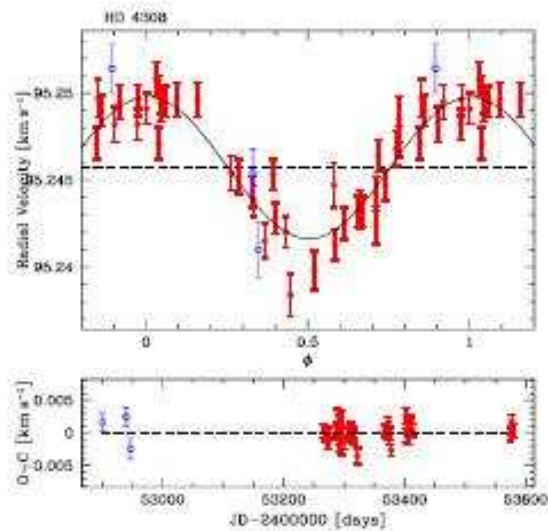


Fig. 6. Phase-folded radial velocities of HD 4308 superimposed on the best Keplerian orbital solution (solid curve). The 3 older points obtained before introducing the new observing strategy (see text) are indicated by open symbols.

Information on statistical properties of the  $\sim 180$  (as of Aug 2006) exo-planets found thus far, see [www.exoplanets.org](http://www.exoplanets.org) for mass, distance from star, and eccentricity distributions, as well as discussions of the fraction of stars with planets, and why this might depend on metallicity of the star, etc.

### Stellar Radii

$R_{\odot} = 6.95 \times 10^{10}$  cm. At  $d = 1$  pc,  $R_{\odot}$  subtends an angle of  $2.2 \times 10^{-8}$  radians, which is  $4 \times 10^{-4}$  arcsec, much too small to be directly measured, even with a telescope in space.

Eclipsing binaries offer a way to measure stellar radii through their light curves. The entrance and egress times in the light curve sketched below corresponding to the 4 points marked in the figure are known, and

$$\frac{t_4 - t_1}{P} = \frac{2(R_1 + R_2)}{2\pi a}, \quad \frac{t_3 - t_2}{P} = \frac{2(R_1 - R_2)}{2\pi a}.$$

A more careful treatment allows for the details of the variation of the illumination across the surface of each star and the gravitational distortions from spherical shape of each star.

Using model atmospheres and limb darkening and multi-color photometry of such a system, we can also derive  $T_{\text{eff}}$  for each of the component stars throughout the period.

Existing interferometers are now good enough that for the nearest and largest stars, some resolution across the stellar surface is possible, and limb darkening can be directly measured.

Direct radius measurements are only possible with interferometry (see brief description below) and only for the nearest and largest stars.

Lunar occultations (a form of interferometry using the limb of the moon as a star is occulted can be used to measure stellar radii. The main problem with this technique is that the orbit of the moon is fixed and only covers a small part of the  $4\pi$  steradian of the sky. Also the stars must be very bright so as to get enough signal in the short crucial time when the moon occults the star, so its actual applicability is limited.

With sufficient time resolution on a bright star, instead of seeing a sharp rise in intensity as the star emerges from behind the moon, one sees a diffraction pattern. The characteristic wavelength of oscillations is  $h^2/(2D) = \lambda/2$ , where  $h$  is the distance of the star from the sharp edge of the moon along its orbit and  $D$  is the Earth-moon distance.  $h = \sqrt{(\lambda D)}$  is the length of 1 Fresnel zone. (This subject is discussed in the Feynman Lectures, vol. 1, pg 30-9.) Measurement shows  $h \approx 14$  m, as we would expect from  $\sqrt{(5000 \text{ \AA})D}$ .

The velocity of the moon around the Earth is  $2\pi D/(30 \text{ days}) = 930$  m/sec. Thus the time for the moon in its orbit to cross 1 Fresnel zone is  $14 \text{ m}/(930 \text{ m/sec}) = 15 \times 10^{-3}$  sec. A very bright star is required to get good signal over  $t < t(\text{Fresnel})/10 \approx 10^{-3}$  sec. A very good fast accurate detector is required as well.

To measure the diameter of the star, the difference between the diffraction pattern of a point source and the slightly rounder edges of a diffraction pattern of an extended source (see sketch) must be detectable. This makes it even harder to measure stellar radii.

We should note that there are many giant and supergiant stars with  $R \sim 100R_{\odot}$ , and for such stars that are reasonably close, direct measurement of  $R$  using an interferometric technique is feasible.

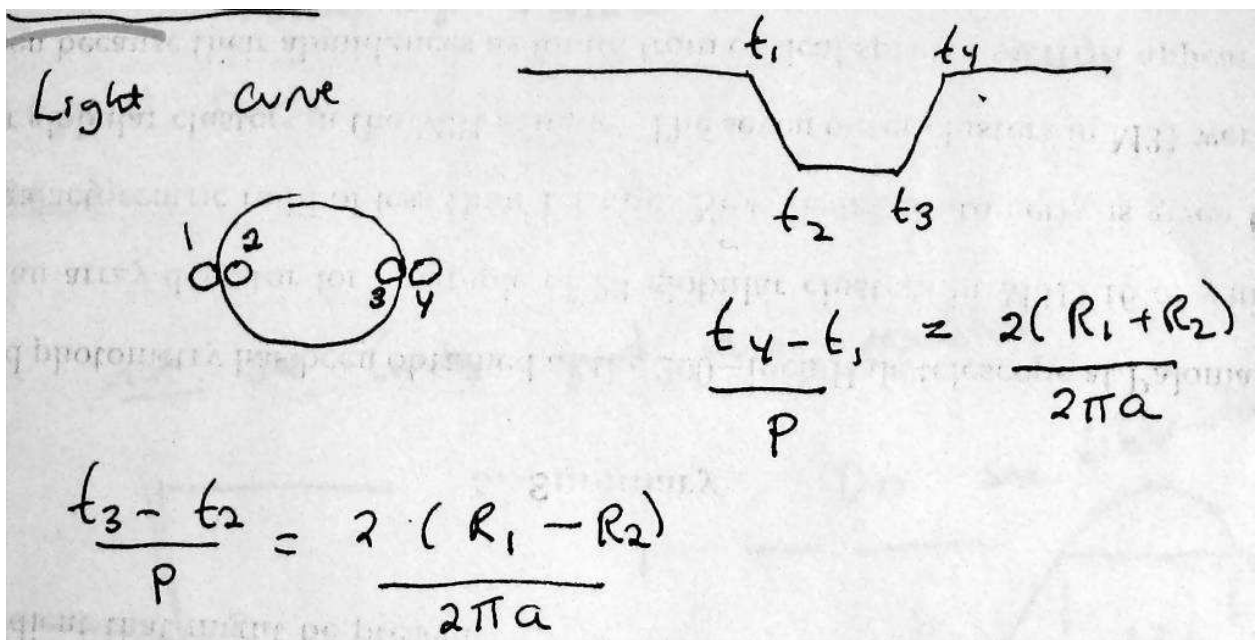


Fig. 6.— A sketch of light curve of an eclipsing binary with key ingress and egress points in the orbit marked.

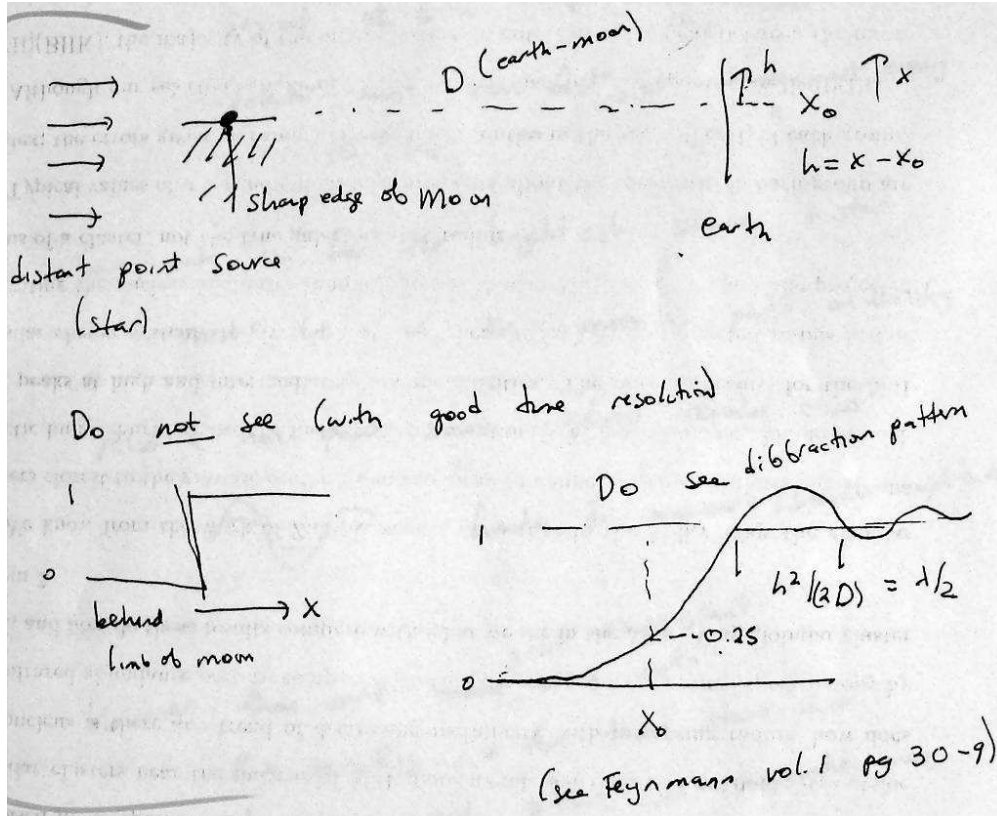


Fig. 7.— Sketches of the principles of using lunar occultations to measure stellar diameters.

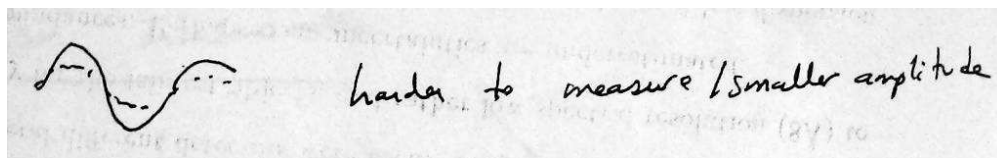


Fig. 8.— Sketch of the diffraction pattern for a point source as compared to an extended source as viewed with lunar interferometry.

## Improving the Seeing for Ground Based Telescopes

To get the orbit of a binary, we must be able to see a star moving on the sky throughout its orbit. If  $a$  at the distance of the binary cannot be resolved, the orbit cannot be determined. Thus the seeing limit forces us to only use binaries that are fairly close to Earth. There are several ways of beating the seeing limit imposed by the Earth's atmosphere and reaching the diffraction limit, which for the 200-inch Hale Telescope at Palomar is  $\theta_d \sim 10^{-6}$  radians, which is  $\sim 0.02''$ , at a wavelength of  $0.5\mu$ .

Speckle interferometry – post-processing of a sequence of very short exposures, with each frame taken on a timescale such that the atmospheric phase error is constant over the maximum possible length (which is the Fried length, called  $r_0$ , with a typical value of 20 cm at  $0.5\mu$ ). This timescale is very short,  $r_0/v(\text{wind}) \sim 0.02$  sec.

The image of a point source as seen with a large telescope is a speckle pattern, a splotch of  $(D/r_0)^2$  images, dancing around rapidly with time, and each of these images (called “speckles”) is diffraction limited image for a size  $r_0$ . Effectively each cell is diffraction limited, but has an additional random phase varying rapidly with time with respect to the other cells, and the total number of such cells is  $(D/r_0)^2$ .

This speckle pattern can be resolved with very short exposures. A cross correlation of the sequence of images will yield the Fourier transform of the image of the source.

MOVIE of dancing speckles –

<http://spiff.rit.edu/classes/phys445/lectures/atmos/seeing.mpg>

With long exposures, the image becomes a big blur corresponding to the normal seeing limit.

Adaptive optics involves deformation of the pupil, divided into cells as above, in a way that corrects the phase errors of each cell, and thus produces a diffraction limited image.

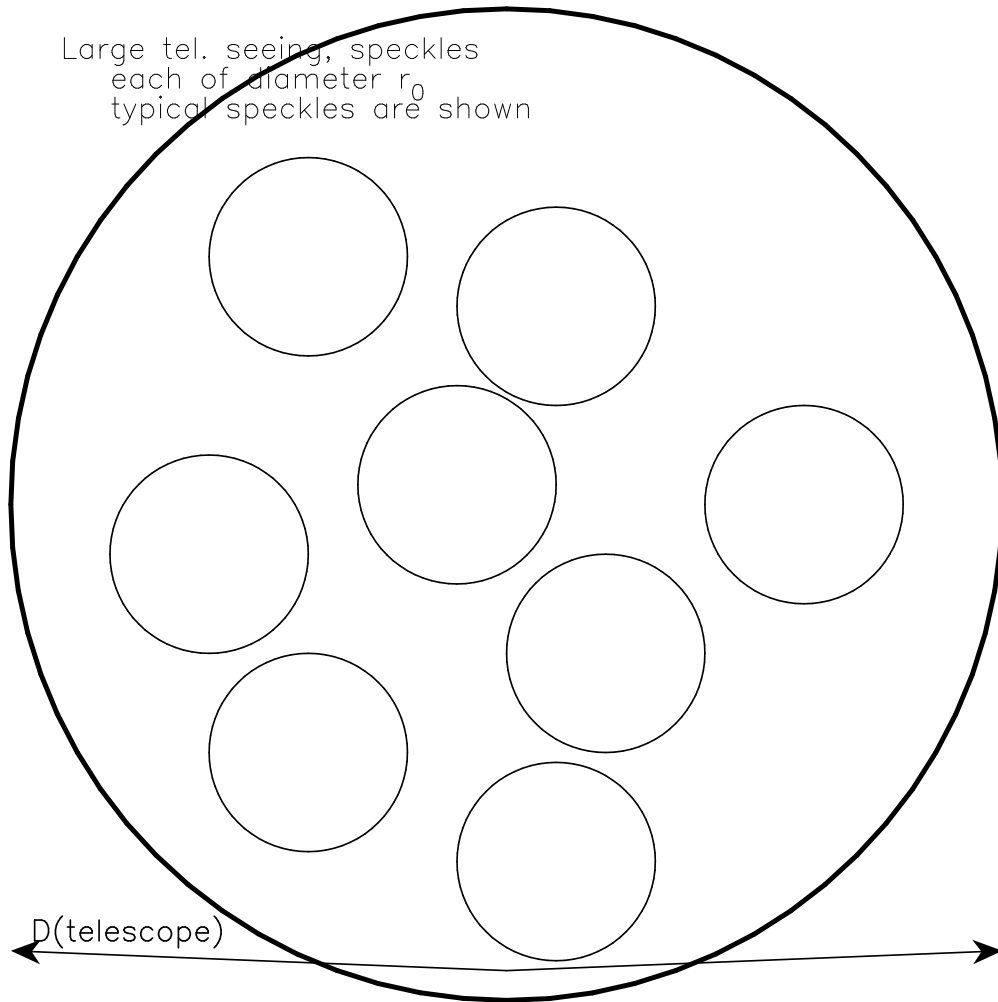


Fig. 9.— A sketch of the origin of the seeing limited image for a large ground-based telescope.

For various reasons, including the possibility that the measurement of the phase error is not perfect, the image is not quite diffraction limited, having a sharp diffraction limited core containing some fraction of the light, the rest being spread out into size of the seeing limited image. Adaptive optics happens in real time during the observations and requires corrections at the necessary rapid rate so that the corrected optical figure will still be appropriate for the atmospheric distortions of the incoming light rays from the star. Complicated miniaturized electronic devices and fast computers are required to implement an adaptive optics system. AO is not adequate for measuring stellar diameters. It is useful for measuring binary orbits and accretion disks of nearby stars can be resolved.

Interferometry is a way of obtaining a very large  $D$  (telescope diameter), by using an unfilled aperture of 2 or more separate telescopes each observing the same source at the same time. A Fourier analysis will produce an image of the source. If the telescopes can be moved with respect to each other (or the rotation of the Earth can also be used here) and observations made at different separation vectors of the telescopes, there are more baselines contributing. The more filled in the aperture is (i.e. the more telescopes and more baselines between them), the more accurate the resulting image of the source will be. While interferometry is most common for radio astronomy, interferometry at optical wavelengths was achieved in 1920 by A.E. Michaelson at Mount Wilson Observatory for the purpose of measuring stellar diameters of the nearest giant stars. He used two beams defined by small mirrors at each end of an optical bench supported from the top ring of a single large telescope (the 100 inch at Mount Wilson), as in the sketch below; the baseline for his instrument was 20 ft, later increased to 50 ft. For a 50 ft baseline used at 500 nm, the minimum angular size that can be detected is  $\theta = \frac{5 \times 10^{-5}}{50 \times 12 \times 2.54} = 0.007$  arcsec, small enough for the closest giant stars. The interference pattern from the two beams determines the stellar diameter.

A number of optical and near-IR interferometers of various design are in use today for this purpose, for determining orbits in binary systems, for measuring parallaxes, etc. The Palomar Testbed Interferometer is one example. The Sydney (Australia) interferometer is another such device, with a baseline of 188m, hence a minimum angular resolution of 0.0007 arcsec.

Whatever technique is used to improve the angular resolution, either by going into space or by one of the complex methods described above, the result is that binaries which have smaller separations and/or are more distant can be resolved and studied.

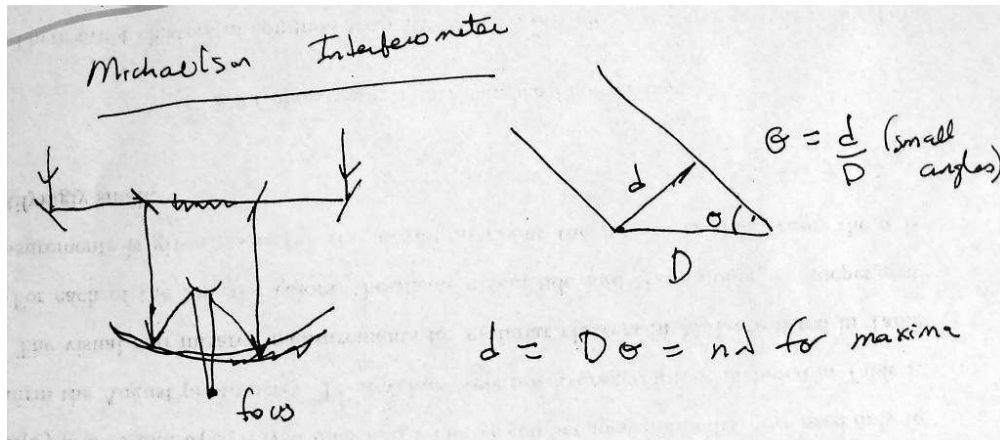


Fig. 10.— A sketch of design of Michaelson's interferometer.

## Astronomical Units

$\text{mag}(\text{nitide}) = -2.5\log_{10}(\text{flux})$ , with 0 mag defined by Vega. Apparent mag  $\equiv$  flux received at Earth (varies with distance of object), absolute mag  $\equiv$  flux at a fixed distance of 10 pc. Notation convention:  $M$  for abs. mag and  $m$  for apparent mag, do not confuse with mass !

$$F(\text{at Earth})/F(10 \text{ pc}) = (10 \text{ pc})^2/D^2, \text{ so } M = m - 5\log(D) + 5.$$

A logarithmic scale is useful because of the large range in  $F(\text{at the Earth})$ , ranging over many powers of 10. The mag scale is logarithmic, but 1 mag is a factor of 2.5, not 10. Also its backward, brighter objects have more negative mag values.

This is all historic, and has to do with ancient Greeks and Romans and the response of the human eye.

In an ideal world, the mag system would no longer be used, and we'd all switch to Janskys (the cgs flux unit). We should try to avoid using them, but they are so ingrained in the consciousness of older optical astronomers...

Bolometric mag: total luminosity, not mag at a particular wavelength,  
 $M_{bol} = -2.5\log(L/L_{\odot}) + M_{bol}(\text{Sun}) = m_{bol} - 5\log(d/10 \text{ pc})$ .  $M_{bol}(\text{Sun}) = 4.74 \text{ mag}$ .  
The term  $M_{bol} - M_{\lambda}$ , where  $\lambda$  is the wavelength of interest/observation, is called the bolometric correction.

Flux is measured in real life through a particular filter with transmission  $T$  over a particular wavelength regime, so  $F \propto \int F_{\nu} T_{\nu} d\nu$ .  $T_{\nu}$  is generally peaked at a particular wavelength/freq., which is the effective  $\lambda$  for that filter/instrument combination. (In real life,  $\lambda_{eff}$  will depend on the spectral energy distribution of the star, and may be perceptibly different for red vs blue stars...)

Interstellar absorption perturbs the intrinsic stellar spectral energy distribution,

reddening it.

Color indices  $X, Y \propto \log[F(\lambda_{eff}, filterX)/F(\lambda_{eff}, filterY)] = 0.4[m(filterY) - m(filterX)] + A$ ,  $A$  is a constant.

Calibration of the flux in a photometric system is done using observations of some terrestrial source of known flux (a crucible of a melting metal, platinum has been used, or a NIST calibrated standard lamp) mounted on a tower, and observed with a small telescope interleaved with observations of Vega or some other very bright star.

In the center of the optical band, at 500 nm,  $V = 0.0$  mag corresponds to a received flux of  $3.8 \times 10^{-9}$  ergs/sec/cm<sup>2</sup>/Å  $\equiv$  1000 photons sec/cm<sup>2</sup>/Å (this is Code's number).

$$1 \text{ Jansky} = 10^{-23} \text{ ergs/cm}^2/\text{sec/Hz}.$$

$$0 \text{ mag at } 500 \text{ nm} \equiv 3.2 \times 10^{-20} \text{ ergs/cm}^2/\text{sec/Hz} \text{ or } 3,200 \text{ Jy}.$$

$$L_{\odot} = 3.9 \times 10^{33} \text{ ergs/sec (emitted luminosity)}$$

$$M_{bol} = 0 \text{ mag} \rightarrow L = 3 \times 10^{35} \text{ erg/sec}$$

$$m_{bol} = 0 \text{ mag} \rightarrow F(\text{at Earth}) = 2.5 \times 10^{-5} \text{ ergs/cm}^2/\text{sec}.$$

Table 1. Traditional Photometric Bands

Name	Central $\lambda$ ( $\text{\AA}$ )	Full Width Half Maximum ( $\text{\AA}$ )
U(ltraviolet)	3650	680
B(lue)	4400	980
V(isual)	5500	890
R(ed)	7000	2200
I(nfrared)	9000	2400
	(microns)	(microns)
J	1.25	0.38
H	1.6	0.4
K	2.2	0.48
L	3.4	0.70
M	5.0	
N	10.2	

Adopted from Allen, *Astrophysical Quantities*, 3rd edition, section 97. The zero points are also given there.

## Stellar Spectral Classes

Spectral types: the names are historical, but the concept is useful, and the classes correspond to different ranges of physical parameters, mostly  $T_{\text{eff}}$ , with the luminosity classes depending on  $\log(g)$  as a secondary parameter.

Spectral classification (based on characteristics of stellar absorption lines) (a good indicator of effective temperature) (The luminosity given is for main sequence stars)

The Sun is a G2V star in this classification scheme.

90% of stars can be satisfactorily classified by the above scheme. The remainder are special cases where rare effects cause the star to appear peculiar, for example peculiar A stars (Ap stars) have strong magnetic fields that inhibit convection and thus have abnormally strong lines of certain elements (Si, Mn, Cr, Sr, Eu...), while very metal poor stars also fail to fit into the above scheme.

## Special classes of stars

### Periodic Variables

**Cepheids** P 1 to 20 days, asymmetric light curve, spectral class F with  $M \sim 5M_{\odot}$  and age  $< 0.1\text{Gyr}$

**Miras** Very long period red variables, P 8 - 1000 days, vary by up to a factor of 50, highly evolved

**RR Lyrae** P  $< 1$  day,  $M_V \sim 0$ , low metallicity, low mass

**W Virginis stars** P  $\sim 7 - 60$  days, old stars

Table 2. Stellar Spectral Classes

Name	$T_{\text{eff}}$ (1000 K)	$\log[L/L_{\odot}]$	Dominant Features
O	> 25	> 5	HeII
B	11–25	2–5	HeI, Balmer HI
A	7.5–11	2–5	Balmer lines dominant
F	6–7.5	0–1	CaII, Fe I, ionized metals
G	5–6	–0.3 – 0	CH, CN, neutral metals, CaII, FeI
K	3.5–5	–1 – –0.3	molecular bands, metals
M	2.5–3.5	< –1	TiO
C			Carbon rich variant of M, CH, CN, C2
L	2.0		Water
T	1.0		Methane
MK	Luminosity	Classes	(indicates surface gravity)
Ia			Most luminous supergiants
Ib			Luminous supergiants
II			Luminous giants
III			Giant
IV			Subgiant
V			Main sequence dwarf

Luminosity discrimination based largely on line width, higher gravity stars have broader lines.

### High mass loss rate stars

**Herbig-Haro stars** Protostars with long jets

**OH-IR Stars** 1612 MHz OH line is seen in emission, high luminosity in the mid-IR

**Wolf-Rayet Stars** Very hot stars, losing mass very rapidly, spectra show broad emission lines of C or N, no hydrogen

### Explosive Variables

**Dwarf Novae** short period binary, white dwarf + G, K or M main sequence star, occasional eruptions by up to a factor of 100 in luminosity

**Novae** Luminosity up by a factor of 500 or more during eruption, close binary, mass containing accretes onto lower mass component and H eventually ignites, ejection velocity  $v_{ej} \lesssim 3000$  km/sec.

**Supernovae** Violent explosion,  $v_{ej} \sim 10,000$  km/sec.

## Effective Temperature

$$L = \text{emitting area} \times \text{energy emitted} = 4\pi R^2 \sigma T_{\text{eff}}^4.$$

For the Sun,  $T_{\text{eff}} = 5780$  K, and  $L = 3.8 \times 10^{33}$  ergs/sec.

The range for stars:  $10^{-3} < L/L_{\odot} < 10^5$  (range of  $10^8$  !)

$10^3$  K  $< T_{\text{eff}} < 10^5$  K (range of only 100)

## Surface Gravity

The local gravity at the surface of a star is the surface gravity,  $g$ ,  $g = GM/R^2$ ,  $g_{\odot} = 2.7 \times 10^4$  cm/sec<sup>2</sup>. Recall that  $g$  at the surface of the Earth is 980 cm/sec<sup>2</sup>.

## Spectral Energy Distributions of Stars

SEDs of stars are *almost* a black body, but not quite. SEDs for galaxies are the sum of many different stars, so further from a single T blackbody, and distinct from those of individual stars.

Planck function, flux emitted by a blackbody,  $B_{\nu} = \frac{2h\nu^3}{c^2} e^{h\nu/kT}$  ergs/cm<sup>2</sup>/sec/Hz

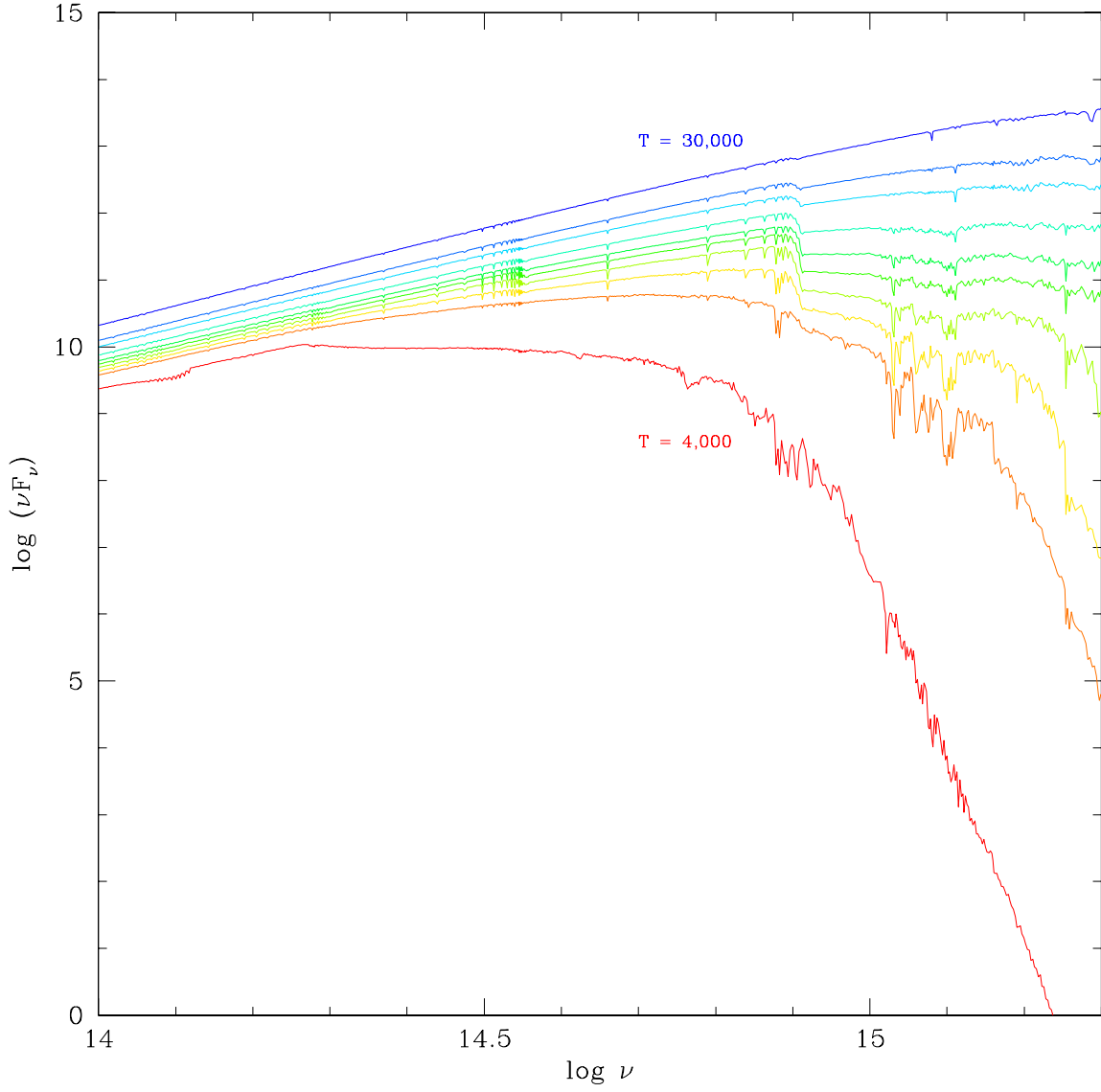


Fig. 11.— The spectral energy distribution of model stellar atmospheres of main sequence stars as a function of  $T_{\text{eff}}$ . This figure is from R.L.Kurucz.

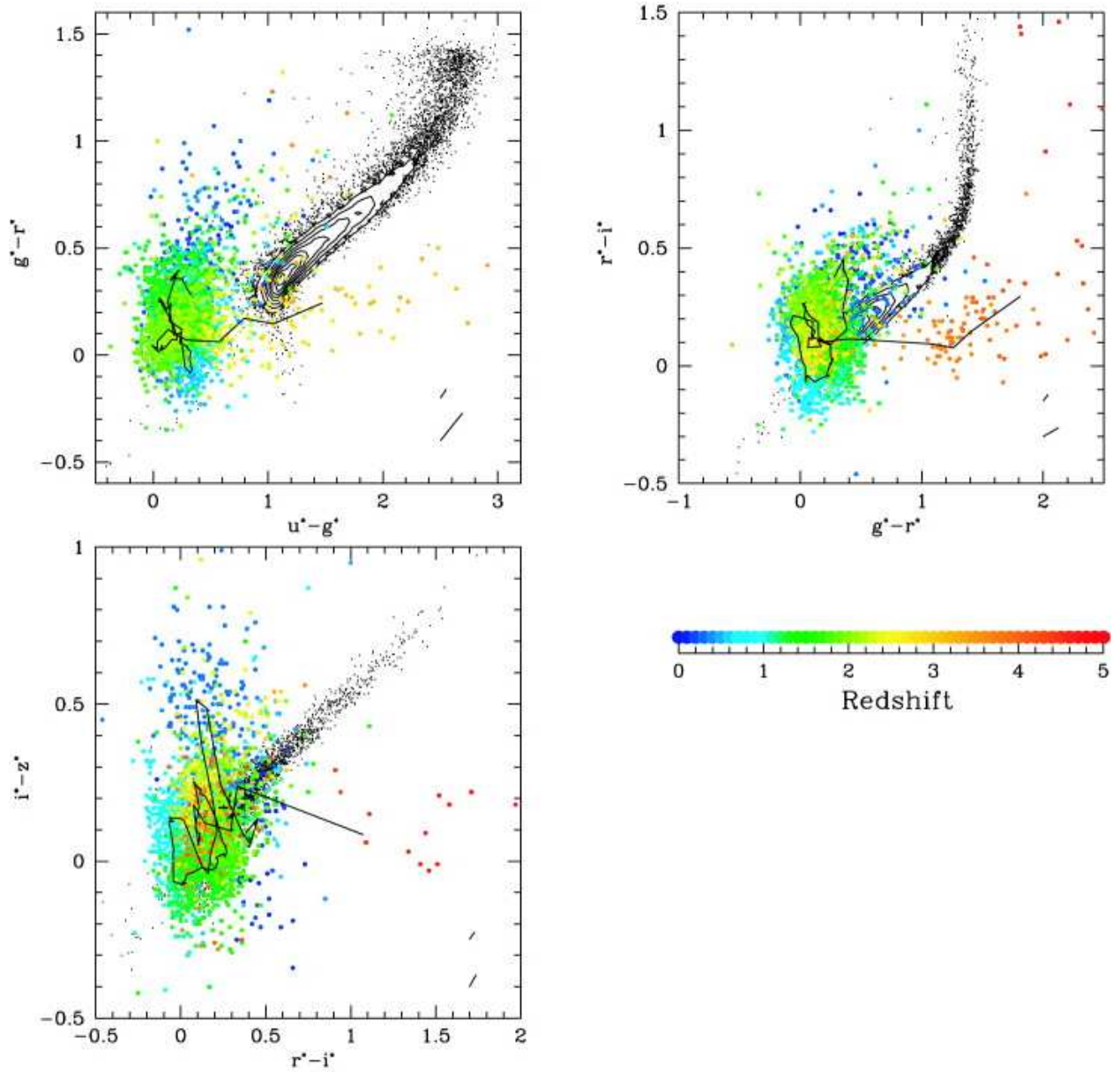


Fig. 12.— Color-color plots (3 different sets of filters) for stars from the SDSS (black points) and for QSOs (color encodes redshift as indicated in the legend). Note the concentration over a relatively range in color of the stellar population. This is figure 4 of Richards et al, Colors of 2625 QSOs from SDSS, AJ, 2001, 2308. (see also the 3D visualization of this in Fan et al, AJ, 117, 254)

## Photometri Radii

Use model atmosphere to match theoretical and observed SED. This gives  $T_{\text{eff}}$  and  $g$ . If the distance to the star is known, then with a bolometric correction, we can get  $L$ . Once  $L$  and  $T_{\text{eff}}$  are known, we can derive  $R$ .

Typical stellar radii: main sequence  $R/R_{\odot} = (M/M_{\odot})^{0.63}$ ,  $0.1M_{\odot}$ ,  $0.2R_{\odot}$ ;  $10M_{\odot}$ ,  $4.3R_{\odot}$ .

White dwarf:  $R = 0.01R_{\odot} = 7000$  km

Neutron star:  $R = 10^{-5}R_{\odot} = 7$  km

Red giant:  $R \sim 1000R_{\odot}$

1 AU = distance Sun to Earth =  $214R_{\odot}$ ,  $1R_J$  (J = Jupiter) =  $1,110 R_{\odot}$ , so a red giant is so large that the orbit of the Earth would be inside the star, and in some cases, the orbit of Jupiter as well.

## Chemical Composition of Stars

For stars which have not mixed any nuclear processed material from their interiors, have not accreted such material from a close companion, etc, i.e. unevolved stars with the initial composition of normal ISM material,

H (X) 0.72 by mass, 0.9 by number

He (Y) 0.26 by mass

Other elements (“metals”) (Z) 0.02 (Sun), range from  $10^{-5} \times$  Sun (very metal poor) to

$3 \times \text{Sun}$  (metal-rich)

There are many types of abnormalities of the chemical elements seen in stars, some caused by mixing of processed material from the interior of the star (intrinsic abnormalities), some by accretion of processed material from a companion, from being born of gas heavily polluted by a nearby SN, from various atmospheric effects which modify the abundances only near the surface of the star (radiative levitation or the opposite, sinking into the star of “metals”), etc.

## 5. Star Clusters

Star clusters have special importance in studying stellar characteristics and evolution. They represent a group of stars formed at approximately the same time, same age, same chemical composition. They are very useful for testing model predictions.

Theoretical evolutionary tracks are predictions of a parameter  $A$  as a function of time for a fixed  $M$ . Isochrones predict  $A$  as a function of  $M$  for a fixed time. So the values of  $A$  within a given cluster should represent the isochrone of  $A$  for  $t =$  the age of the cluster.

In general, young clusters are not fully virialized and may not be bound against the Galactic tidal field or passages through the Galactic plane. Young clusters contain stars of a wide range in mass.

Old “globular clusters”, found in the Galactic halo, are massive clusters which are dynamically evolved and have ages of  $\sim 10$  Gyr, so they probe the evolution of low mass stars, all older stars having evolved through stellar death.

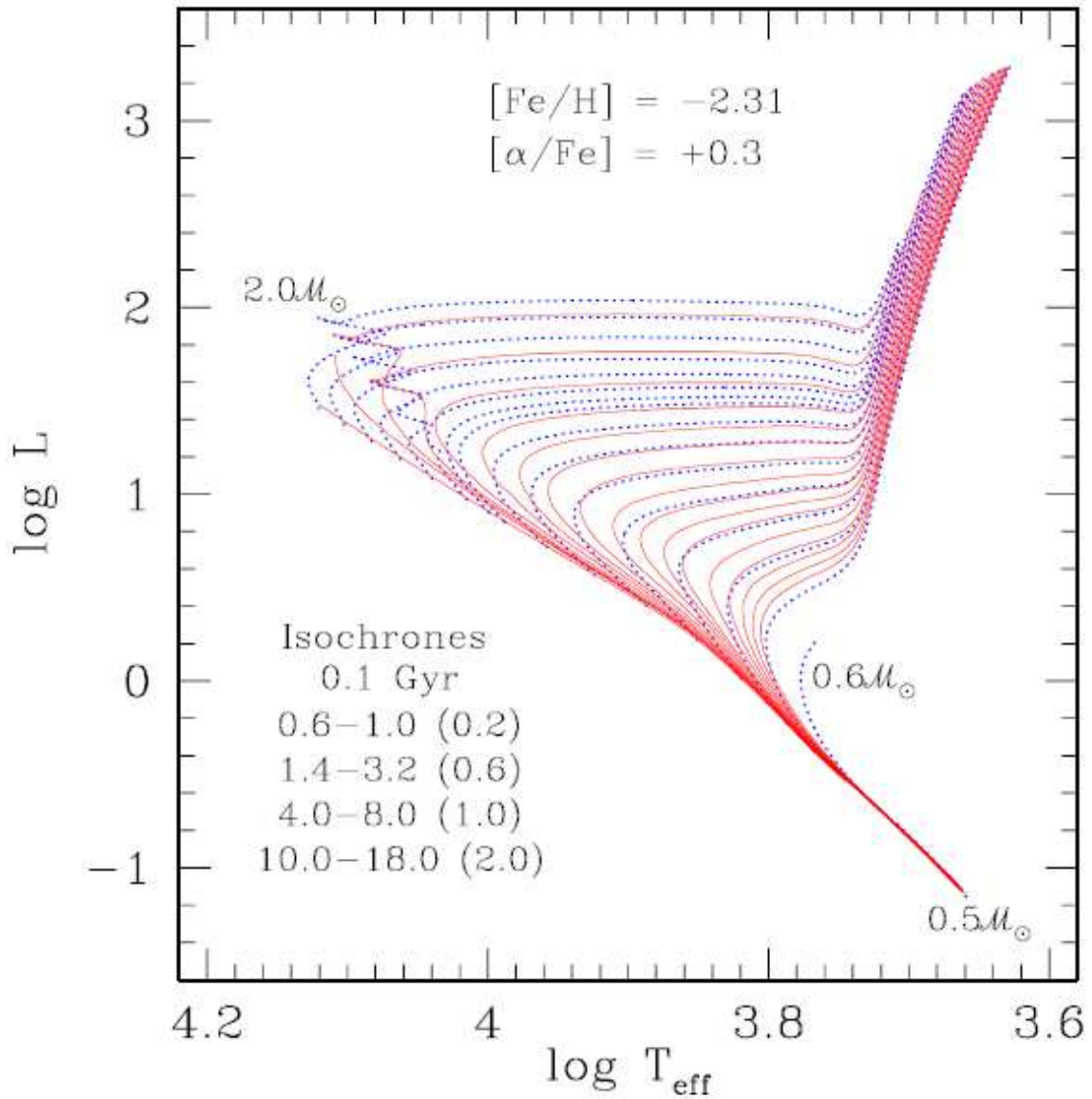


Fig. 9.— The complete grid of evolutionary tracks for the  $[\text{Fe}/\text{H}] = -2.31$ ,  $[\alpha/\text{Fe}] = +0.3$  grid are plotted with the dotted blue lines. The tracks are spaced at  $0.1M_{\odot}$  intervals with an additional track at  $1.44M_{\odot}$ , which is the transitional mass for the development of the blue hook. The isochrones, plotted as solid red lines, span the range of ages from 0.1 to 18.0 Gyr, as indicated.

Fig. 13.— Isochrones for stars of low metal content with ages from 0.1 to 18 Gyr. Note the overlap of all tracks for the lower main sequence where stars of low mass have not yet exhausted their H.

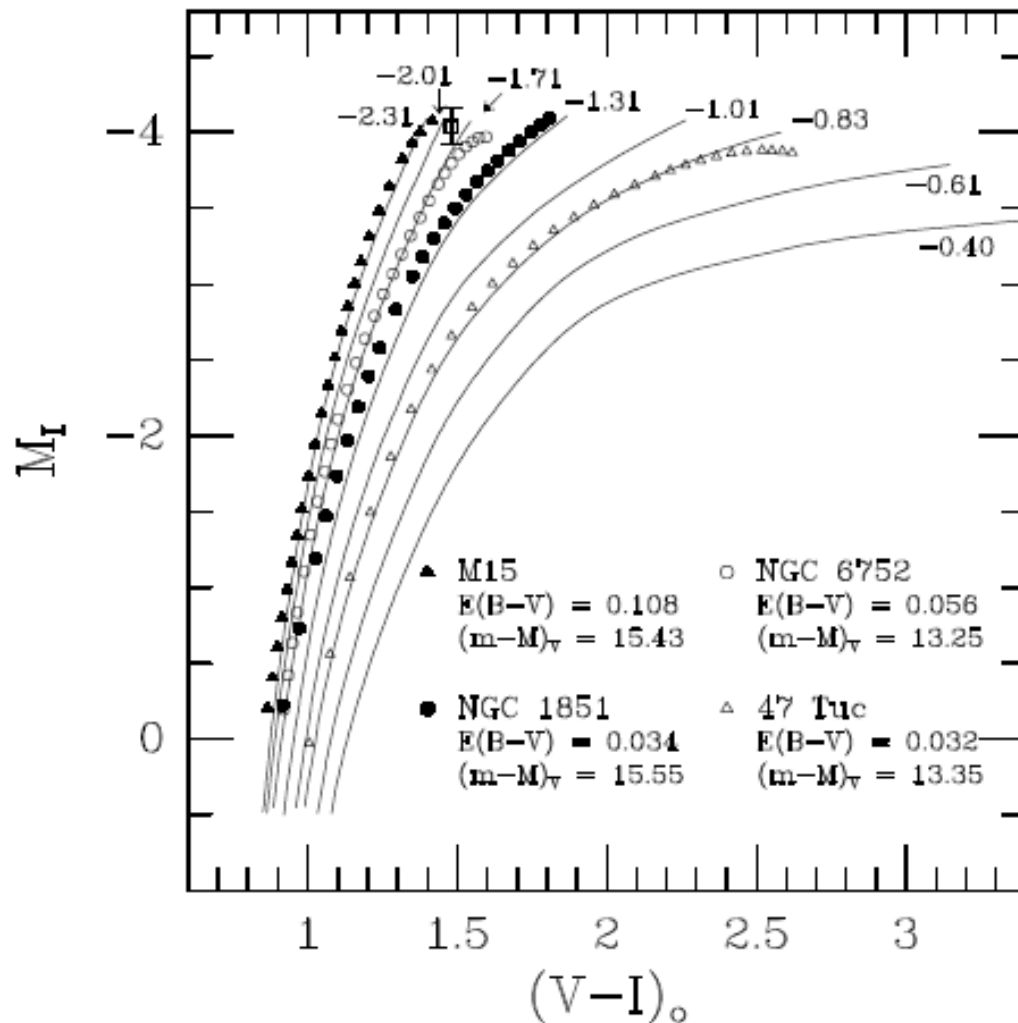


Fig. 13.— Overlay of the fiducial sequences derived by Da Costa & Armandroff (1990) for M15, NGC 6752, NGC 1851, and 47 Tucanae, on the assumption of the indicated reddenings and apparent distance moduli, onto the giant-branch segments of 13 Gyr isochrones for  $[Fe/H]$  values ranging from  $-2.31$  to  $-0.40$ , assuming  $[\alpha/Fe] = 0.3$  in each case. Note that, as shown by Vandenberg (2000), turnoff ages close to 13 Gyr are consistent with the adopted  $(m - M)_V$  values only for M15 and NGC 6752: somewhat younger ages ( $\approx 11.5$  Gyr) are obtained for NGC 1851 and 47 Tuc. The large open square, with error bars, gives the absolute  $I$  magnitude of the RGB tip stars in  $\omega$  Centauri determined by Bellazzini et al. (2001); see the text.

Fig. 14.— Isochrones for stars of a range of metal content with a fixed age of 13 Gyr. Only the region of the red giant branch is shown. Fiducial lines for 4 well studied Galactic globular clusters are shown. Overlapping theory and data requires specifying the distance and the interstellar reddening of each cluster.

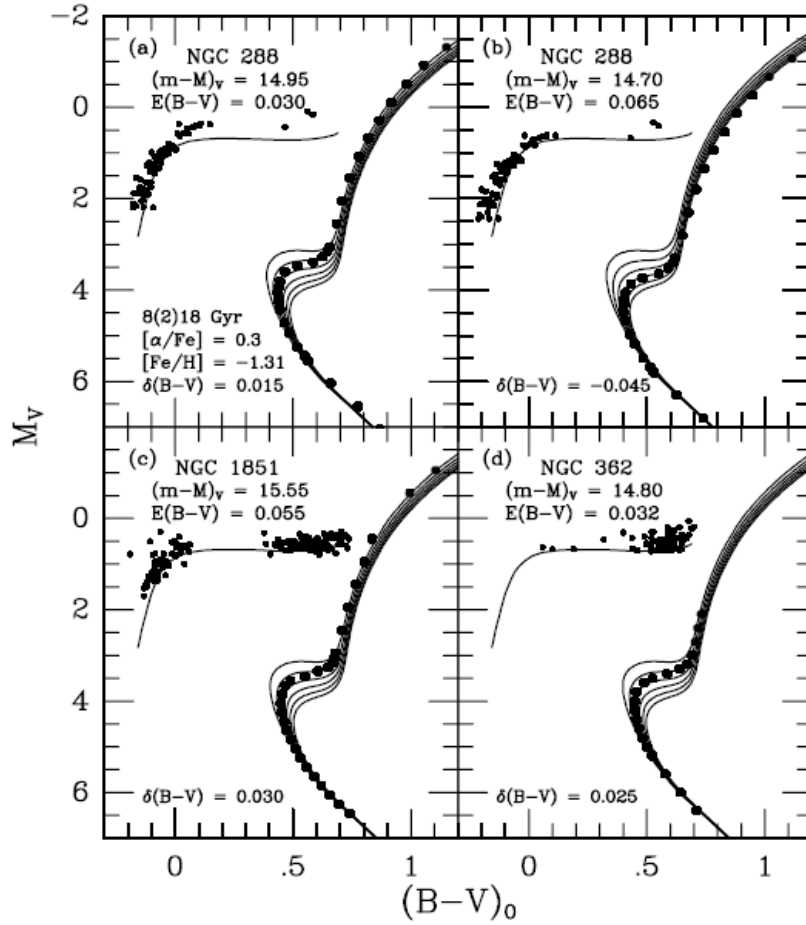


FIG. 28.—Panels (a) and (b) illustrate two of many possible fits of our ZAHB and isochrones for the indicated parameters to the NGC 288 CMD. The fiducial for the main sequence and giant stars is as defined in Fig. 23: the HB photometry is from Bergbusch (1993), but corrected by +0.01 mag in  $B-V$  and  $-0.0635$  mag in  $V$  to place those data on the system of Bolte (1992) photometry. Panels (c) and (d) show how well the same models match the NGC 1851 (Walker 1992b) and NGC 362 (VBS, Harris 1982) CMDs, respectively. The adopted reddening for the latter is from the SFD maps, while that for the former is based on the fit of the ZAHB to the blue HB stars (see the text for some discussion of this point).

Fig. 15.— Isochrones for stars of a range of metal content with a fixed age (13 Gyr ?). Fiducial lines for 4 well studied Galactic globular clusters are shown including the lower main sequence and the RGB. Individual horizontal branch stars are shown as well.

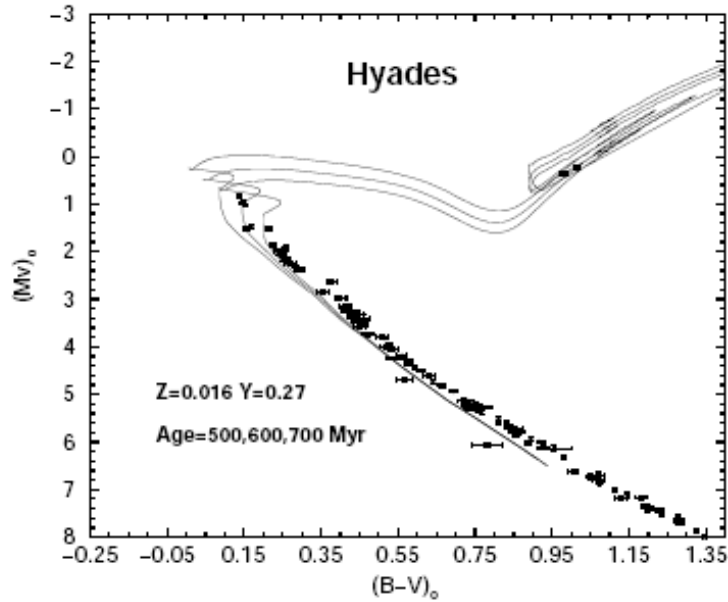


FIG. 3. The CMD for the Hyades, using the parallax values from Madsen et al. (2002). Visual, spectroscopical and suspected binaries are excluded, see also Madsen et al. (2000). Error bars indicate observational errors as given by Madsen et al. (2002) for the parallax and by the Hipparcos catalog (at the node <http://astro.estec.esa.nl/Hipparcos/HIPcataloguesearch.html>) for the colors. Observational data are compared with present theoretical isochrones for  $Z=0.016$   $Y=0.27$   $\alpha=1.9$ . Color transformations and bolometric corrections from Castelli (1999).

Fig. 16.— Match of observations and theoretical isochrones for stars in the nearby Hyades cluster, believed to have an age of about 400 Myr. Note the presence of stars significantly more luminous (i.e. massive) than the Sun still on the main sequence.

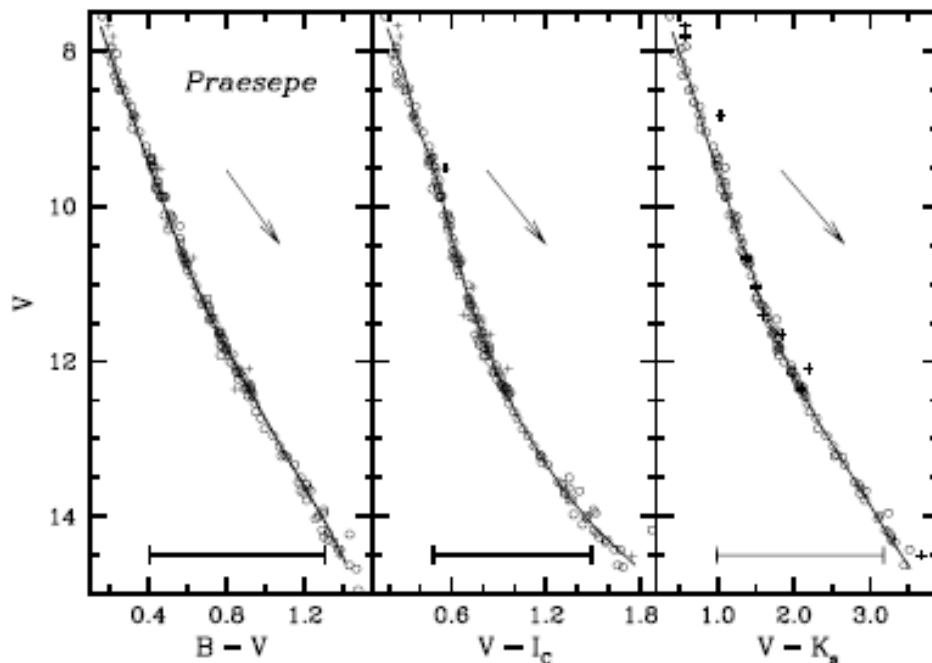


FIG. 6.— CMDs of Praesepe, after excluding known binaries and stars with low membership probability. Pluses are photometrically rejected data points from the filtering algorithm, and open points are those remaining. The solid lines are empirically calibrated isochrones with spectroscopic metallicity (Table 2), which were adjusted for the reddening with the literature value (Table 3). Fitting ranges are shown as horizontal bars. The arrow denotes the direction of reddening vectors.

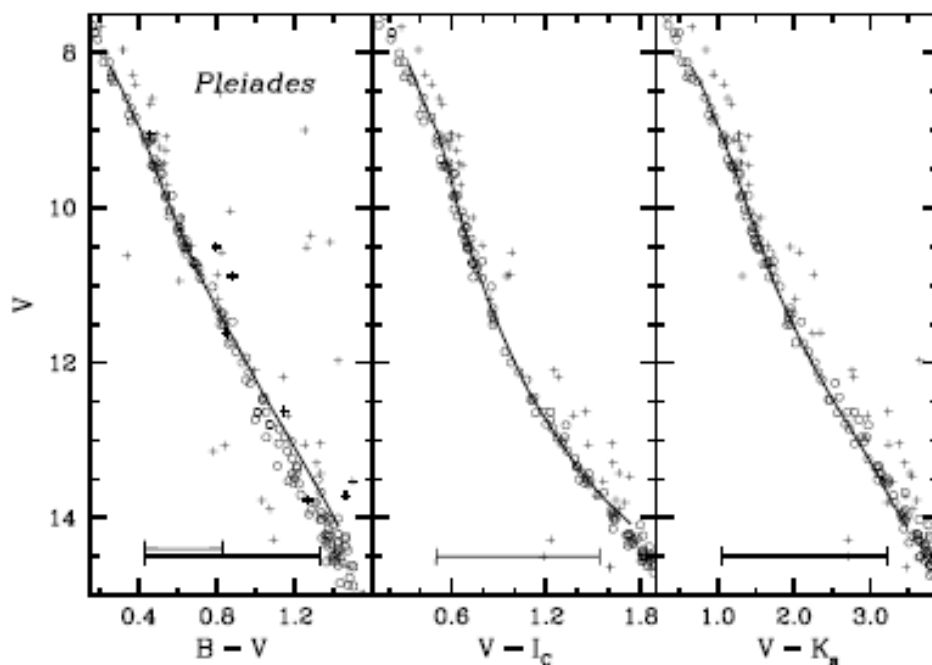


FIG. 7.— Same as Fig. 6 but for the Pleiades. As discussed by [Stauffer et al. \(2002\)](#), the Pleiades K dwarfs are bluer than the given isochrone in  $(B - V, V)$ , but not in the other two CMDs (see text). The distance modulus in  $(B - V, V)$  was derived at  $0.4 \leq (B - V)_0 \leq 0.8$  as shown by the shorter horizontal bar.

Fig. 17.— Match of observations and theoretical isochrones for stars on the lower main sequence of the nearby young clusters Pleiades and Praesepe. Note the very tight locus of the

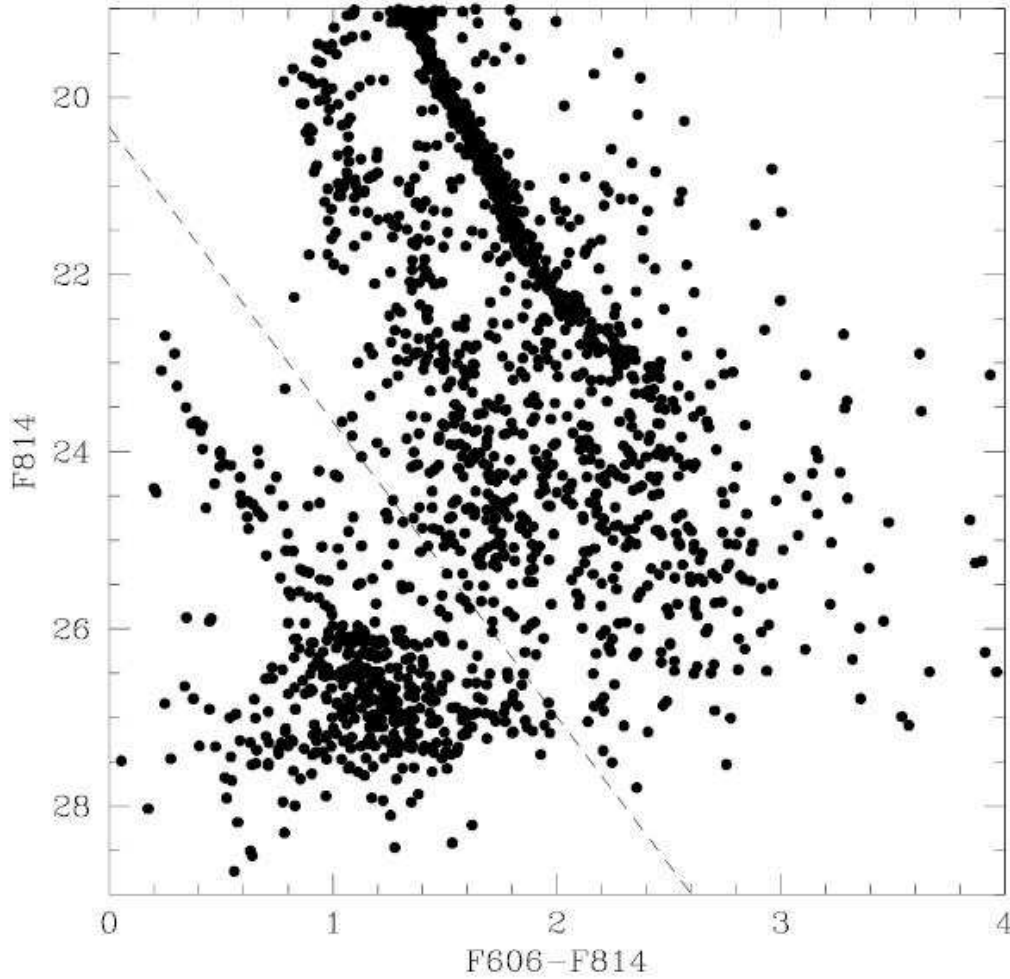
## 6. The Endpoint of Stellar Evolution

Eventually nuclear fuels are completely consumed, at least in the hot central regions of the star. At that time, low mass stars more or less cease nuclear burning, and gradually fade in a quiescent fashion, becoming white dwarfs. These stars have central regions made mostly of He or of the CNO elements, with at most a thin outer layer of H.

The manner of “stellar death”, for single non-rotating stars, depends on the initial mass of the star and on the amount of mass lost during its evolution. There will be extensive mass loss during the AGB, supergiant, and/or planetary nebula phases. Higher mass stars explode, becoming supernovae of various types. The remnant may be a neutron star (a pulsar perhaps), a black hole, or in the case of certain violent explosions, no remnant left at all. The nature of the remnant depends on the stellar mass (and to a lesser extent, its metallicity).

The material ejected from the supernova into the interstellar medium contains heavily processed material, both from nuclear burning prior to the SN, and nuclear burning during the SN itself, which enriches the ISM in metals. The heavy metal content of the ISM rises with time due to mass loss from evolved stars (AGB, PN nuclei, etc) as well as from SN.

Type Ia SN, which do not show any lines of H in their spectra, are believed to arise from interactions in evolved binary stellar systems. These involve lower mass stars in general. Since the timescales for stars increase rapidly as the stellar mass decreases, there will be a delay in the appearance of Type Ia SN compared to the Type II SN occurring as the endpoint of stellar evolution for single massive stars. Since these contribute different sets of elements to the ISM, this delay can in principle be observed by studying the chemical evolution of the ISM with time, measured through unevolved stars of different ages.



JPG

Fig. 18.— The lower main sequence and the sequence of low luminosity small radius white dwarfs in the globular cluster M4. The white dwarf sequence basically represent a cooling locus.

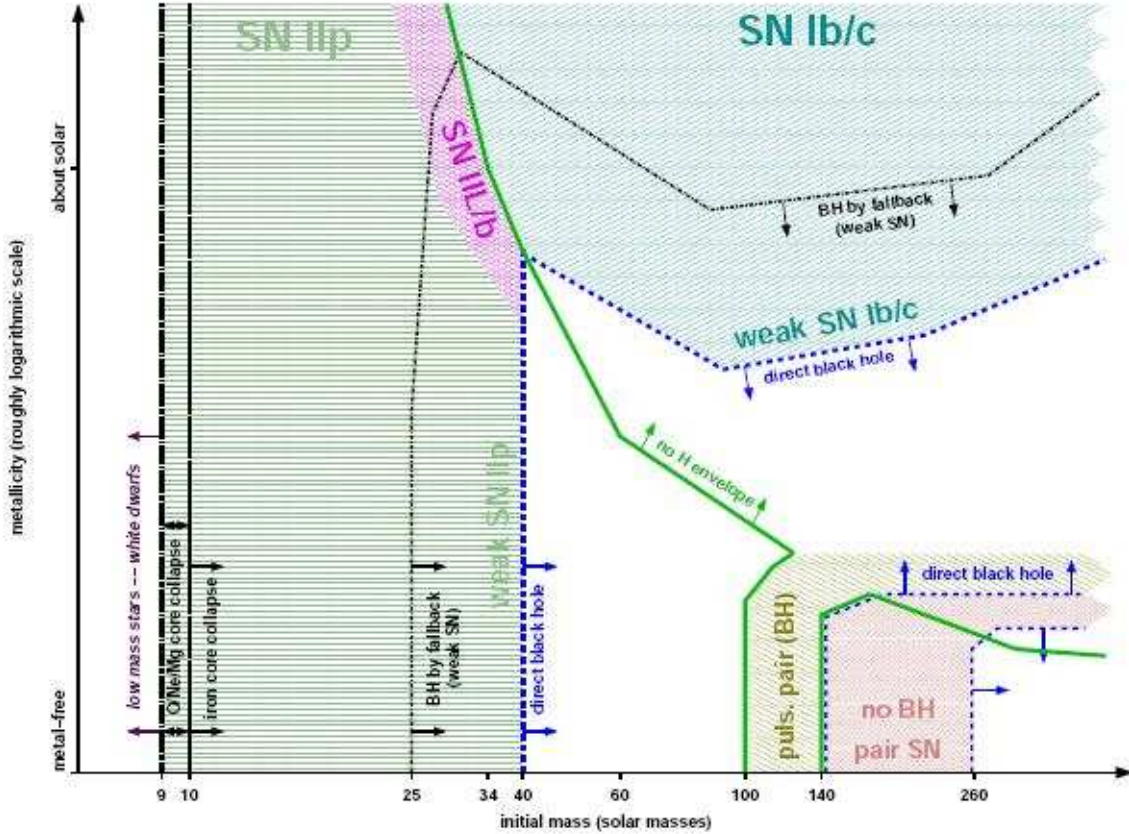


Fig. 2.— Supernovae types of non-rotating massive single stars as a function of initial metallicity and initial mass. The lines have the same meaning as in Fig. 1. *Green horizontal hatching* indicates the domain where Type IIp supernovae occur. At the high-mass end of the regime they may be weak and observationally faint due to fallback of  $^{56}\text{Ni}$ . These weak SN Type IIp should preferentially occur at low metallicity. At the upper right edge of the SN Type II regime, close to the *green line* of loss of the hydrogen envelope, Type III/b supernovae that have a hydrogen envelope of  $\lesssim 2 M_{\odot}$  are made (*purple cross hatching*). In the upper right quarter of the figure, above both the lines of hydrogen envelope loss and direct black hole formation, Type Ib/c supernovae occur; in the lower part of their regime (middle of the right half of the figure) they may be weak and observationally faint due to fallback of  $^{56}\text{Ni}$ , similar to the weak Type IIp SNe. In the direct black hole regime no “normal” (non-jet powered) supernovae occur since no SN shock is launched. An exception are pulsational pair-instability supernovae (lower right corner; *brown diagonal hatching*) that launch their ejection before the core collapses. Below and to the right of this we find the (non-pulsational) pair-instability supernovae (*red cross hatching*), making no remnant, and finally another domain where black hole are formed promptly at the lowest metallicities and highest masses (*white*) where nor SNe are made. White dwarfs also do not make supernovae (*white strip* at the very left).

Fig. 19.— The mode of stellar death as a function of initial stellar mass. Low mass stars become white dwarfs, while stars with  $M \gtrsim 8 M_{\odot}$  become SN of various types.

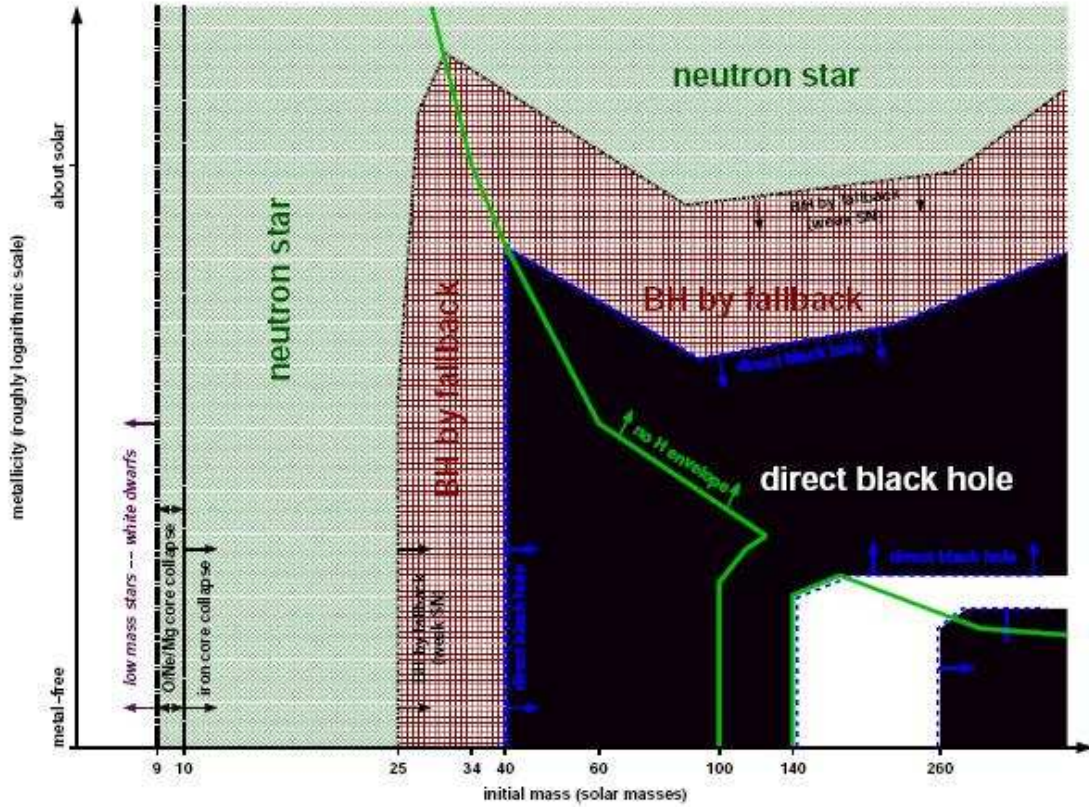


Fig. 1.— Remnants of massive single stars as a function of initial metallicity ( $y$ -axis; qualitatively) and initial mass ( $x$ -axis). The *thick green line* separates the regimes where the stars keep their hydrogen envelope (left and lower right) from those where the hydrogen envelope is lost (upper right and small strip at the bottom between 100 and 140  $M_{\odot}$ ). The *dashed blue line* indicates the border of the regime of direct black hole formation (*black*). This domain is interrupted by a strip of pair-instability supernovae that leave no remnant (*white*). Outside the direct black hole regime, at lower mass and higher metallicity, follows the regime of BH formation by fallback (*red cross hatching* and bordered by a *black dash-dotted line*). Outside of this, *green cross hatching* indicates the formation of neutron stars. The lowest-mass neutron stars may be made by O/Ne/Mg core collapse instead of iron core collapse (*vertical dash-dotted lines* at the left). At even lower mass, the cores do not collapse and only white dwarfs are made (*white strip* at the very left).

Fig. 20.— The remnant at the end stage of the evolution of a star is shown as a function of mass and metal content. Possibilities include white dwarfs, neutron stars, black holes, no remnant left at all, etc.