

## 1. Nuclear Reactions - Summary (for AY123)

Free neutrons have no charge, they are captured instantaneously. Free neutrons are thus very rare in normal stellar interiors.

$p$  or heavier nucleus + ionized atom - Coulomb repulsion corresponds to a very high energy. For  $T$  typical of normal stellar interiors, the probability of a reaction occurring using a classical physics calculation is very low.

Height of Coulomb barrier,  $V(r) = Z_A Z_B e^2 / r$ . At temperature  $T$ , the mean velocity atom can penetrate to  $r_{min}$  such that  $Z_A Z_B e^2 / r_{min} = kT$ .

$r_{min} \approx 1000 r_{nuc}$ , too far away. Quantum mechanical tunneling is required for nuclear reactions to occur. We need to determine the wave function, and find the probability for the particle to be in the region close to  $r_{nuc}$ .

Rate of nuclear reaction  $A(B, C)D \equiv A + B \rightarrow C + D$  is a function of the relative velocity of  $A$  and  $B$ ,  $v_{AB}$ ,

$$R_{AB}(v_{AB})(\text{reactions/cm}^3/\text{sec}) = \frac{n_A n_B \sigma_{AB}(v_{AB}) v_{AB}}{1 + \delta_{AB}}$$

Energy generation rate  $\epsilon_n$  (ergs/sec/gm) =  $R_{AB} Q / \rho$

$$\epsilon_n = \frac{\rho X_A X_B v_{AB} \sigma_{AB} Q_{AB}}{m_H^2 A_A A_B (1 + \delta_{AB})}$$

$\sigma(E) = (S/E) \exp[(-2\pi Z_A Z_B e^2) / (\hbar v_{AB})]$ . The exponential is the Gamow factor, or tunneling probability from QM.

We need to average  $\sigma$  over the Maxwellian distribution of velocities to get:

$$\langle \sigma v \rangle = \frac{8\hbar}{3^{5/2} \pi e^2 Z_A Z_B \mu} S_0 \tau^2 e^{-\tau}$$

where  $\tau = 42.5[Z_A^2 Z_B^2 \mu]^{1/3} T_6^{-1/3}$ .  $S$  is a constant, with units keV-barns. A barn is a unit of area (cross section) in nuclear physics; it is  $10^{-24}$  cm<sup>2</sup>.

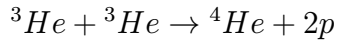
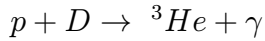
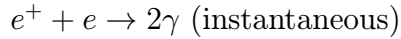
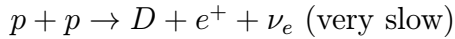
$$\langle \sigma v \rangle_{AB} = \frac{0.72 \times 10^{-18}}{\mu Z_A Z_B} S_0 \tau^2 e^{-\tau} \text{cm}^3/\text{sec}$$

Power law expansion around  $T$ ,

$$\langle \sigma v \rangle (T) = \langle \sigma v \rangle (T_0) (T/T_0)^{\tau(T_0)/3 - 2/3}$$

If we want a power law form for  $\epsilon$ ,  $\epsilon = \epsilon_0 \rho T^\eta$ , then  $\eta = \tau(T_0)/3 - 2/3 \approx \tau(T_0)/3$ .

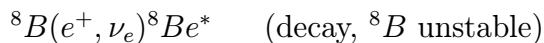
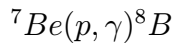
Example: main  $p - p$  chain:

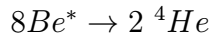


For the first reaction, adopting  $\mu = 1/2$ ,  $T_6 = 10$ ,  $\tau = 15.7$ , then  $\epsilon \propto T^\eta \propto T^{4.6}$ .

For the CNO cycle,  $\tau = 50.3$ ,  $\epsilon \propto T^{16}$ , a very high power of  $T$ .

In addition to main  $p - p$  chain, several less probable alternatives, one of which produces a 7.2 MeV neutrino.





This side chain occurs  $\sim 10^{-3}$  times as often as the main  $p - p$  chain.

The CNO cycle produces  $^4He$  from 4 protons.  $^{12}C$  is used as a catalyst, its abundance is not altered, but  $^{13}C$ ,  $N$  and  $O$  can be affected. Rates are in equations 6.77 and 6.78 of HKT.

Problem: after producing  $^4He$ , it is impossible to proceed further by just adding protons as there is no stable nucleus with atomic mass 5 or 8. Thus to produce  $^{12}C$  we must use 3  $^4He$  nuclei.

For nuclei heavier than Fe, the binding energy per nucleon decreases as the atomic number increases, and no energy can be generated through fusion, only through fission.

Slow neutron capture (in AGB stars) and rapid neutron capture (in SN) form the elements heavier than Fe. At the thermal energies typical of stellar interiors,  $\sigma v \approx \text{constant}$  for neutron capture reactions.